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P.S.S.C. PHYSICS

TEACHER'S RESOURCE BOOK AND GUIDE

PART THREE

D. C. HEATH AND COMPANY

INDIAN
EDITION

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

P.S.S.C. PHYSICS

TEACHER'S RESOURCE BOOK AND GUIDE

PART THREE

INSTITUTE OF EDUCATION FOR WOMEN
2412(2)
Dept. of Extension
SERVICE
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EDITION**

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

REPRINTED IN INDIA
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INDIAN EDITION 1964.

PUBLISHED BY
National Council of Educational
Research and Training, New Delhi.

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The Chief Publication Officer,
Publication Unit,
National Council of Educational
Research and Training,
114, Sundar Nagar, New Delhi.

PRINTED IN INDIA BY
the Photo-Litho Wing of the
Government of India Press, New Delhi.

FOR SALE AND DISTRIBUTION WITHIN INDIA ONLY.

PRICE Rs. 2.50

Published by D. C. HEATH AND COMPANY
under arrangements with
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INTRODUCTION TO PART III

Part III deals with dynamics. It attacks the general question, "Why do things move as they do?" The discussion naturally leads to a study of force and mass and to a development of the important parameters, momentum and energy.

In general there are two approaches to a problem in dynamics. One is a detailed, point by point, treatment which we might call a differential approach. It begins with a specified initial position, speed and force field. The force field is used to determine the resultant acceleration, and knowledge of the acceleration makes it possible to construct a trajectory, bit by bit. The second approach might be called an integral approach. It ignores the details of the interaction and deduces some of the characteristics of the final state of a dynamical system through the use of conservation laws which relate final state parameters to initial state parameters.

A great many of the dynamics problems that are of interest in the day to day work of a physicist can be handled best by the integral approach. This is why Part III emphasizes the conservation laws. Considerably more emphasis has been given these laws than often has been the case in secondary school courses. Conservation laws lie at the heart of much of contemporary physics. It is important that students get a feeling for their origin, their usefulness and the extent of their validity.

On the other hand, whatever intuitive feelings a student may have about dynamics are sure to be associated with his familiarity with force and motion. Therefore it is logical that dynamics be introduced through the differential approach, leading finally to an understanding of the usefulness of the integral techniques.

Within this framework, Part III begins, in Chapter 20, with the development of Newton's law, $\vec{F} = m\vec{a}$. The approach is partially historical since the methods developed by Galileo and Newton in solving this basic problem are the fountainhead of physics as we know it. There is ample opportunity in Chapter 20 to develop an appreciation for the method that was evolved as well as for the physical understanding that sprang from its application.

Chapter 21 treats a class of dynamical problems in which the force field is particularly simple and well-known. In the case of the gravitational force, attention is confined principally to motion at the earth's surface where the force can be treated as constant. Near the end of the chapter evidence is introduced which shows that the gravitational force falls off with increasing distance. This leads toward Chapter 22 where man's solution of the planetary motion problem is developed in a historical context.

The remaining four chapters of Part III develop two important conservation laws of mechanics, conservation of momentum and conservation of energy. Chapter 23 establishes the conservation of momentum from experiment, thus developing it as a new physical law, not as a deduction from Newton's "second" and "third" laws as is sometimes done. In fact, at the stage where conservation of momentum is treated, Newton's third law has not yet been introduced. It is important that your treatment be consistent with this point of view. A mixture of the two possible approaches is almost sure to cause confusion for the students. The validity of conservation of momentum, like all other physical laws previously established, is based on the synthesis of a number of experimental observations.

The last three chapters of Part III provide a broad treatment of the concept of energy. These chapters depart rather sharply from the standard development that has been used in many introductory courses. For this reason it may be worthwhile to discuss briefly the general nature of how energy is developed in these chapters.

Importance of the Energy Concept. Application of the law of conservation of energy is a powerful method for solving a broad range of problems, hence energy is one of the most basic concepts of physics. It is therefore important for students to understand both energy and the conservation of energy.

Omission of a Definition. The text makes no attempt to give a specific, short definition of energy. There is none! Rather than beginning with a definition, a better eventual comprehension and appreciation of the significance of the idea of energy can be achieved through building an understanding of how the concept of energy can be applied to many phenomena. In pursuing such a development, the text presents the following ideas:

The term energy is applied to many widely-different characteristics of the motion and relative position of matter. A single term is useful because these widely-varying characteristics are all connected through a single conservation law. In order to emphasize the breadth of the energy concept and to stress energy conservation, the text purposely avoids an over-simplified energy definition such as "the capacity to do work". Instead, various forms of energy are introduced and the convertibility of energy from one form to another is discussed. The concept of work takes its rightful place as simply one mode of energy transfer. Work is defined on a quantitative basis; it is then possible to formulate a quantitative description of several forms of energy, starting from the amount of work done to produce them.

Plan of Attack. The strategy of the text is to introduce the three forms of energy that appear: kinetic, potential, and thermal. With these three forms, it is possible to discuss the transfer of energy from one form to another, showing that the total energy is always conserved.

Illustrative Examples. Examples may illustrate several difficulties that arise if energy is simply defined as "the capacity to do work".

Consider a baseball moving horizontally with a velocity, v. Relative to the earth, it has a kinetic energy of $1/2\ mv^2$. But relative to an observer on a moving train, it has a different kinetic energy. Therefore the amount of work that the baseball can do depends on whether it does work on something on the ground or on the train.

If the ball were thrown horizontally from the edge of a cliff, a person at the bottom of the cliff would say that the ball had both kinetic and potential energy. Hence the ability of the ball to do work depends not only on the horizontal speed, but also the height (or depth) of the system on which it is to do work. (If the man at the bottom of the cliff were to dig a hole in the ground, he could get still more work from the ball.)

To complicate the situation even more, one might set the ball on fire after it lands. He could then accomplish work by running a heat engine with the heat liberated by the release of chemical energy. Measuring the energy of the ball by its capacity to do work now requires the specification of not only its horizontal velocity and height, but also its heat content. Even then, however, unless the temperature of the exhaust gases be at absolute zero, not all of the thermal energy can be recovered as work; much of it will be lost in exhaust heat.

As a final example, the substance of the ball could be annihilated by anti-matter, thus liberating energy in radiant form.

From even this brief sample of the many forms in which energy is manifested, it is clear that no brief definition can give an adequate feeling for the true breadth of the concept. The important thing is to develop some feeling for the broad utility of the conservation law in dealing with the many widely-different guises of the thing we call energy. If you can give your students some appreciation for this idea, they will have something that is much more powerful than a neatly packaged, but incomplete, five-word definition.

RELATED MATERIALS FOR PART III

Laboratory. As with other parts of the course, the experiments for Part III are intended to carry a significant share of the instructional load. Good scheduling and use of the laboratory will contribute not only to an increased understanding of various principles and their application, but to a better perception of how principles are derived from experiments. The yellow pages at the back of this volume of the Guide include recommendations on scheduling and handling each of the experiments, and answers to questions in the students' Laboratory Guide.

Films. The PSSC films appropriate for each chapter are described in the chapter summaries of the Guide. The films related to Part III of the course are listed below with brief comments on possible scheduling.

"Forces" is a general introduction to mechanics. It can be scheduled as early as the first day of classwork on Part III.

"Inertia" and "Inertial Mass" are related films which show the experimental base of Newton's law with nearly ideal (low-friction) apparatus. "Inertia" relates to Sections 1–4 of Chapter 20; "Inertial Mass" to Sections 5 and 6. These films are probably best used as a basis for class discussion after students have performed Experiments III-2, 3 and 4 in the laboratory.

"Falling Bodies" shows the proportionality of gravitational and inertial mass. The film can be used with Sections 2, 3, and 4 of Chapter 21.

"Deflecting Forces" shows the validity of Newton's law in vector form through an analysis of circular motion. This film is related to Section 5 of Chapter 21.

"Periodic Motion" analyzes experiments to show the force-distance relationship and the period in simple harmonic motion. This film can be used with Section 8 of Chapter 21.

"Frames of Reference" demonstrates how different frames of reference lead to different descriptions of motion and how an appropriate choice of frame of reference simplifies dynamical problems. This film can be used to good advantage with Sections 9, 10 and 11 of Chapter 21.

"Universal Gravitation" reconstructs the logic behind the law of universal gravitation through imagining how the law might have been discovered on a planet in another solar system which has only the one planet. This film is closely related to Sections 6, 7 and 8 of Chapter 22.

"Elliptic Orbits" uses Kepler's law of areas to show that an inverse square force law follows from an elliptic orbit. This film can be used in connection with discussion of Sections 7 and 8 of Chapter 22.

"Energy and Work" explores the relationship between work and kinetic energy, closing with a lead toward thermal energy. Depending on the class discussion that is developed around the film, it can be used at various points in Chapters 24 and 25.

"Mechanical and Thermal Energy" uses models and experiments to show the interconnection between energy of bulk motion and thermal energy of random motion. This film is best scheduled with Sections 3 through 7 of Chapter 26.

"Conservation of Energy" illustrates energy conservation by tracing gross energy changes in a power plant. This film can be used with the last sections of Chapter 26.

Science Study Series. Currently available titles that deal with subjects related to Part III of the course are:

The Universe at Large, Herman Bondi;
The Birth of a New Physics, I. Bernard Cohen;
The Watershed, Arther Koestler.

Books on gravity, cosmology, general relativity, special relativity, satellite motion, and a number of other subjects related to Part III are planned.

SCHEDULING PART III

Two possible schedules for Part III are indicated in the table on the following page. The nine-week schedule is intended as a rough guide for those who are programming a 36-week course. The fifteen-week schedule indicates how one might allot any additional available time, as well as providing a guide for a three-semester course.

Chapter	15-week schedule for Part III			9-week schedule for Part III		
	Class Periods	Lab Periods	Exp't	Class Periods	Lab Periods	Exp't
20	4	4	III-1 III-2 III-3 III-4	4	2	III-2 III-3
21	14	2	III-5 III-6	8	1	III-6
22	5	1	III-7	3	0	-
23	11	3	III-8 III-9 III-10	8	2	III-8 III-9
24	9	1	III-11	6	1	III-11
25	7	2	III-12 III-13	4	1	III-12
26	8	1	III-14	4	0	-

Chapter 20 — Newton's Law of Motion

This chapter introduces Newton's law of motion which is fundamental to the study of mechanics. In view of the variety and complexity of the phenomena that can be unraveled with this law, its elegant simplicity is almost surprising. To appreciate the scope and power of Newton's law, students should become familiar with its application. Examples are given in the HDL section. However, too early or too great emphasis on the formality of applying Newton's law, especially in some of the more complicated problems, may detract from students' basic understanding of the relations between force, time, velocity and mass. For this reason, the formal statement of Newton's law is deferred in the text until after the analysis of simple experiments which, holding two of these variables constant, display the relation between the other two. This approach is intended to avoid the simple memorization of $F = ma$ without real understanding of the fundamental relationships between force, time, velocity and mass. Early laboratory work will contribute to this purpose.

Newton's law of motion is based on a wide range of experiments and observations, which are described in this and succeeding chapters. Sections 8, 9, and 10, for completeness of presentation, cite properties of the law which will not be verified by actual comparison with experiment until Chapter 21. The HDL problems relating to these sections are nonetheless very useful. They serve as valuable reviews of earlier vector work, and as introductions to mechanics. Subjects in the text for which the number of appropriate HDL problems may appear to be insufficient (inertial and gravitational mass, varying forces) will be found to reappear in later chapters, to be treated in more detail.

CHAPTER SUMMARY

Introductory Section and Sections 1 and 2. Galileo's principle of inertia is presented after students have been encouraged to think independently about force-free motion. The emphasis is on the reasoning which Galileo used in drawing conclusions from an idealization of a physical situation.

Sections 3 and 4. The quantitative relationship between force and motion is formulated directly from simple experiments. With a constant mass and constant force, $\Delta v \propto \Delta t$. With a constant mass and a constant time, $\Delta v \propto F$. These two facts are summarized by $\Delta v \propto F\Delta t$, or $\frac{\Delta v}{\Delta t} \propto F$.

Force is introduced intuitively as being related to a push or pull. The constancy of the exerted force is based on the reasonable assumption that a constant force results if a spring (or rubber band) is stretched by a constant amount. The relative force scale rests on the assumption of additivity (i. e., two "identical", equally-stretched springs pulling in parallel in the same direction exert twice the force that would be exerted by either one acting alone.

Sections 5 and 6. The concept of inertial mass is introduced and its characteristics are investigated. Experiment shows the proportionality between inertial and gravitational mass. The kilogram is introduced as the standard of mass.

Section 7. The proportionalities between the various dynamic and kinematic variables which have been developed in previous sections are synthesized in a single equation $F\Delta t = m\Delta v$ (or $F = ma$). With the units of mass and acceleration already defined, this equation defines a unit of force, the newton in the MKS system.

Sections 8 through 11. Newton's law is further generalized by citing empirical evidence for its validity in the case of a varying force. The vector nature of force and acceleration and of the equation $\vec{F} = m\vec{a}$ are also developed empirically.

The applicability of Newton's law can be extended to cover forces which vary with time, and the motion produced when several forces act simultaneously. In the latter situation the resulting acceleration is the same as if the several forces were replaced by their vector sum. This is an experimental verification that forces add as vectors. This leads to the idea that Newton's law is true in vector form: $\vec{F} = m\vec{a}$. To determine $\vec{a}$, it is important to know the net force, including resistive forces such as friction. Newton's law was based on known forces and is particularly useful for predicting motion when all forces are known. A less obvious, but equally important application (especially in the study of physics), involves deducing an unknown force from the observed acceleration it produces. From such investigations we have learned a great deal about the forces in nature. When we can describe them in our terms, we can then predict the motions that will occur under their influence.

Comments on Developing the Logic of Chapter 20. So many key ideas are summarized by the misleadingly simple expression, $F = ma$, that if students are to grasp something of its full power, they should approach it gradually with a developing awareness of the meaning of the underlying relationships. Otherwise, they will memorize the equation quickly, be able to apply it to simple problems, and miss most of its meaning. Through study of the text, laboratory work, films and class discussion, the three fundamental ideas – the principle of inertia, Newton's law, and inertial mass – should be developed fully before the equation $F = ma$ is introduced.

At the risk of repeating some of the summary of the chapter, it is worth considering the logical development of the chapter from the standpoint of the sequence of ideas to be taught:

1. The principle of inertia requires neither a quantitative concept of mass nor a quantitative measure of force. Inertia can be introduced through work in the laboratory (Experiment III - 1) and class discussion even before students have read the first few sections. Some students will better appreciate Galileo's contribution if they try to reason independently about the nature of motion before they study his simple but elegant arguments.

2. A constant force produces a constant change in velocity, independent of any prior velocity. (The constant stretch of a spring or a rubber band is used as an obvious indication of a constant force; at this point neither a definite nor a relative force scale is needed.) Using Figures 20 – 5 and 20 – 6 as background, the class can use Figure 20-7 as a source of data on which to base the conclusion that $\Delta v \propto \Delta t$.

3. The change in velocity of an object is proportional to the net force acting on it. (The required relative force scale is based on the reasonable assumption that two identical springs pulling side by side exert twice the force of either alone.) $\therefore \Delta v \propto F$. Summarizing, since Δv is proportional to both Δt and F, $\Delta v \propto F\Delta t$, or $F \propto \frac{\Delta v}{\Delta t}$.

4. The inertial mass of an object is defined by the change in the object's motion as a result of the action of a given force. Thus mass ratios are defined in terms of the ratios of observed accelerations resulting from the action of a given force.

5. After the above ideas are understood, students are in a sound position to understand the mass standard, the definition of units, and the equation $F = ma$. You will note that, although the proportionality between F and $\Delta v/\Delta t$ is reached quite early, and students will know from Chapter 5 that $\Delta v/\Delta t$ is acceleration, the text does not use the concept of acceleration as such until after the effect of mass is considered. At this point $F\Delta t = m\Delta v$ is formulated and the equivalent form, $F = ma$, is noted. If the earlier material is covered too quickly, the reasoning may seem quite circular to students who, remembering that the newton was defined from $F = ma$, are confused about whether $F = ma$ is a law or a definition.

6. One can infer the net force on an object by observing its acceleration. This fact is used to show that two forces add as vectors.

7. $\vec{F} = m\vec{a}$ is a vector relation; it does not depend on the velocity, $\vec{v}$, of the object.

SCHEDULING CHAPTER 20

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2	1	1	III-1	1	0	
Secs. 3, 4	1	2	III-2 III-3	1	2	III-2 III-3
Secs 5, 6	1	1	III-4	1	0	
Secs. 7, 8, 9, 10, 11	1	0		1	0	

RELATED MATERIALS FOR CHAPTER 20

Laboratory. The Laboratory Guide includes four experiments related to Chapter 20. If you do not have time to do all four, Experiments III-2 and III-3 should be given first priority; III-4, second priority; and III-1 third priority. If necessary, III-1 can be handled as a class demonstration. Experiments III-2 and III-3 should precede class discussion of the related ideas. Experiment III-1 can be used to introduce the chapter. See the yellow pages for further comments on scheduling and for suggestions on conducting the experiments.

Experiment III-1, A Variation on Galileo's Experiment, employs a pendulum to provide a simple parallel to Galileo's idealization of the action of objects moving on inclines.

Experiment III-2, Changes in Velocity with a Constant Force, uses the ticker tape timer to provide a record of the change in velocity of a small cart which is subject to a constant force. A constant force is provided by the constant stretch of rubber strands.

Experiment III-3, The Dependence of Acceleration on Force and Mass, uses the same apparatus as Experiment III-2.

Experiment III-4, Inertial and Gravitational Mass, uses a simple inertial balance and ordinary weighing to show the proportionality of inertial and gravitational mass. The operation of the inertial balance is shown to be independent of gravity.

Home, Desk, and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Section	Easy	Medium	Hard	Class Discussion
2	1*, 3	2		2
3		4*		4
4	5	6*, 7		
5	8*, 9, 10			
6	14			
7	11*, 12*	13, 15*, 16	17, 18, 22	16, 22
8	19, 21*		20	19, 20
9 and 10	23	24, 25*, 26		
11		27*	28	
General and review		29*	30	30

Films. "Forces", by Jerrold R. Zacharias of MIT, is an introduction to Part III of the course. The film examines the general nature of gravitational and electric (both electrostatic and magnetic) forces and touches briefly on nuclear forces. The gravitational force between relatively small objects is demonstrated with a Cavendish balance. Running time: 22 minutes.

"Inertia", by Edward M. Purcell of Harvard University, develops the idea of inertia, considers frictional effects and introduces the "frictionless" puck which has a gas bearing. From stroboscopic photographs of the puck moving under a constant force, the proportionality of force and acceleration is found. One of the best ways to use this film is as the basis of a summarizing discussion of the ideas of Sections 1 through 4. Running time: 27 minutes.

"Inertial Mass", by Edward M. Purcell of Harvard University, is a continuation of "Inertia". Using the same techniques as in the earlier film, acceleration is found to be inversely proportional to mass. Thus, with a constant force, acceleration is a measure of inertial mass. The proportionality of inertial and gravitational mass is shown. This film may be used in connection with a discussion of Sections 5 and 6. Running time: 20 minutes.

Introductory Section
Section 1 – Ideas about Force and Motion
Section 2 – Motion Without Force

PURPOSE To distinguish between kinematics and dynamics. To raise the question: What is the cause of motion? To discuss pre-Galilean attempts to understand dynamics; and to introduce Galileo's law of inertia.

CONTENT a. Kinematics is the systematic description of motion.

b. Dynamics is the analysis of the causes of motion.

c. The idea of force is generalized from that of a push or pull.

d. When no net force acts on an object, its state of motion remains unchanged.

EMPHASIS Very little class discussion will be needed if these sections have been assigned for outside reading. However, you may prefer to discuss the first few sections of the chapter before you give any assignments.

DEVELOPMENT If you choose to present this material through classroom discussion, you might ask for, and list on the blackboard, examples of moving objects which are neither pushed nor pulled by an obvious agent. Next to each example, describe the motion briefly as in the table below:

Object	Motion	Type
Automobile (out of gas and without brakes).	Slows down and stops.	1
Book which has been pushed so that it slides on desk.	Slows down and stops.	1
Chalk (or any object) which is dropped.	Speeds up until it hits floor.	2
Ball rolling down hill.	Speeds up until it reaches bottom.	2
Moon revolving around the earth.	Goes around at constant speed.	3
Earth rotating on its axis.	Rotates at constant speed.	3

After you have several examples of each type, call students' attention to the three categories: 1) slowing down, 2) speeding up, and 3) constant speed.

You can get a good idea of the information your students have by asking them to give reasons for or explanations of the motion. You will probably have students who already think of gravity and friction as external agents. There is no reason to avoid mentioning these mechanisms briefly.

After students understand the three categories, concentrate the discussion on how one could make an object which is slowing down go further. Your class will be interested in and will give good examples of reducing friction by lubrication. Some students may even know about gas bearings from popular science articles or from household gadgets such as the tank vacuum cleaner which partly rides on the exhaust air.

Finally, ask what would happen to the motion if the friction were reduced to zero. Ask them how they could convince a skeptic of this. It should be possible to get the class to see that one could never really perform such an experiment and that reasoning such as is given in connection with Figures 20–3 and 20–4 is needed.

* * *

It will be desirable to make sure that students understand how to interpret quantitatively such stroboscopic pictures as Figures 20-5b, 20-7, 20-9, and 20-16. You can discuss the experimental arrangement using either some analogous demonstration equipment or the explanatory figures (20-5a, 20-6, 20-8, 20-11 and 20-13).

You might have students determine the speed of the disc in Figure 20-5b. Using a compass (as a pair of dividers) they can easily find that the speed is about 12.7 cm per flash or 127 cm per second. They should be able to measure this to 1% or 2%. You could ask the students for the factor by which the figure was reduced (6.4) or for the diameter of the disc (about 10 cm).

Students will not be able to measure the speed in Figure 20-5 with enough accuracy to detect the deceleration. This deceleration (as implied by the last sentence in Section 2, page 310) is only 0.01 ft/sec^2. Therefore, during the seven intervals or 0.7 seconds shown on the photograph the speed should have changed only by 0.007 ft/sec which is unobservable since it is only about 1/6 per cent of the measured speed.

(Note: The deceleration or frictional force was actually determined by measuring the reduction of oscillation when one of these discs was driven back and forth across the table by a spring whose internal losses were known accurately. If an object were moving in only one direction on a perfectly flat table, this deceleration would be caused if one side of a 1 meter table were only 0.32 mm (or 0.032 cm) higher than the other. In order to do experiments which exploit fully this very low frictional force, one must use an extremely flat table and level it very carefully. Furthermore, the table and the floor supporting it must be rigid. In addition, the table surface must be very clean. For example, ice crystals formed from water vapor in the air would interfere with the motion. This difficulty was avoided by using a metal table (aluminum) which conducts heat well enough to prevent local freezing.)

COMMENTS With your help in leading the discussion, your class will be able to give you examples of the three apparently different types of motion even though they have not read the text. An introductory class discussion of this type can add considerable freshness and student interest to this material. Keep the discussion short, however. The idea is to illustrate the way that the problem was attacked before Galileo, and to show the difficulties of the problem. Then the incisive thinking of Galileo may be more readily appreciated.

Many students are familiar with Galileo's principle of inertia; most of them can learn it quickly and would be completely convinced by experiments with dry ice bearings or satellites. However, before you proceed to convince them through the use of modern techniques be sure that they understand how Galileo approached the problem. Problems 1, 2, and 3 present useful starting points for discussion.

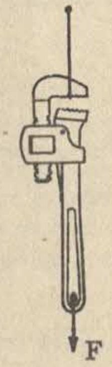
Try to emphasize Galileo's tremendous intellectual achievement, not only in the original enunciation of the law of inertia, but even more in his development of a new method of reasoning in which he combined the observation of real experiments with idealizations of these to simpler and more tractable situations.

LABORATORY AND DEMONSTRATION Experiment III-1 is one actually devised by Galileo, and beautifully illustrates his arguments. It can be used to introduce these sections. If you do not have time to do Experiment III-1 in the laboratory, it can be done as a demonstration to show the tendency of a fallen object to return to its original height. You can use inclined planes as well as pendulums. Emphasize that the total path length is irrelevant if friction is small. Students will enjoy both making measurements and guessing how high the object will go it you change conditions slightly.

CAUTION The standard demonstrations used in connection with the law of inertia (such as pulling a tablecloth from under glasses) show merely that an object which is at rest tends to remain at rest. All students (including those who studied with Aristotle) are aware that an object does not tend to move if it is at rest. If you are enough of a showman the memory of such a demonstration may somehow remind your students of the law of inertia, but demonstrations which get at the tendency of a body in motion to remain in motion, such as those mentioned above, are much more desirable. Pulling a tablecloth from under glasses shows only that the frictional force was not great enough to accelerate the glasses appreciably; few students understand the point of these standard demonstrations.

If you do wish to do such a demonstration, an amusing one which is somewhat more instructive is the following:

A weight of several pounds is suspended with a length of thread. (The weight could be some reasonably heavy tool such as a pipe wrench.) Tie another piece of thread below the weight. Now pull gradually, harder and harder, on the lower piece of thread. Ask students to guess which thread will break. Eventually the thread above the wrench will break. (Look out for your knuckles!) The upper thread breaks first because the force on it will be the applied force plus the weight of the wrench.



Next, suspend the wrench as before, but this time, starting with no tension in the lower thread, pull with a sharp jerk. If you do this quickly enough, the lower thread will break and the top one will remain intact. The explanation is that in order to transmit force to the upper thread, (since the thread is somewhat elastic) the wrench must move, increasing the stretch of the thread. The tendency of the wrench to remain stationary prevents the force of your jerk from being transmitted instantaneously to the upper thread. Before the wrench has time to move far enough to transmit this force, the lower thread breaks.

Section 3 – Changes in Velocity When a Constant Force Acts

Section 4 – Dependence of Change of Velocity on Magnitude of Force

PURPOSE To show the relationship between force and change of velocity.

CONTENT a. For a given object, a constant force imparts a change in velocity which is directly proportional to the time interval during which the force acts.

b. For a given object, the change in velocity during a given time interval (i.e., the acceleration) is directly proportional to the applied force.

EMPHASIS These are very important ideas, but should not require much class discussion if they are introduced, as recommended, through performing Experiments III-2 and III-3 in the laboratory before discussing them in class.

DEVELOPMENT If after their laboratory work, you feel that your students need more practice in interpreting this kind of data, you might ask them to make their own measurements of Figure 20-7 (and/or 20-9), and to set up tables similar to Tables 1 and/or 2 in the textbook. Inasmuch as an error of 0.5 cm in the distance increment in Figure 20-7 contributes an error of 2.5 cm/sec to the speed, the values which students get for the acceleration may differ appreciably from those shown in Table 1, page 312.

Be sure that students realize that although the data derivable from these figures are limited to about 5% accuracy, Newton's law is known to a much higher accuracy from other experiments. In Chapters 21 and 22 some of these will be discussed.

COMMENTS In these sections the text emphasizes Δv and Δt rather than explicitly using the acceleration, a. This was done to emphasize the relation between F, Δv and Δt; to make clear how the relation is determined from observation and measurement; and to avoid masking that relation behind an incompletely understood concept of acceleration. It will be wise not to substitute a for $\Delta v/\Delta t$ until your students are sure of these ideas.

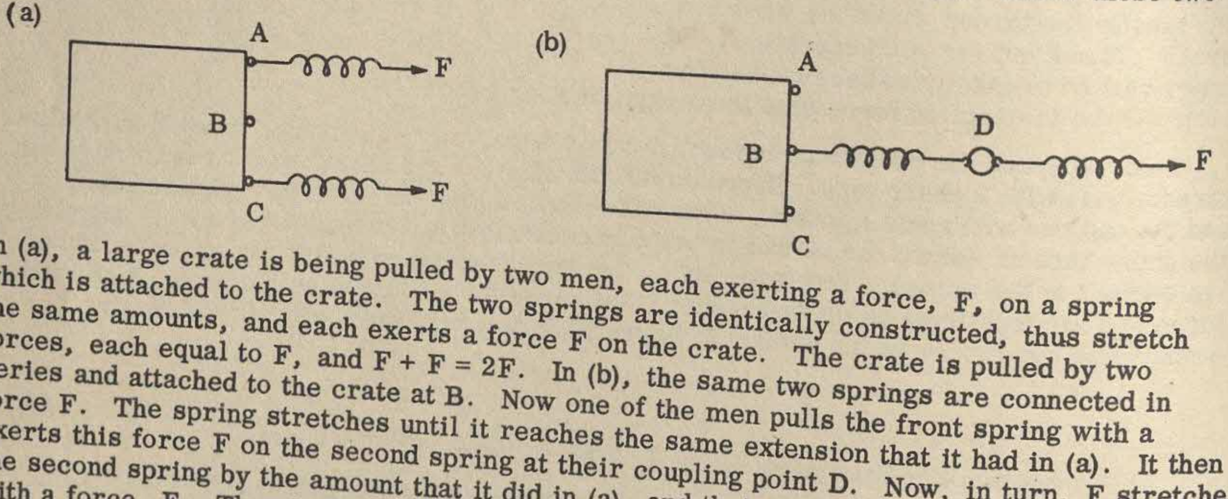
* * *

With reference to Table 1, some students wonder how such precise values as 5.68 cm and 9.48 cm can be determined. Greatly enlarged photographs were used, and extremely careful measurements were made.

* * *

The simple additivity of two parallel forces is assumed in the third paragraph of Section 4. There are many ways in which this fact can be treated. Probably the most direct method is to treat it as a part of the definition of a force, F + F = 2F.

It may be instructive for you to raise questions about the difference between the effects of applying two forces to an object in parallel and in series. In the figure below these two arrangements are shown.



In (a), a large crate is being pulled by two men, each exerting a force, F, on a spring which is attached to the crate. The two springs are identically constructed, thus stretch the same amounts, and each exerts a force F on the crate. The crate is pulled by two forces, each equal to F, and F + F = 2F. In (b), the same two springs are connected in series and attached to the crate at B. Now one of the men pulls the front spring with a force F. The spring stretches until it reaches the same extension that it had in (a). It then exerts this force F on the second spring at their coupling point D. Now, in turn, F stretches the second spring by the amount that it did in (a), and that spring pulls on the crate at B with a force, F. Thus in (b), although both springs are stretched the standard amount, one simply serves to transmit the one single force, F, to the other spring which in turn transmits it to the crate. Only one man is pulling. The force on the crate is only 1 × F.

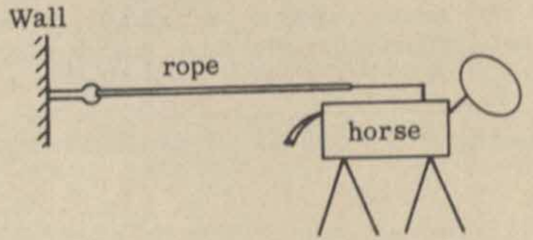
* * *

Some students intuitively feel that a spring will exert twice as much force if it is extended twice as far from its equilibrium, non-extended, length. Although this is approximately true for many springs (and is known as Hooke's law), it is only approximately true at best and is far from the truth in many cases. For this reason, it has not been used in this development, and should not be introduced as a significant factor in classroom discussion.

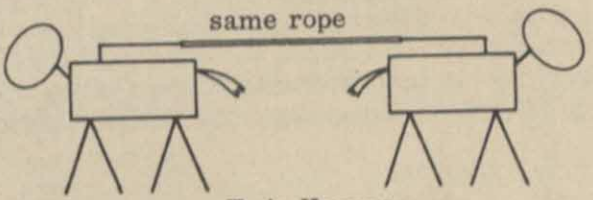
In order to avoid confusion it will be wise to sharpen students' appreciation of the importance of being explicit about both the object on which the force acts and the agent exerting the force. In any physical problem, it is meaningless to discuss a force unless both the object and the agent are known. Of course, there are many times when both the object and agent are so obvious that they need not be stated explicitly. But from time to time you might ask a student who is dealing with a force for the complete description. (i.e., Which force? What exerts the force? On what is the force exerted?)

QUIZ QUESTIONS (or questions to stimulate classroom discussion)

1) It is never safe to assume that forces in series are well understood. You might try this: A farmer with twin horses has a rope which is so strong that neither horse pulling alone can quite break it.



Horse almost but not quite strong enough to break the rope.



Twin Horses

Will the rope break?

Some students will expect the rope to break when an extra horse is involved. They fail to realize that the second horse merely replaces the wall.

2) Another stock question which is often a puzzler is the following:

A horse is hitched to a wagon with a rope. The horse pulls on the rope with a force, F. The rope, then, pulls on the horse with a force, F. Then why is it that the wagon starts to move?

The important point here is that in considering the acceleration of any object one must consider those forces which are acting on that object. The wagon starts to move because the pull of the rope on the wagon is greater than the backward force of friction. The horse starts to move because the forward force exerted by the ground on his feet is larger than the backward force exerted by the rope on him – and so forth.

Section 5 – Inertial Mass

Section 6 – Inertial and Gravitational Mass

PURPOSE To define operationally the idea of inertial mass. To point out that inertial and gravitational mass might be expected to be independent because the operations used to define them are entirely different. To cite the empirical evidence for the direct proportionality between inertial and gravitational mass.

CONTENT a. Inertial mass is the ratio of the net force on an object to the acceleration it produces.

$$m = \frac{F}{a} \text{ or } m = \frac{F\Delta t}{\Delta v}.$$

The ratio F/a is closely the same for all speeds of motion of the mass that are small compared to the speed of light.

b. Inertial masses are additive.

c. There apparently is no logically necessary relationship between gravitational and inertial effects, but experimentally, the pull of gravity on various objects is observed to be directly proportional to their inertial masses.

EMPHASIS These ideas are centrally important. You may need to spend a full period presenting and untangling the concepts of gravitational mass, inertial mass and weight. Even then, you may need to comment from time to time to help keep the ideas straight.

APPROPRIATE LABORATORY Experiment III-4 should follow class discussion of these sections. It can be omitted if you are pressed for time.

COMMENTS Students will feel more at home with inertial mass if you can give them an intuitive understanding to supplement the formal definition based on force and acceleration. When students think of something massive they tend to think of weight, and if asked to judge mass, they might "heft" it. They will get a much better idea of inertial mass if you can get them to think of moving an object from side to side (i.e., judging the mass by the effort required to "shake" it).

* * *

Students will have been introduced to the equivalence of mass and energy in Section 6 of Chapter 7 and from other sources. (See Teacher's Guide for Part I, page 7-6.) The concept of energy is the subject of the latter chapters of Part III. Detailed class discussion should be avoided at this point, but if you are asked, you might tell the questioner that mass and energy are regarded as equivalent – they can be converted into each other, but in any process the total mass plus the energy divided by the square of the speed of light remains constant, i.e.,

$$m_1 + m_2 + m_3 + \ldots + \frac{E}{c^2} = \text{constant}$$

Thus if light leaves a flashbulb, the bulb will have less mass and the light will have the remaining mass – both as measured gravitationally or as measured inertially. Because this mass difference is such a small fraction of the mass of the flashbulb, it would not be detectable by any practical balance, although the mass of the light can be detected. (See solution to Problem 10.)

* * *

Experiments which measure mass by measuring the acceleration and the force, illustrate the meaning of inertial mass. However, such experiments cannot be done with high precision. Hence, wherever possible, they are replaced by other more precise measurements. For ordinary-sized objects, comparison of weights (gravitational masses) using an equal arm balance is preferred to an acceleration experiment. However, the validity of such a procedure hinges upon a knowledge of the proportionality of inertial and gravitational mass.

Some students will have trouble trying to distinguish between the two different kinds of mass associated with any object. They may accept more readily the independence of gravitational force and inertial mass. The gravitational force between any object and the earth is what we call the object's weight. If object A weighs twice as much as object B, this signifies that the pull of the earth is twice as great on A as it is on B. Now if we were to do another, completely independent set of experiments, in which we measured the accelerations of A and B under the action of various forces, we would find that any given force would accelerate A just half as rapidly as it would B. From these acceleration experiments we could conclude that the inertial mass of A is twice that of B. This result could not have been predicted from the observations of the gravitational forces. No basic principle of classical physics relates the two sets of experiments, but the ratio of gravitational forces on two different objects at the same place defines the ratio of their gravitational masses. Consequently, there is also no basic principle relating the gravitational masses of the two objects. The fact that the two mass ratios are identical is an experimentally-established relationship. *

Actually the two kinds of experiments described above could not be performed with an extremely high degree of accuracy. A more accurate method, which is directly sensitive to the ratio $m_{inertial}/m_{gravitational}$, is the measurement of the constancy of acceleration of different masses in free fall. Such a method will be treated in detail in Chapter 21, Section 2.

More precise experiments based on this same idea involve the oscillation frequency of a pendulum. The gravitational force tends to push the pendulum to its lowest position; the inertial mass resists the acceleration produced by this force. The pendulum frequency is proportional to the square root of the ratio, (m_g/m_i); it is also proportional to $\sqrt{g/\ell}$ where g is the acceleration produced in free fall and ℓ is the pendulum length. Very precise experiments have been made using pendulums of the same length but different materials.

Even more precise measurements are currently underway which involve measuring the speed of revolution of the moon (or an artificial earth satellite) using a very precise time standard established by an "atomic clock" (i.e., a clock which is stabilized by characteristic frequencies of particular atoms). The best experiments thus far show that the ratio of m_g to m_i varies by less than 3 parts in 10^{10}.

This apparently coincidental relationship, the proportionality of m_g and m_i, is proposed, in Einstein's general theory of relativity, as a universal truth, an inherent property of matter. This "principle of equivalence" is one of the premises upon which the theory is built.

Section 7 – Newton's Law: Dynamical Measurement of Force; Units

PURPOSE To identify Newton's law and to establish a unit of force which can, in turn, be used to measure masses.

CONTENT The relationship, $F\Delta t = m\Delta v$, or $F = ma$, is identifies as Newton's law of motion. This law provides a basis for defining a unit of force. Acceleration can be measured in terms of distance and time – meters and seconds, and we can get known amounts of mass by comparison with the standard kilogram, so a unit of force – the newton – can be defined as that amount of force which will cause a 1-kg mass to accelerate at 1 m/sec^2. With forces so defined, we can then measure masses in terms of the quantity, F/a.

EMPHASIS Get across the idea that Newton's law provides a meaningful, non-arbitrary, basis for defining a unit of force. Don't stop for extensive practice on the use of the new unit here, as it will come up naturally in the rest of this chapter and in the next two chapters.

COMMENT This section describes another step forward in the physicists' attempt to work with things that can be derived from first principles. Newton's law is one of these first

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* You might like to think about the following argument although it is not appropriate for class discussion at this point in the course. The force on an electrically charged body in an electrical and gravitational field can be described as

$$\vec{F} = q_e\vec{E} + q_g\vec{g}$$

where $q_g = m_g$. There is no a priori reason we know of that q_g, the "gravitational charge" (or gravitational mass) should be the same as the inertial mass, any more than q_e should be the inertial mass. Experimentally, however, q_g is proportional to m_i and can be chosen equal to it by an appropriate choice of standards.

principles. Our concept of the magnitude of a force is thus defined in terms of one of the first principles. We could define a unit of force in terms of how much it would stretch a well-preserved spring kept in some national bureau of standards. But this definition would be completely arbitrary. The unit of mass seems arbitrary enough, but actually it now can be defined in terms of the mass of the hydrogen atom; 5.98×10^{26} of them make a kilogram. The standard could as well be the mass of a single hydrogen or oxygen atom except for its being an impractically small unit of mass.

* * *

As is pointed out in the footnote on page 317, the text refers to one law as Newton's law of motion. Newton is often credited with three laws. The law of inertia, frequently called Newton's first law, was stated by Galileo and is simply a special case of the "second law" – it covers the case where $F = ma = 0$. The "second law" is just $F = ma$ and is called here Newton's law of motion. The "third law" relates to the equality of forces of action and reaction, and is discussed in Chapter 24, Section 8; it is not really a law of motion.

Section 8 – Forces That Change, and Newton's Law

PURPOSE To show how Newton's law can be applied if the force is not constant.

CONTENT a. If a force changes, its effect can be calculated by considering what happens in each of many very small successive time intervals. (These time intervals can be chosen to be so small that the force is essentially constant during the interval.) For each time interval, Δt, the change in motion is described by the equation $F\Delta t = m\Delta v$.

b. $F\Delta t = m\Delta v$ is true, independent of any initial speed the object may have. (The equation does not hold at speeds approaching the speed of light.)

EMPHASIS It is important that students understand the idea that we are able to use $F\Delta t = m\Delta v$ to analyze changes in motion during a particular time interval without being concerned with the source or nature of the motion the body may have at the beginning of that time interval. However, once pointed out, the idea should be clear. The statement of the law restricts it, so to speak, to the change in velocity of a particular mass acted on by a constant force for a particular time interval. If we wish to analyze motions involving changing forces, we simply select our time intervals in such a way that within them the force is (essentially) constant.

COMMENT From what previously has been established from experimental observations (i.e., that $F = ma$ if F is constant), it is not logically possible to deduce, by argument alone, what happens if F is not constant. The fact that we do experiments in which a force is applied and then removed means that we have seen situations in which the force varied with time and no striking departures from the law $F = ma$ were observed. To be more confident that the acceleration depends only on F/m instead of, for example $\frac{1}{m}\left(F + \frac{dF}{dt}\right)$ we have to do experiments. Collisions, discussed in Chapter 21, and planetary motions are excellent phenomena with which to check that the instantaneous acceleration equals the instantaneous force divided by the mass, without any added terms that depend on the rate at which the force varies. All such experiments have continued to verify Newton's law of motion except in the case of motions with speeds near the velocity of light where the law changes to the extent that the effective mass varies with the speed.

Section 9 – How Forces Add; The Net Force

Section 10 – The Vector Nature of Newton's Law

PURPOSE To consider the effect of several simultaneous forces, and to extend Newton's law to such cases.

CONTENT a. Forces add as vectors.

b. Newton's law applies to the vector sum of the forces acting on an object.

c. Newton's law can be written as the relation between two vectors: the net force and the acceleration.

EMPHASIS These results require experimental proof, which is given in Chapter 21. As long as the point is accepted by students that Newton's law retains its extremely simple form in even the most complicated situations, but not much time need be spent on detailed discussion at this stage.

COMMENTS Newton's law has been established as a relationship between the acceleration of an object and a single force acting upon that body. When several forces act, at any instant, there is still only a single observable acceleration. The single effective force which satisfies Newton's law can then be found: $ma = F_{\text{effective}}$. This $F_{\text{effective}}$ must be related to the several independent forces which are acting. It is found experimentally that $F_{\text{effective}}$ is simply the vector sum of all of the forces that are acting and is called the

A finer point: The relationship $F = ma$, as suggested in this section, has more content than has been shown up to this point. $F = ma$ implies that the acceleration is always proportional to the force, no matter what the angle between the force and the velocity, since velocity itself does not appear in the equation. For example, Newton's law should hold if we shove something sidewise to its motion. This will be verified experimentally in Chapter 21.

Section 11 – Forces in Nature

PURPOSE To indicate that, though forces in nature are often complex, the consequent motions can be analyzed with Newton's law.

CONTENT To learn about the many forces in nature, we can study motions to find the accelerations, and then apply Newton's law to deduce the forces. Once we have learned about the forces, we can predict any other motions produced by these forces.

EMPHASIS Treat briefly. The ideas of this section lead toward Chapter 22 which gives an exciting example of the application of Newton's law.

COMMENT While not of great immediate consequence, it is worth noting the point that air resistance increases with speed. Understanding this now will reduce the number of ideas to be assimilated when students analyze the Millikan experiment (p 469-474), and must understand that constant velocity of a particle moving through air means no net force, and since air resistance is proportional to velocity, different constant velocities represent different applied forces. In the range of velocities used in Millikan's experiment the velocity is proportional to the applied force – it is not so for all velocities.

Chapter 20 – Newton's Law of Motion

For Home, Desk and Lab – Answers to Problems

The ability of students to analyze and solve problems is a good measure of their understanding of the ideas in Chapter 20. The analysis of typical problems in class will be helpful.*

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 20-16 to 20-32.

Section	Easy	Medium	Hard	Class Discussion
2	1*, 3	2		2
3		4*		4
4	5	6*, 7		
5	8*, 9, 10			
6	14			
7	11*, 12*	13, 15*, 16	17, 18, 22	16, 22
8	21*, 19		20	19, 20
9 and 10	23	24, 25*, 26		
11		27*	28	
General and review		29*	30	30

SHORT ANSWERS

1. (a) 10 cm.
   (b) 100 cm.
   (c) 200 cm.
2. See discussion on page 20-17.
3. See discussion on page 20-17.
4. 4.0 m/sec.
5. 30 cm/sec^2.
6. (a) 1.5
   (b) 0.9 m/sec.
7. See discussion on page 20-18.
8. (a) The first object.
   (b) 0.36

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*Problem 15 is a particularly instructive problem, but if you assign it you should take account of:

1. In the first printing of the textbook, the caption for Figure 20-16 was omitted. The caption is included in the second printing. The caption indicates that the scale is in cm, and the interval between flashes is 0.2 sec.

2. Some students may have difficulty in seeing that they must deal with average speed in successive time intervals. See the SPECIAL NOTE under the discussion of Problem 15 on page 20-21.

9. (a) 0.239
   (b) Aluminum.

10. See discussion on page 20-19.

11. 2 nt.

12. 5m/sec^2.

13. 6m/sec^2.

14. 1.9m/sec^2.

15. (a) See discussion on page 20-21.
    (b) See discussion on page 20-21.
    (c) See discussion on page 20-21.
    (d) Probably, yes.
    (e) 1.0 nt.

16. (a) 2.0 kg.
    (b) Use a balance.
    (c) There was friction.

17. (a) 4.0 sec.
    (b) 0.35 m/sec, 2.8 m/sec.

18. (a) 1.0 sec.
    (b) 12 m/sec.

19. Zero force.
    See discussion on page 20-24.

20. (a) Acceleration increasing, proportional to t.
    (b) Force increasing, proportional to t.

21. (a) 0.40 m/sec.
    (b) 0.10 m/sec.
    (c) In the same direction as initially.

22. (a) 3.0 m/sec^2.
    (b) 9.0 m/sec.
    (c) 6.0 m.
    (d) 12 m.

23. (a) $\frac{1}{6}$ m/sec^2.
    (b) $\frac{1}{4}$ m/sec^2.
    (c) There was a frictional force of 2/3 nt.

24. (a) 0.70 m/sec^2 along the bisector.
    (b) 1.0 m/sec^2, 0.

25. (a) 2.0 m/sec.
    (b) 25° to the 4 nt force.

26. 520 nt.

27. $F_{\text{applied}} = 8200$ nt.

28. (a) See discussion on page 20-30.
    (b) 3.0 m/sec, 6.0 m/sec.
    (c) Change in mass does not affect maximum velocity.
    (d) The maximum velocity would decrease.

29. See discussion on page 20-31.

30. See discussion on page 20-32.

COMMENTS AND SOLUTIONS

PROBLEM 1

A ball is released from rest on the left-hand incline of Fig. 20-4 at a height of 10 cm above the lowest point.

(a) If there is no friction, how high vertically will it rise on the right-hand incline?

(b) If the right-hand incline rises 1 cm for every 10 cm of horizontal distance, how far will the ball travel horizontally on it?

(c) If the incline rises only $\frac{1}{2}$ cm for every 10 cm of horizontal distance, how far will the ball go?

a) The ball will rise to the same height, i.e., 10 cm.

b) Since there is 1 cm rise for 10 cm horizontal travel, in rising 10 cm the ball will travel a horizontal distance of 100 cm.

c) The slope is halved. The vertical height reached is still 10 cm, so the horizontal distance is now 200 cm.

We are progressing toward Galileo's ideal case of zero slope, when the ball will travel an indefinitely large distance.

PROBLEM 2 Why is it dangerous to step from a moving vehicle onto the ground? In what direction would you tend to fall?

This is dangerous because when your feet touch the ground they will be brought to rest. However, the remainder of your body will tend to maintain the motion it had in the vehicle. If the center of the body moves forward beyond your support (your feet), you will fall. You would tend to fall in the direction in which the vehicle is moving.

To prevent falling you must be able to keep your center of weight over its support points, i.e., keep your feet under you. To do this you must be able to run as fast as the vehicle is moving when you step off. It is easier to wait for the bus to stop.

PROBLEM 3 Why is it particularly dangerous to drive on an icy highway?

A force is required to produce the acceleration associated with turning, slowing, and speeding up. This force is developed between a car and the earth through the friction between tires and road surface. The force of friction between tires and ice can be less than one tenth what it might be between tires and a dry road. The relatively small frictional force that is developed on an icy road makes it difficult to change appreciably either the magnitude or direction of a car's speed. Thus, on an icy road, a car often perversely maintains its speed and direction despite brakes, throttle or steering. In such cases, eventual contact with a telephone pole, a roadside ditch, or another car may supply an accelerating force.

PROBLEM 4 A certain force exerted for 1.2 sec raises the speed of an object from 1.8 m/sec to 4.2 m/sec. Later, this same force is applied for 2.0 sec. How much does the speed change in the 2-sec period? (In both cases, the force is applied in the direction of motion.)

In this problem we are, in effect, given a few bits of data like that in Table 1. Since the force is the same in the two cases, the change in speed is proportional to the time.

$$\frac{\Delta v_1}{\Delta v_2} = \frac{\Delta t_1}{\Delta t_2}, \text{ and } \Delta v_2 = \frac{\Delta t_2}{\Delta t_1} \Delta v_1 = \frac{2.0}{1.2} 2.4 = \underline{4.0 \text{ m/sec}}.$$

PROBLEM 5 A body is pulled across a smooth horizontal surface by a spring that is kept stretched by a constant amount. It is found that the body is accelerated at 15 cm/sec². What will be the acceleration of the body if it is pulled by two springs, each just like the first spring, side by side and stretched by the same amount? [See Fig. 20–8(a).]

Both springs pull on the body. Since the springs are stretched by the same amount, they pull with equal force. Therefore, $F_{total} = F_1 + F_2 = 2F_1$. Since the force has been doubled, the resultant acceleration must also be doubled:

$$\text{If } \frac{F_1}{m} = 15 \text{ cm/sec}^2, \ \frac{2F_1}{m} = 2 \times 15 \text{ cm/sec}^2 = \underline{30 \text{ cm/sec}^2}.$$

PROBLEM 6

An object sliding on a low-friction bearing is pulled with a constant force. In a time interval of 0.3 sec the speed changes from 0.2 m/sec to 0.4 m/sec. In a second trial, the object is pulled with another force. In the same length of time the speed now changes from 0.5 m/sec to 0.8 m/sec.

(a) What is the ratio of the second force to the first?

(b) If the body is pulled with the second force for 0.9 sec, what change in speed results? (Note that the forces are in the direction of motion.)

a) Because the masses and the time intervals are the same in the two trials, the ratio of the forces is equal to the ratio of the changes of speed:

$$\frac{F_2}{F_1} = \frac{\Delta v_2}{\Delta v_1} = \frac{0.8 \text{ m/sec} - 0.5 \text{ m/sec}}{0.4 \text{ m/sec} - 0.2 \text{ m/sec}} = \underline{1.5}.$$

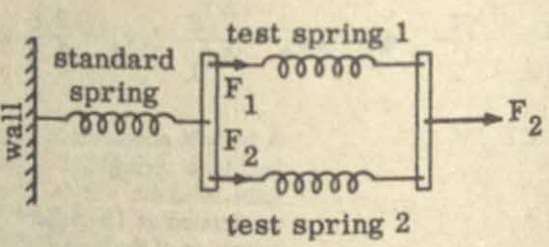
b) When a given mass is accelerated by a given force, the change in speed will be proportional to the length of time the force is applied. In the second trial, since the time is three times longer, the change in speed will be three times greater.

$$\frac{\Delta t_2}{\Delta t_1} = \frac{\Delta v_2}{\Delta v_1}, \text{ and } \Delta v_2 = \Delta v_1 \frac{\Delta t_2}{\Delta t_1} = 0.3 \text{ m/sec} \times \frac{0.9 \text{ sec}}{0.3 \text{ sec}} = \underline{0.9 \text{ meters/sec}}.$$

PROBLEM 7

In Section 20-4 we discussed a way in which a stretched spring could be calibrated so that it exerts twice as much force as a standard spring stretched a given amount. Be ready to discuss in class how you would determine when a stretched spring exerts one-half as much force as the standard.

One way to do this would be to yoke two identical springs to a standard spring. This system of springs can be pulled until the standard spring has its standard stretch. When the system of springs is held steady at this point, the yoke does not move (its acceleration is zero) hence the yoke is subject to no net force and the total force exerted by the pair of test springs must be equal to the standard force. $F_s = F_1 + F_2$.



If the two test springs are identical, they will each have the same stretch, and F_1 and F_2 will be equal. Then $F_1 = F_2 = \frac{F_s}{2}$.

In order to test these half-standard forces, we can perform experiments in which the springs successively apply a force to the same body. Applying F_s, F_1 and F_2 separately, each for the same length of time, observe the increase in velocity of the test body. If $F_1 = F_2 = 1/2\ F_s$, then when F_1 and F_2 act separately on the same body, they should each produce only half the velocity change produced by F_s.

PROBLEM 8

Suppose you accelerate an object with a steady force and find that the change in speed during a time interval Δt of 1 sec amounts to 2.4 m/sec. You now repeat the measurement using the same force with a second object. It gains 3.3 m/sec in 0.5 sec.

(a) Which body has the greater inertial mass?

(b) What is the ratio of the inertial mass of the second object to that of the first?

a) The change of speed of the first object is 2.4 m/sec in one second. The change of speed for the second object is 3.3 m/sec in 0.5 seconds, or 6.6 meters in one second. Since the forces are the same, the first object must have the greater inertial mass.

b) Since the forces are the same in both cases, $m_1a_1 = m_2a_2$.

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{2.4\ \text{m/sec}^2}{6.6\ \text{m/sec}^2} = \frac{4}{11} = \underline{0.36}.$$

PROBLEM 9

(a) Two blocks, made of different metals, identical in shape and size, are acted upon by equal forces which cause them to slide across a frictionless horizontal surface. The acceleration of the second block is found to be 4.18 times that of the first. What is the ratio of the mass of the second block to that of the first?

(b) The first block is known to be made of lead. Using the densities given in Table 2, Section 7–4, decide of what material the second block might be made.

a) Since the applied forces are equal, $m_1a_1 = m_2a_2$.

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{a_1}{4.18\ a_1} = \frac{1}{4.18} = \underline{0.239}.$$

b) Since the density of lead is 11.3, the density of the second object must be $\frac{11.3}{4.18} = 2.70$. From Table 2, the second block may be made of aluminum.

PROBLEM 10

Why is the flask in Fig. 20–10 closed? Be prepared to explain in class.

The flask is closed so that no product of the chemical reaction can escape from the system. (Actually no gas is generated in this reaction.) Thus the mass of the flask plus contents remains constant.

Note: Rigorously, if heat, light or sound energy are given off during the reaction, there will be a corresponding decrease in mass as dictated by the mass equivalence of energy. However, this change is negligibly small, and it is probably best not to discuss the point unless the question is raised by a student. A change of mass Δm kg corresponds to a change of energy $E = \Delta mc^2 = \Delta m\ (3 \times 10^8\ \text{m/sec})^2 = 9 \times 10^{16}\ \Delta m$ joules. Very good heating coal gives about 3×10^7 joules of heat energy per kg. This corresponds to a mass change of only $\frac{3 \times 10^7}{9 \times 10^{16}} = 3 \times 10^{-10}$ kg for each kg burned; this constitutes a change of only one part in 3×10^{10} parts.

PROBLEM 11 A body with a mass of 0.5 kg is accelerated at 4 m/sec². How large a force is acting?

Since the mass is given in kilograms and the acceleration in meters/sec^2, the force will be expressed in newtons.

$$F = ma = 0.5\text{ kg} \times 4\text{ m/sec}^2 = \underline{2\text{ newtons}}.$$

PROBLEM 12 A force of 3 newtons is applied to a mass of 0.6 kg. How fast does it accelerate?

$$F = ma, \quad a = \frac{F}{m} = \frac{3\text{ newtons}}{.6\text{ kg}} = \underline{5\text{ meters/sec}^2}.$$

PROBLEM 13 A force of 5 newtons gives a mass m_1 an acceleration of 8 m/sec², and a mass m_2 an acceleration of 24 m/sec². What acceleration would it give the two when they are fastened together?

This problem may be done in several ways. If it is handled step by step, we can note that

$$m_1 = \frac{F}{a_1} = \frac{5}{8}\text{ kg, and}$$

$$m_2 = \frac{F}{a_2} = \frac{5}{24}\text{ kg.}$$

Hence, $m_1 + m_2 = \frac{5}{8} + \frac{5}{24} = \frac{5}{6}$ kg, and

$$a = \frac{F}{m_1 + m_2} = \frac{5}{5/6} = \underline{6\text{ m/sec}^2}.$$

In following these steps, it is apparent that F drops out. This might have been anticipated by remembering that force is proportional to both mass and acceleration. We could have written

$$m_1 = F/a_1, \quad \text{and } m_2 = F/a_2.$$

$$a = \frac{F}{m_1 + m_2} = \frac{F}{F/a_1 + F/a_2} = \frac{F}{F/8 + F/24} = \frac{F}{F/6} = 6\text{ m/sec}^2.$$

PROBLEM 14 You have two objects, *A* and *B*, which balance each other when placed on opposite sides of an equal-arm balance. When you place both these objects on one side, they balance a third object C on the other side. Object *A* accelerates at 3.8 m/sec² when you apply a certain force. Suppose you now apply this same force to *C*. What is its acceleration?

Since $m_a = m_b$, and $m_a + m_b = m_c$, it follows that $m_c = 2\,m_a$. Since a given force accelerates m_a at a rate of 3.8 m/sec^2, the same force will accelerate m_c at one half that rate, or $\underline{1.9\text{ m/sec}^2}$.

While balancing is concerned with gravitational mass, and acceleration is concerned with inertial mass, the experimentally determined equivalency of inertial and gravitational mass makes the above result what one would expect and, indeed, observe.

PROBLEM 15

In Fig. 20-16 measure Δx, the distance traveled in each interval, for intervals 5 through 10.

(a) What is the speed $\Delta x/\Delta t$ in each interval?

(b) What are the changes in speed Δv in each interval?

(c) What is the acceleration $\Delta v/\Delta t$ in each interval?

(d) Has a constant force been acting?

(e) Assuming the disc has a mass of 2 kg, what is the average applied force?

The caption for Figure 20-16 was inadvertently omitted in the text. The caption states that the scale is in cm, and the time interval between flashes is 0.2 sec.

a), b), and c) The values below were taken from a student's measurement. They are typical, not necessarily "correct".

Interval number	Δx (cm)	$\frac{\Delta x}{\Delta t}$ (cm/sec)	Δv (cm/sec)	a (cm/sec^2)
5	8.9	44.5		
			10.0	50.0
6	10.9	54.5		
			10.0	50.0
7	12.9	64.5		
			9.5	47.5
8	14.8	74.0		
			10.5	52.5
9	16.9	84.5		
			10.0	50.0
10	18.9	94.5		

d) Variation in the value for acceleration can easily be due to small errors in measurement. A constant force probably has been acting.

e) $a_{\text{average}} = 50\ \text{cm/sec}^2 = 0.50\ \text{m/sec}^2$. $F = 2 \times 0.50 = \underline{1.0\ \text{newtons}}$.

Note that 1 cm on the scale in the picture is slightly less than 1/16 inch. Hence, even careful students may make errors of 0.3 cm in measuring Δx. This corresponds to an error of 1.5 cm/sec (or 15%) in Δv. If errors are made in adjacent Δv values, errors of 30% or 15 cm/sec^2 may be common in the acceleration, a.

Note also that certain errors are compensated through the procedure of taking successive differences. If one Δx were measured too large, this would result in too large a value for $\Delta x/\Delta t$ in that interval. This will affect the calculations of two of the Δv's, raising one value and lowering the other by the same amount. Then in taking a final average, the effects of such an error would tend to cancel out.

SPECIAL NOTE: In solving this problem, some students may have difficulty in seeing that they must deal with average speed in successive time intervals. For this difficulty, you may want to let them struggle, then straighten them out in class discussion, or you may want to give them a few pointers. If you want to provide hints, you can suggest that, in

Part a) "speed, $\Delta x/\Delta t$ in each interval" means average speed.

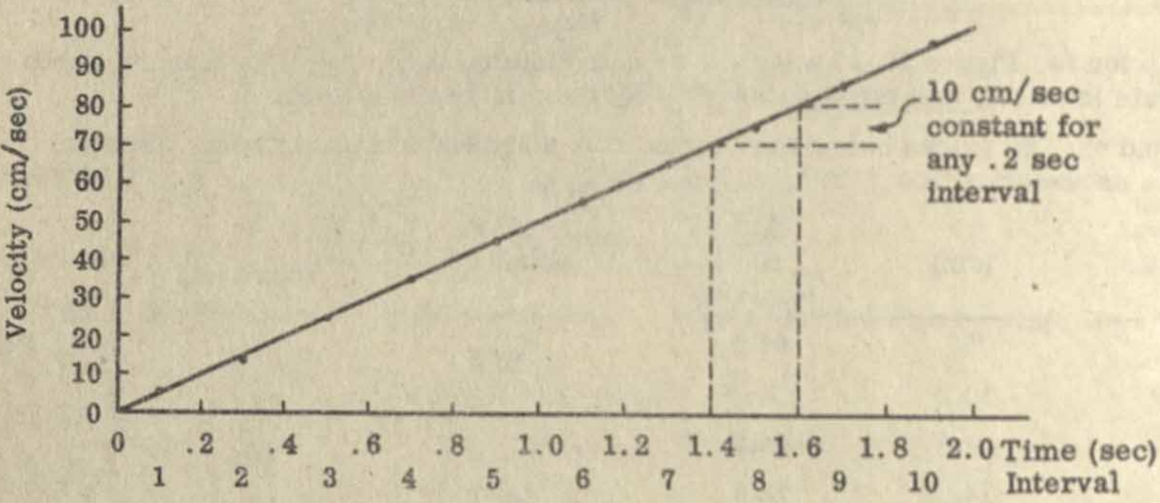
Part b) "changes in speed, Δv, in each interval" should be interpreted as changes in average speed between the successive intervals.

Part c) "acceleration $\Delta v/\Delta t$, in each interval" should be based on changes in average speed between the successive time intervals through assigning the average speed for the interval to the mid-point in time of each .2 sec interval.

In any case, it will be instructive for many students, after they have worked this problem, to graph on the blackboard the velocity-time data for this problem (see below), and remind them that

1) Δv for a .2 sec interval is constant (approximately 10 cm/sec) for any .2 sec interval no matter what instant is chosen for the start of that interval, and

2) that the acceleration $\Delta v/\Delta t$ in cm/sec^2 is simply Δv for a 1 sec interval instead of a .2 sec interval, and while the values for acceleration were derived from changes in average speed between successive time intervals, the acceleration is constant (the graph is straight) no matter where one chooses to "start" a one-second interval.



PROBLEM 16

A force of 3.0 newtons is exerted on an object and it speeds up at 1.5 m/sec².

(a) Assuming that this is the only force on the object, what is the mass?

(b) How would you make an independent measurement of the mass?

(c) Suppose this measurement indicates that the mass determined by the first method is greater. What might you suspect?

a) $F = ma$. $m = \dfrac{F \text{ newtons}}{a \text{ m/sec}^2} = \dfrac{3.0}{1.5} = \underline{2.0 \text{ kg}}$.

b) One possible way is to place the object on an equal arm balance and balance it against standard masses.

c) You might suspect that a frictional force was acting on the object in opposition to the measured force that was intentionally applied, resulting in the net force being smaller than 3.0 newtons.

Note: If friction were found not to be a reasonable explanation, one of the following errors should be suspected.

(1) One of the standard masses might be damaged.
(2) The force standard (3.0 newtons) may be incorrectly calibrated.
(3) The measuring stick might be incorrectly calibrated.
(4) The timing device, used in measuring the acceleration, might be in error.
(5) A mistake may have been made in observing or recording data.
(6) There may be some residual gravitational force acting if the surface is not completely level.

Items (1), (2), (3), and (4) could be checked by comparing each standard against a second standard. (2) is the one that is most likely to be in error. In order to check (5), the experiment can be repeated. To investigate (6), the experiment could be done twice, in opposite directions. Taking the average of the two results would eliminate the effect of a tilted surface.

PROBLEM 17

Two bodies, one of mass 8.0 kg and the other of mass 2.0 kg, stand at rest on a smooth table top side by side. The 8-kg mass is accelerated from rest by a force of .70 newton and the 2-kg mass is accelerated from rest in the same direction by a force of 1.4 newtons. Both bodies start accelerating at the same instant of time.

(a) How long does it take before the separation of the bodies is 5.0 m?

(b) What is the velocity of each at this instant of time?

a) When a body moves from rest with a constant acceleration, (Chapter 6, Part I), $d_1 = \frac{1}{2} a_1 t^2$, and $d_2 = \frac{1}{2} a_2 t^2$. In this problem the time of motion, t, is the same for the two bodies.

We are interested in knowing how much time will elapse before the absolute value of the difference between d_1 and d_2 is equal to 5.0 meters. Since a_2 is greater than a_1, d_2 will always be greater than d_1. Hence, we wish to find the value of t under the condition, $d_2 - d_1 = 5.0$ meters.

Substracting the first equation from the second, above, we find:

$$d_2 - d_1 = \frac{1}{2} a_1 t^2 - \frac{1}{2} a_2 t^2 = \frac{1}{2} (a_2 - a_1) t^2.$$

From this expression,

$$t = \left(\frac{2 \times (d_2 - d_1)}{a_2 - a_1} \right)^{1/2}$$

We are given that $a_1 = \frac{.70}{8.0}$ m/sec^2, and $a_2 = \frac{1.4}{2.0}$ m/sec^2. Also, $d_2 - d_1 = 5.0$ m.

Substituting,

$$t = \left(\frac{2 \times 5.0}{\frac{1.4}{2.0} - \frac{.70}{8.0}} \right)^{1/2} = \left(\frac{10}{4.9/8} \right)^{1/2} = \left(\frac{80}{4.9} \right)^{1/2} = \left(\frac{800}{49} \right)^{1/2} = \left(\frac{400 \times 2}{49} \right)^{1/2}$$

$$t = \frac{20}{7} \times \sqrt{2} = \underline{4.0 \text{ sec.}}$$

(We can check algebra by finding

$$d_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} \times \frac{.70}{8.0} \text{ m/sec}^2 \times \frac{400 \times 2}{49} \text{ sec}^2 = 0.71 \text{ meters, and}$$

$$d_2 = \frac{1}{2} a_2 t^2 = \frac{1}{2} \times \frac{1.4}{2.0} \text{ m/sec}^2 \times \frac{400 \times 2}{49} \text{ sec}^2 = 5.71 \text{ meters.)}$$

b) To find the velocity of each body at the instant they are 5.0 meters apart, knowing their accelerations and the time, we make use of the relation v = at.

$$v_1 = \frac{.70}{8.0} \times 4.0 = \frac{2.8}{8.0} = \underline{0.35 \text{ m/sec.}}$$

$$v_2 = .70 \times 4.0 = \underline{2.8 \text{ m/sec.}}$$

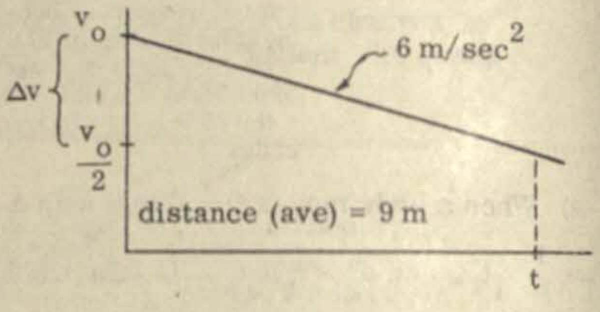
PROBLEM 18

A block of mass 3.0 kg is moving along a smooth horizontal surface with a speed v_0 at an instant of time $t = 0$. A force of 18 newtons is applied to this body opposite to the direction of its motion. This force reduces v_0 to one-half its value while the body moves 9.0 m.

(a) How long does it take for this to occur?

(b) What is v_0?

a) and b) A simple velocity-time graph of the motion of the 3.0-kg mass in slowing from v_o to $v_o/2$ during a time, t, will be helpful. We are given the acceleration and distance (the slope and the area of the graph). We must find v_o and t, the two linear dimensions. We know that $d = v_{ave}t$. It is clear from the graph that $v_{ave} = \frac{3}{4}v_o$. Hence, we can write $d = \frac{3}{4}v_o t$.



In $d = \frac{3}{4}v_o t$, we know d, the area. We will be able to solve this problem if we can express either of the dimensions, v_o or t, in terms of the other. What do we know? First we know that $\Delta v = v_o/2$. Second, we also know that $\Delta v = at$. Hence,

$$\frac{v_o}{2} = at = 6\,\text{m/sec}^2 \times t, \text{ and}$$

$$v_o = 12\,\text{m/sec}^2 \times t.$$

From $d = \frac{3}{4}v_o t$, $t = \frac{4d}{3v_o}$. Substituting,

$$t = \frac{4d}{3v_o} = \frac{4 \times 9\,\text{m}}{3 \times 12\,\text{m/sec}^2 \times t} = \left(\frac{36\ \text{m}}{36\,\text{m/sec}^2}\right)^{1/2} = \underline{1.0\ \text{sec}}.$$

Since $v_o = 12\,\text{m/sec}^2 \times t$, $v_o = \underline{12\,\text{m/sec}}$.

PROBLEM 19

If the distance covered by a moving object varies directly as the time, what conclusions could you draw about the motion and the forces?

If the distance covered varies directly as the time, this means that the increment in distance, Δx, is proportional to the increment in time, Δt. Expressed analytically; $\Delta x/\Delta t = \text{constant}$.

But $\Delta x/\Delta t$ is simply the speed of the object. Therefore, it must be moving with constant speed, and we may conclude that there is no net force acting in the direction of the motion. (A force might be acting perpendicular to the direction of motion but this would not affect the speed. With a constant perpendicular force, motion at constant speed in a circular path would result.)

PROBLEM 20

You observe an object covering distance in direct proportion to t^3, where t is the time elapsed.

(a) What conclusion might you draw about the acceleration? Is it constant? Increasing? Decreasing? Zero?

(b) What might you conclude about the forces? Be prepared to discuss in class.

a) If an object moves with constant speed, from $d = vt$, we know that $d \propto t$. If an object moves with constant acceleration, from $d = \frac{1}{2}at^2$, we know that $d \propto t^2$. Hence our given object cannot have zero acceleration or constant acceleration. Further, when $d \propto t^3$, d is increasing with t more rapidly than when $d \propto t^2$. Therefore the acceleration must be increasing with t.

What relation between a and t must hold for $d \propto t^3$? Compare the case of constant acceleration with our given case. If instead of $d \propto t^3$, we write $d \propto t \times t^2$, it may be a bit easier to see the relation.

$$\text{Given case: } d \propto t \times t^2.$$
$$\text{Constant accel: } d \propto a \times t^2.$$

For the case of: $d \propto t \times t^2$, if the acceleration is proportional to t instead of being constant, d will be proportional to t^3. Hence, if $d \propto t^3$, $\underline{a \propto t}$. The acceleration is increasing uniformly with time.

b) From $F = ma$, $F \propto a$. If $a \propto t$, $\underline{F \propto t}$. The accelerating force must be increasing uniformly with time.

* * *

Some students may require a more concrete approach to this problem. We can write $x = Kt^3$, and construct a table of time, distance covered, speed, change in speed and acceleration. To do this we must assume some arbitrary value for the constant, K, and choose an arbitrary time interval. For simplicity, we shall set $K = 1\ \text{m/sec}^3$ and $\Delta t = 1$ sec.

t (sec)	x (meters)	Δx	$v = \Delta x/\Delta t$ (m/sec)	Δv (m/sec)	$\Delta v/\Delta t$ m/sec^2
0	0				
		1	1		
1	1			6	6
		7	7		
2	8			12	12
		19	19		
3	27			18	18
		37	37		
4	64			24	24
		61	61		
5	125			30	30
		91	91		
6	216			36	36
		127	127		
7	343				

We see, by inspection, that the acceleration is increasing linearly with time.

PROBLEM 21

You pull a sliding disc with a constant force, starting from rest. The speed increases 0.10 m/sec in each Δt of 0.30 sec.

(a) How fast is the body moving after 1.2 sec?

(b) Now you begin pulling with the same force in the opposite direction. You continue pulling in the reverse direction for 0.90 sec. How fast is the body moving?

(c) In what direction is it moving?

a) Since the disc started from rest at t = 0, the instantaneous speed at any time, t seconds later, will be given by $v = at$.

$$v_{1.2} = at = \frac{\Delta v}{\Delta t} \times t = \frac{0.10\ \text{m/sec}}{0.30\ \text{sec}} \times 1.2\ \text{sec} = \underline{0.40\ \text{m/sec}}.$$

b) Applying the same force in the opposite direction produces a reversed (negative) acceleration. The change in speed (not the absolute speed since the object is not starting from rest) is given by:

$$\Delta v = at = -\frac{\Delta v}{\Delta t} \times t = -\frac{0.10 \text{ m/sec}}{0.30 \text{ sec}} \times 0.90 \text{ sec} = \underline{-0.30 \text{ m/sec.}}$$

The final resultant speed will be the vector sum of the initial speed at the time of the force reversal, and the change in speed that occurs during the 0.90 seconds thereafter:

$$v_f = v_{1.2} + \Delta v = 0.40 - 0.30 = \underline{+0.10 \text{ m/sec.}}$$

c) The positive sign of v_f indicates that the final speed is still in the same direction as the initial motion of the disc.

You might ask your class what the final speed would be if the disc was pulled forward for only 0.30 sec, the difference in time between the forward and backward pulls. They should see that the result would be the same. This approach can be used because the force is the same for both the forward and backward pull.

PROBLEM 22

A block of mass 2.0 kg is pulled on a frictionless table by a constant force of 6.0 newtons. The block starts from rest.

(a) What is the acceleration of the block expressed in meters/sec²?

(b) What is the speed of the block 3.0 sec after the force starts acting?

(c) How far does the block travel in 2.0 sec?

(d) If at the end of 3.0 sec, the block splits in two equal pieces — one piece still being pulled by the force of 6.0 newtons, and the other free — how far apart will the two pieces be 2.0 sec after the break occurs?

a) F = ma

$$a = \frac{6.0}{2.0} = \underline{3.0 \text{ meters/sec}^2}.$$

b) Since the block starts from rest, we have, after 3.0 seconds,

$$v = at = 3.0 \times 3.0 = \underline{9.0 \text{ m/sec}}.$$

c) Again, since the motion starts from rest, we have,

$$d = \frac{1}{2} at^2 = \frac{1}{2}\, 3.0 \times 2.0^2 = \underline{6.0 \text{ meters}}.$$

d) At the end of the first 3.0 seconds both pieces are moving with a speed of 9.0 m/sec. After two more seconds, the unaccelerated half will still be moving at this speed and will have traveled 9.0 × 2.0 = 18 meters further. The acceleration of the accelerated half will be $a = \frac{F}{m} = \frac{6.0}{1.0} = 6.0 \text{ m/sec}^2$; therefore its speed at the end of the two additional seconds will be $6.0 \text{ m/sec}^2 \times 2.0 \text{ sec} = 12 \text{ m/sec}$ greater than at the time of the split. Thus it will be moving at 9 + 12 = 21 m/sec. Its average speed during the 2.0 second period must have been $\frac{9 + 21}{2.0} = 15$ m/sec. Traveling at this average speed for two seconds, the accelerated half of the block must cover a distance of 30 meters in the two second interval. Then the distance between the two pieces will be 30 - 18 = 12 meters.

* * *

Part (d) can be solved more directly by recognizing that the difference in distance between the two pieces of blocks will be given by the relative acceleration × time. After the split, the acceleration of one block is $\frac{F}{m} = \frac{6.0 \text{ newtons}}{1.0 \text{ kg}} = 6.0 \text{ m/sec}^2$. The acceleration of the other block is zero. Hence, 6 m/sec^2 is the relative acceleration between the two blocks — it is the rate at which their relative velocity is changing. In 2.0 sec. this relative acceleration would result in a distance separation of:

$$d = \frac{1}{2}at^2 = \frac{1}{2} \times 6\,\text{m/sec}^2 \times (2.0\ \text{sec})^2 = \underline{12\ \text{meters}}.$$

PROBLEM 23

A block of mass 8.0 kg, starting from rest, is pulled along a horizontal table top by a constant force of 2.0 newtons. It is found that this body moves a distance of 3.0 m in 6.0 sec.

(a) What is the acceleration of the body?

(b) What is the ratio of the applied force to the mass?

(c) Since your answer to part (b) is not equal to that of part (a), (at least, it shouldn't be), what conclusions can you draw about this motion? Give numerical results, if possible.

a) The block starts from rest and is acted upon by a constant force. Assuming that the net force acting upon the block was constant, the acceleration will also be constant. Then the distance traveled will be:

$$d = \frac{1}{2}at^2, \quad \text{and } a = \frac{2d}{t^2}$$

$$a = \frac{2 \times 3.0\ \text{m}}{36\ \text{sec}^2} = \underline{\frac{1}{6}\ \text{m/sec}^2}.$$

b) Since F = 2.0 newtons, and m = 8.0 kg, $\frac{F}{m} = \frac{2.0}{8.0} = \underline{\frac{1}{4}\ \text{m/sec}^2}$.

c) The observed acceleration, $\frac{1}{6}$ m/sec^2, was less than the acceleration computed from the ratio $\frac{F}{m}$, $\frac{1}{4}$ m/sec^2. Therefore, some other force (retarding) must also have acted. It could be assumed to have been a friction force. In order to evaluate this force, we must assume that it also was constant. Then we can solve for the net effective force which must have been acting to produce the observed acceleration.

$$F_{\text{applied}} - F_{\text{friction}} = ma, \quad \text{and } F_{\text{friction}} = F_{\text{applied}} - ma$$

$$F_{\text{friction}} = 2 - (8 \times \tfrac{1}{6}) = \underline{\tfrac{2}{3}\ \text{newtons}}.$$

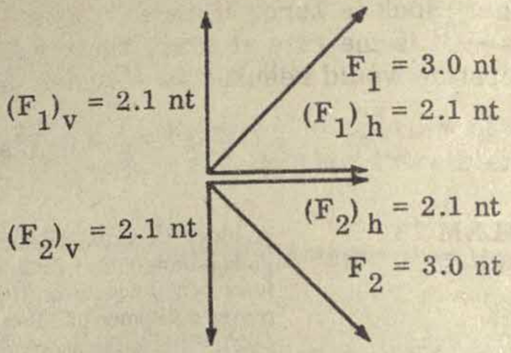
The force of friction is opposed to the motion of the block.

PROBLEM 24

A 6.0 kg object is acted upon by two forces of 3.0 newtons each.

(a) If these forces act at an angle of 90° with each other, what is the magnitude and the direction of the acceleration?

(b) What is the acceleration if the forces act in the same direction? In opposite directions?



a) Newton's law applies to the vector sum of all forces acting on the object. F_1 and F_2 may each be resolved into two components, one horizontal and one vertical. The horizontal components will both be forces directed toward the right. The vertical component of F_1 will be directed upward, whereas that of F_2 is directed downward. Since the angles are all 45°, the magnitude of each individual component is $\frac{3.0\text{ nt}}{\sqrt{2}} = \frac{3.0}{1.41} = 2.1$ nt. The net vertical force will be zero. The horizontal force will be $F_h = 2\frac{3.0\text{ nt}}{\sqrt{2}} = 4.2$ newtons to the right.

Therefore, $a = \frac{F}{m} = \frac{4.2}{6.0} = 0.70\text{ m/sec}^2$ along the bisector of the 90° angle between F_1 and F_2.

b) Since F = 6.0 newtons if the forces act in the same direction, $a = \frac{6.0}{6.0} = 1.0\text{ m/sec}^2$.

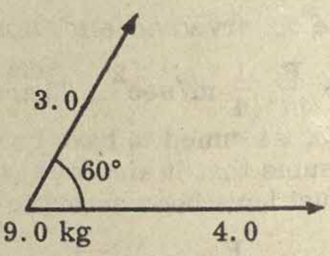
Since F = 0 if the forces act in opposite direction, $a = \frac{0}{6} = 0$.

PROBLEM 25 A 3.0-newton force and a 4.0-newton force act on a 9.0-kg mass. The two forces act at an angle of 60° with each other.

(a) If the object starts from rest, how fast will it be moving after 3.0 sec?

(b) In what direction will it be moving?

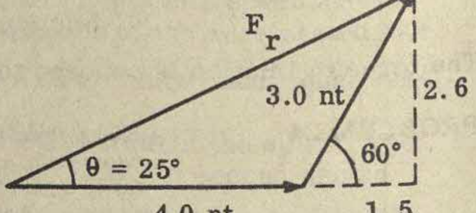
a) The first problem here is to obtain from the two forces which are given, one single resultant force of known magnitude and direction. This can be done graphically with protractor and ruler.



Such a construction gives $F_r = 6.1$ newtons directed at an angle θ = 25° from the 4.0 newton force. The force of 6.1 newtons will produce a constant acceleration of:

$$a = \frac{6.1}{9.0}\text{ m/sec}^2.$$

The resultant speed after 3.0 seconds will be $v = at = \frac{6.1}{9.0} \times 3.0 = 2.0\text{ m/sec}$.



b) This velocity will be in a direction between the 3.0 newton force and the 4.0 newton force, at an angle of 25° from the 4.0 newton force.

Note: This vector problem can also be solved analytically by making the construction indicated in the second figure. We can see that the 3 newton force has a component of $3.0 \times \cos 60° = 3.0 \times \frac{1}{2} = 1.5$ newtons in the direction of the 4 newton force (the x direction). Its perpendicular component is $3.0 \times \sin 60° = 3.0 \times .866 = 2.6$ newtons. Then the components of acceleration are

$$a_x = \frac{F_x}{m} = \frac{4.0 + 1.5}{9.0} = \frac{5.5}{9.0}\ \text{m/sec}^2,\ \text{and}$$

$$a_y = \frac{2.6}{9.0}\ \text{m/sec}^2.$$

After 3.0 seconds,

$$v_x = 3.0\ a_x = \frac{5.5}{3.0}\ \text{m/sec},$$

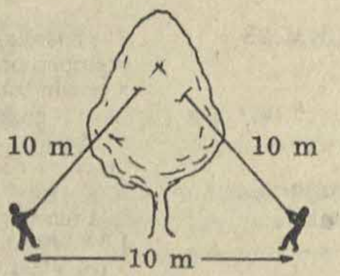
$$v_y = \frac{2.6}{3.0}\ \text{m/sec}$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \frac{1}{3.0}\ \sqrt{30.25 + 6.76} = \frac{1}{3.0}\ \sqrt{36.01} = \frac{6.0}{3.0} = \underline{2.0\ \text{m/sec.}}$$

PROBLEM 26

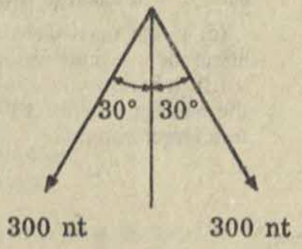
Two men wish to pull down a tree by means of a rope fastened near the top. If they use one rope, the tree will come down on top of them. To prevent this, they tie two ropes 10.0 meters long to the same point and stand on the ground 10.0 meters apart when they pull. If each pulls with a force of 300 newtons, what is the force exerted by the ropes on the tree?

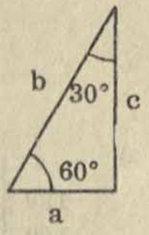
Since each rope is 10.0 m long and the two men stand 10.0 m apart, the three lines form an equilaterial triangle, and the angle between the ropes is equal to 60°.



The two forces are equal, so the resultant must be along the line which bisects the 60° angle. To find the component of each of the forces along this line, note that we have a 30°, 60°, and 90° triangle. The ratio of the sides a:b:c: = 1:2:√3. Therefore, $\frac{b}{c} = \frac{300}{F_r}$,

$$F_r = \frac{300\,c}{b} = \frac{300 \times \sqrt{3}}{2} = 260 \text{ newtons.}$$



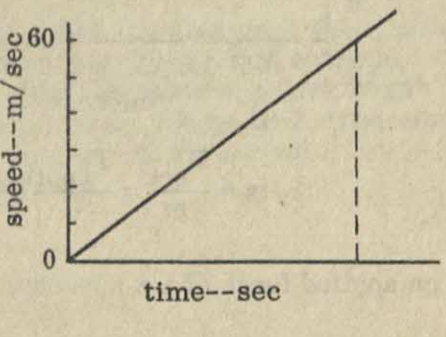


Each of the ropes pulling on the tree exerts a force which has a component of 260 newtons in the resultant direction. Thus, the force exerted by the ropes on the tree is 520 newtons.

PROBLEM 27

Find the applied force required to accelerate a 450-kg rocket from a standing start to a velocity of 60 m/sec along a 100-meter horizontal track. The retarding force of friction is 93 newtons.

In order to solve this problem we must assume that the thrust of the rocket is constant and therefore that the acceleration is constant. Then, on a speed – time graph, the speed will increase linearly from 0 to 60 meters/second, as shown at the right.



To find the time that is required for this acceleration, we note that the area of the triangle, the distance traveled, is given as 100 meters, and that during the time interval required to accelerate to 60 m/sec, the average speed is 30 m/sec.

$$d = v_{\text{average}}t,\ \text{and } t = \frac{d}{v_{\text{average}}} = \frac{100\ \text{m}}{30\ \text{m/sec}} = 3.33\ \textbf{seconds.}$$

Then, $F_{\text{net}}\,\Delta t = m\,\Delta v$, and $F_{\text{net}} = \frac{m\,\Delta v}{\Delta t} = \frac{450 \times 60}{3.33} = 8100$ newtons.

$$F_{\text{net}} = F_{\text{applied}} - F_{\text{friction}},\ \text{and } F_{\text{ap}} = F_n + F_f = 8100 + 93 = \underline{8200\ \text{newtons}}.$$

* * *

When this problem is discussed in class, you can point out that it is not necessary to solve for t. Without making a special point of it at this time, you can give some students a bit of a headstart on energy through seeing that F is proportional to $mv^2/2$.

$$F_{\text{net}}\,\Delta t = m\,\Delta v.$$

$$\text{But } d = v_{\text{ave}}t,\ \text{and } v_{\text{ave}} = \frac{\Delta v}{2}.\quad \text{Hence } \Delta t = \frac{2d}{\Delta v}.$$

$$\therefore\ F_{\text{net}} = \frac{mv^2}{2d}.$$

PROBLEM 28

The retarding force of air resistance on a balloon is proportional to the square of the velocity. For a certain balloon, inflated a certain amount, this force is given in newtons by $F_R = .2v^2$ where v is the velocity in m/sec. The balloon and the air inside have a combined mass of 10 gm.

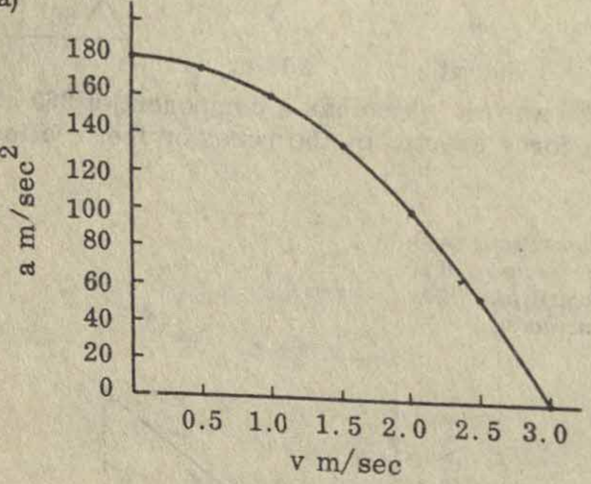
(a) Draw graphs of the balloon's acceleration as a function of velocity when you pull it with a 1.8-newton force and with a 7.2-newton force.

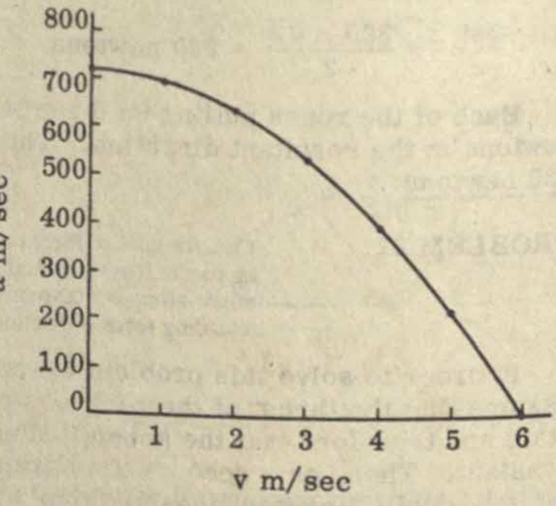
(b) What is the maximum velocity that the balloon will reach in each case?

(c) If the mass were 5.0 gm, how would this affect the maximum velocity?

(d) What do you think would be the effect on the maximum velocity if you inflated the balloon to a larger volume?

a)





$$a = \frac{F_{\text{net}}}{m} = \frac{F_{\text{applied}} - F_{\text{resistance}}}{m} = \frac{F_{\text{applied}} - .2v^2}{m}.$$

For an applied force of 1.8 newtons, $a = \frac{1.8 - 0.2v^2}{10^{-2}} = 180 - 20v^2\ \text{m/sec}^2.$

For a 7.2 newton force, $a = \frac{7.2 - .2v^2}{10^{-2}} = 720 - 20v^2 \text{ m/sec}^2$.

b) The maximum velocity is that velocity for which a = 0. In the first case, the balloon reaches a terminal velocity of 3.0 m/sec. In the second case, it reaches a velocity of 6.0 meters/sec.

c) A change in balloon mass would not affect the velocity. The resistive force of the air depends on the velocity only, not on the mass. Thus the resistive force will equal the applied force at some particular velocity, independent of the mass. This can be demonstrated analytically as follows:

$$a = \frac{F_{\text{applied}} - F_{\text{resistance}}}{m}$$

$$ma = F_{\text{applied}} - F_{\text{resistance}} = F_{\text{applied}} - 0.2v^2$$

To satisfy the maximum velocity condition we must set a = 0. Thus,

$$0 = F_{\text{applied}} - 0.2v^2_{\text{max}}$$

$$v^2_{\text{max}} = 5\,F_{\text{applied}}$$

$$v_{\text{max}} = \sqrt{5\,F_{\text{applied}}} \quad \text{(independent of m)}$$

Where v is in m/sec and F is in newtons.

d) If the balloon were larger, it would present a greater area to the air, and the force of air resistance would be larger at any given velocity. (This means that the coefficient of v^2 would be larger.) The result would be that the resistive force would become equal to the applied force at a lower velocity than before. Therefore v_{max} would be decreased.

PROBLEM 29

Aristotle taught that a constant force was required to produce a constant velocity and from this he concluded that, in the absence of force, bodies would come to rest.

(a) Name several situations where a constant force seems to produce a constant velocity.

(b) How do you explain each of the situations in (a) in the light of Newton's law of motion?

a) Situations in which a constant force seems to produce a constant velocity:

1. A horse pulls a wagon with a constant force at constant velocity.

2. A car attains a constant speed for any given gas pedal position (constant force).

3. An airplane flies at a constant air speed when its engine speed and propeller pitch remain constant.

4. The balloon of Problem 28 would move with a constant velocity if a constant force were applied to it.

b) In each example of this kind, careful inspection shows that, in addition to the applied force, there are other forces which oppose the applied force. In all cases of constant velocity, the applied force is balanced by opposing forces. [Usually these opposing forces are not constant but are proportional (but not directly) to v.] When the applied force exceeds the opposing forces at any given v, positive acceleration occurs. If the applied force is smaller than the opposing forces, negative acceleration results.

In the examples given in part a), when the object is moving with constant velocity, the sources of the forces which oppose and balance the applied force are:

1. For the horse and wagon, the friction of wheels on bearings and wheels on the road surface.

2. For the car, road and bearing friction and (especially at high speeds) air resistance.

3. For the airplane, air resistance.

4. For the balloon, air resistance.

PROBLEM 30 How would you define a unit of mass if people had placed a standard spring at Sèvres instead of a standard mass?

Since this is a chapter on dynamics, our first thought might be of a dynamical method of defining a mass unit.

A direct way to establish a unit of mass would be to perform an acceleration experiment. The unit of mass would be the mass of an object which would be accelerated at a specified rate by application of the standard force (and that force only). While the condition that no other force act on the body is a very difficult one to satisfy, the "frictionless" pucks described and used in the films and text could be employed in such a way that the net force is given by the spring and nothing else.

A practical experimentalist, knowing the equivalence of gravitational and inertial mass might simply hang a mass from the spring. He would have to recognize that the result would be sensitive to his position relative to other massive objects. Consequently his position on the earth would have to be standardized. For great accuracy, his position relative to the sun and moon also would have to be standardized. He would also have to recognize that an appreciable change in the configuration of the earth would destroy his standard. Still, this would be a more practical standard than that determined by an acceleration experiment.

A more sophisticated procedure would be to define the mass standard in terms of a frequency of oscillation of that standard when set into motion hanging from the standard spring, in a vacuum. The frequency of an oscillating mass varies inversely as the square root of the oscillating mass. This method would be by far the most practical. Furthermore, it is a direct measure of inertial mass.

While most students will not be familiar with simple harmonic motion at this point in the course, this type of answer may be suggested by the students' experience with the inertial balance (Experiment I-1). A brief discussion of this idea can make it seem reasonable without going into an analysis of simple harmonic motion.

Chapter 21 – Motion at the Earth's Surface

PURPOSE 1. To generalize the study of dynamics to motion in more than one dimension, starting the exploration of forces in nature and giving evidence supporting the validity of Newton's law as a vector law.

2. To use Newton's law in some simple situations which, however, are more complicated than produced by a constant force acting on an object confined to move on a straight line. The situations considered are:

a. A constant force which is not in the direction of motion (Sections 3 and 4).

b. A force constant in magnitude but changing direction so as to be perpendicular to the velocity at all times (Sections 5 through 7).

c. A force confined to a single direction but varying in magnitude and changing direction (i.e., algebraic sign). (Section 8)

3. To build up background for the dynamical consideration of planetary motions in Chapter 22.

CONTENT

Section 1 discusses weight and mass. On the earth the weight of any mass is about 10 nt/kg.

Section 2 treats free fall with no horizontal component of velocity. This motion is due to a constant force in the direction of motion and hence is an extension of the material in Chapter 20. It is given special emphasis because free fall is one component of the projectile motion to be discussed as the first vector example.

Section 3 and 4 deal with projectile motion. The key idea is that the components of motion can be treated separately:

$$x = v_{ox}t$$

$$y = v_{oy}t - \frac{1}{2}gt^2.$$

Sections 5 through 7 discuss circular motion. The velocity is $v = \frac{2\pi R}{T}$. The centripetal acceleration is: $a = \frac{2\pi v}{T}$ which can also be written as $\vec{a} = -\left(\frac{2\pi}{T}\right)^2 \vec{R}$, or $a = \frac{v^2}{R}$.

Applying Newton's law to this derived acceleration predicts that a centripetal force must act in order to produce uniform circular motion.

Section 8 discusses simple harmonic motion by relating it to a projection of circular motion. It stresses the force producing simple harmonic motion.

Sections 9 through 11 deal with the importance of using an unaccelerated coordinate system in applying $\vec{F} = m\vec{a}$.

EMPHASIS This chapter has two very important ideas. One is that Newton's law is a vector law. This is verified for projectile and circular motion.

The second idea is that Newton's law can be used either:

a. to predict motion if the forces are known or,

b. to find the forces if the motion is known.

As you cover this chapter, these points will be emphasized alternately. Verification of the vector nature of Newton's law will be the subject of those discussions centered about the developments in the text. The use of Newton's law to find the motion or the force is the subject of the problems. Students should find both these aspects of the material stimulating. In this, the familiarity of the material arouses much interest.

As they progress from free fall, to projectiles, to circular motion, to simple harmonic motion, students should be reminded of the wide variety of situations in which

Newton's law applies. When you finish the chapter, it may be worth while as a review to list the characteristics of the forces and motions which have been studied:

Type of Motion	Magnitude of Force	Direction of Force	Motion
Free Fall	constant	constant in line of motion	uniform acceleration
Projectile	constant	constant but not in line of motion	parabolic, uniform acceleration
Circular Motion	constant	varying to be perpendicular to motion	uniform circular motion
Simple Harmonic Motion	proportional to displacement	restoring, i.e., opposite to displacement	oscillatory

In dealing with projectile motion the independence of the components of force should be emphasized. This is the key to the vector nature of the force. One reason for stressing free fall and doing practice problems is to get students to appreciate the great simplification which is possible when the components are separated. Taking components is also essential in relating simple harmonic motion to circular motion.

In discussing circular motion, it will be wise to take time for a kinematic development of centripetal acceleration; students should themselves determine this acceleration graphically before they apply Newton's law to circular motion.

SCHEDULING CHAPTER 21

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Sec. 1	1/2	0	-	1/2	0	-
Sec. 2	2	0	-	1	0	-
Secs. 3, 4	2 1/2	1	III-5	2	0	-
Secs. 5, 6, 7	5	1	III-6	3	1	III-6
Sec. 8	3	0	-	1	0	-
Secs. 9, 10, 11	1	0	-	1/2	0	-

RELATED MATERIALS FOR CHAPTER 21

Laboratory. Experiment III-5, Forces on a Ball in Flight, involves analysis of velocity changes from flash photographs. The experiment can be done after Section 4.

The yellow pages include suggestions on handling these experiments.

Home, Desk and Lab. Analyzing and solving problems in class will be especially useful in connection with Chapter 21. Students should be encouraged to sketch and roughly graph the force relationships and motions involved.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Problems 17 and 18 are probably more easily handled after students have gained a little maturity with the ideas of the chapter than if handled with Section 2.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1*, 2			1*, 2	
2	3	4, 5, 8*, 7, 10*, 17, 18	6, 9, 11	4, 5, 6, 7, 8*, 9, 10*, 11, 17, 18	5
3, 4	12*, 15*	13, 14, 16		12*, 13, 14, 15*	12
5, 6, 7	20, 21*, 22*	19, 23, 24*, 25, 26, 27*		19, 21*, 22*, 23, 24*, 25, 26, 27*	
8	31	28*, 29, 30		29, 30	28
9, 10, 11			32	32	

Films: " Falling Bodies", by N.H. Frank of the Massachusetts Institute of Technology. From observation of the constant acceleration of a falling body and F = ma, the proportionality of gravitational and inertial mass is shown. For objects in free fall, the independence of the vertical and horizontal components of the motion is considered, then is demonstrated with the monkey-and-gun experiment. This film can be used with Sections 2, 3, and 4. Running time: 30 minutes.

"Deflecting Forces", by N.H. Frank of the Massachusetts Institute of Technology, shows how a force acting at a right angle to the direction of motion changes the direction of motion but not the speed. A constant deflecting force (perpendicular to the direction of motion) results in circular motion with uniform speed. Using vectors, $F = \frac{mv^2}{R}$ is derived from F = ma. The same magnitude of force is used to produce accelerations in both circular and straight line motion. The equality of the magnitude of the accelerations shows the validity of Newton's law in vector form. This film is related to Section 5. Running time: 29 minutes.

"Periodic Motion", by Donald Ivey and Patterson Hume of the University of Toronto. An object in simple harmonic motion is used to trace a displacement vs. time graph. Analysis of this graph shows that $\vec{a} \propto -\vec{x}$; from Newton's law it follows that $\vec{F} \propto -k\vec{x}$. This is verified by an experiment. The relationship between simple harmonic motion and the vertical component of uniform circular motion is observed and analyzed, leading to the conclusion that the period for simple harmonic motion is given by $T = 2\pi\sqrt{m/k}$, where the motion results from a force whose force constant is k. This relationship is then tested and confirmed by experiment. This film is for use with Section 8. Running time: 30 minutes.

"Frames of Reference", by Donald Ivey and Patterson Hume of the University of Toronto, demonstrates how different frames of reference lead to different descriptions of motion. The motion of a ball which falls from a moving cart, when the cart moves first with constant speed then under constant acceleration, is observed both from a reference frame which moves with the cart and from a fixed frame. These observations are used to distinguish between inertial and non-inertial frames of reference, and to show how "fictitious" forces arise in a non-inertial frame. Experiments with a dry-ice puck on a rotating turntable show the fictitious nature of centrifugal force in a rotating frame of reference. The film closes with a brief study of the evidence provided by the Foucault pendulum for the rotation of the earth in a Newtonian frame of reference. This film can be used to good advantage with Sections 9, 10 and 11. Running time: 26 minutes.

Section 1 – Weight and the Gravitational Field of the Earth

PURPOSE To distinguish between mass and weight.

CONTENT a. The weight of an object is the magnitude of the gravitational force on that object.

b. While mass is independent of position, the weight per unit mass varies from place to place, but at any one place weights are proportional to the masses.

c. The weight of an object at the surface of the earth is always very close to 9.8 nt/kg, but the exact value of this proportionality factor changes slightly with position on the earth.

d. The gravitational force per kilogram at a given place defines the value of the gravitational field, $\vec{g}$, at that place. The total gravitational field is the pattern of such values of $\vec{g}$ throughout space.

EMPHASIS These ideas are important but should not require much class time.

DEVELOPMENT Problems 1 and 2 can be done in class. Arithmetic can be simplified by using $g \approx 10$ nt/kg, as suggested in the green pages for Problem 1, after students understand that $\vec{g}$ varies slightly at different places. To get a feeling for the magnitude of a newton, students can compute their own approximate weights in newtons; have them get the weight from their masses expressed in kilograms.

COMMENTS Defer discussion of why $\vec{g}$ varies from one place to another until the next chapter, which deals with the law of universal gravitation.

Perhaps no single subject in physics has been confused by more students than the distinction between weight and mass. It is important that students clearly understand the distinction between these quite different properties (see Chapters 7 and 20).

Since the weight of an object is the force, F, exerted by gravity on the object's mass, m, it is most convenient, when thinking in these terms, to express $\vec{g}$ as 9.8 newtons/kg. On the other hand, when dealing with acceleration in a free fall problem, $\vec{g}$ is most simply considered in the equivalent units $9.8\,\text{m/sec}^2$. At this point in the chapter the 9.8 nt/kg form should be used.

In this section the concept of a force field is introduced. While a brief discussion of what is meant by a field is certainly in order, the idea does not require emphasis at this point. This subject will be further explored in the next chapter and in Part IV.

Section 2 – Free Fall

PURPOSE To study vertical motion at the earth's surface as a simple application of Newton's law.

CONTENT a. In the presence of a gravitational force alone, an object undergoes a constant acceleration $a = -9.8\,\text{m/sec}^2$, down.

b. For objects moving in the atmosphere, air resistance is often negligible; but as speed increases, air resistance increases. Eventually, at some specific downward speed (the terminal velocity), the resistive air force equals the gravitational force, and acceleration ceases.

EMPHASIS This material is very important. It shows how motion can be determined when a constant force acts along the same line as the motion. The time you should spend on this now depends on how many similar problems you treated in Chapter 20 and the degree to which your class needs a review of kinematics. They should be adept in handling free fall problems before they encounter the additional complications of projectile motion.

FILM "Falling Bodies" by N. H. Frank, (see description in chapter summary) can be used to help pull together the ideas in Sections 2, 3, and 4. The film is probably best scheduled near the end of class work on these sections

DEVELOPMENT Both in discussing the text and in solving problems, students should be reminded of the three distinct steps involved in determining the motion which results from the action of one or more known forces:

1. Determine the net force, the vector sum of all the forces acting.
2. Using Newton's law, find the acceleration.
3. With this known acceleration a find the motion by applying the kinematic equations.

This separation is particularly worthwhile because it stresses the tremendously wide applicability of Newton's law. Sometimes the forces may be complex; indeed the steps are often reversed so that the force is found from the motion.

In illustrating these techniques in this section, the force is simplified by making the approximation that the air resistance, F_a, is negligible. Further, one assumes that g is constant over the region in which the experiment is performed. (A more detailed treatment of the gravitational field will be given in Chapter 22.)

All compact, dense objects are observed to undergo the same acceleration when falling in air, and all objects fall with the same acceleration in a vacuum. Newton's law relates acceleration to force, $F = m_i a$, where m_i is the inertial mass of the object. But if F is the force of gravity on the object, by definition $F = km_g$ where m_g is the object's gravitational mass. Then $a = k\frac{m_g}{m_i}$, and since a is observed to be a constant (g) for all objects, $\frac{m_g}{m_i}$ must also be constant for all objects. Thus the observation of equal accelerations, under gravity, for all objects, is verification of the equivalence of gravitational and inertial mass. In practice, we set the constant $k = g$, and then the proportionality of m_g and m_1 becomes a numerical equality.

In finding the motion from the known acceleration, the cookbook approach of formula substitution should be avoided. Each new situation should be treated individually, drawing a schematic diagram, selecting appropriate sign conventions and graphing both a and d as a function of t. Figure 21-2 and Table II are a good reminder of the relation between motion and acceleration.

Problems 8 and 10 are particularly recommended. A qualitative discussion of Problem 9 provides a good example of the utility of graphs in understanding a situation. Problems 17 and 18 can be done at this point although they are more difficult and might be saved for review purpose.

Note: The question asked about terminal velocity in the text (page 327) turns up again as part (c) of Problem 5. When an object is moving down with a speed greater than its terminal velocity, it slows down because the retarding force due to the air exceeds the accelerating force due to gravity.

DEMONSTRATION – Constancy of Acceleration

The PSSC film on free fall begins with a convincing demonstration of the equality of the acceleration of all objects, light or heavy, when they fall freely under the influence of only the force of gravity. If you do not use the film, you can show this demonstration yourself in class. It goes as follows:

First take a book and a sheet of paper (a little smaller in area than the book); hold them side by side and drop them simultaneously. The force of air resistance is comparable to the gravitational force on the paper, while it is much smaller than the larger gravitational force on the book. Hence, the paper soon has a smaller net force acting on it and falls more slowly. (The paper quickly reaches its terminal velocity whereas the book continues to accelerate.)

Ask for suggestions for explaining the lack of constancy of the acceleration and for making the paper fall faster. Two common suggestions involve dropping the paper either after holding it edge down or after having crumpled it. Try these.

If no one suggests it ask about holding the paper above or below the book. In either case they will fall together. Putting the book above the paper is not convincing because the book obviously pushes the paper down. But when the paper is placed on the book (without protruding beyond the edges), the book merely shields the paper from the resistive air force. The two will drop together with the paper remaining on top of the book throughout the fall.

DEMONSTRATION – A Mechanical Graph of $y = -\frac{1}{2} gt^2$.

(Particularly useful later in showing projectile motion and trajectories.)

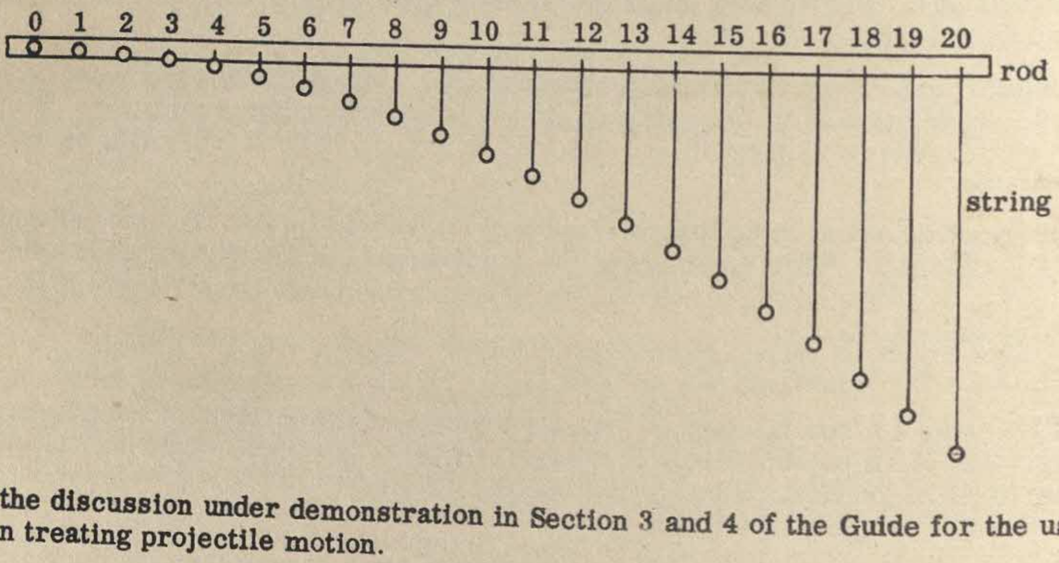
Use a long rigid rod as the t axis; for this graph the rod should be supported in a horizontal position. The values of y will be shown by lengths of string or thread supported at equal intervals. Arrange to fix the support point of each thread by using a notch or a hole in the rod. (In later applications, this rod must be tilted and the threads will tend to slip along the rod if they are not firmly positioned.) Each string should be cut to a length corresponding to the y value appropriate to the t value at which the string will be hung. Attach small weights (such as washers or nuts) to the ends of the strings to keep them vertical and to make the ends more easily visible.

At least ten strings should be used; there is little point in using more than 20.

$t = 0$	1	2	3	4	5	6	7	8	9	10
$-y = 0$	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	1.00

$t = 11$	12	13	14	15	16	17	18	19	20
$-y = 1.21$	1.44	1.69	1.96	2.25	2.56	2.89	3.24	3.61	4.00

If the y values are in meters, each unit of t is actually 1/22.2 seconds.



See the discussion under demonstration in Section 3 and 4 of the Guide for the uses of this in treating projectile motion.

Section 3 – Projectile Motion: The Vector Nature of Newton's Law of Motion
Section 4 – Projectile Motion: Determination of the Path

PURPOSE An investigation of the vector nature of Newton's law, by considering general motion of a particle under gravity. (An extension of Section 2 to the case where the motion has a horizontal as well as a vertical component.)

CONTENT a. In free fall, vertical motion is observed to be independent of horizontal motion.

b. This is consistent with viewing Newton's law as a vector relationship, $\vec{F} = m\vec{a}$

c. For an object projected with an initial velocity v_o in the horizontal direction

$$x = v_o t$$
$$y = -\frac{1}{2} gt^2.$$

EMPHASIS This is important material which should be covered in detail. Most students find it interesting and can master the problems if they are given enough practice.

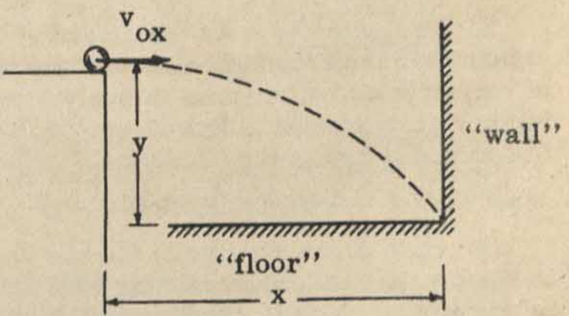
LABORATORY Experiment III-5, Forces on a Ball in Flight, can be done after these sections are completed. See the yellow pages.

FILM "Falling Bodies" can be used in connection with these sections.

DEVELOPMENT The independence of the vertical and horizontal motions should be stressed. This is the essence of the vector nature of Newton's law; each component of the motion obeys Newton's law. First, be sure that students observe this independence. Then, be sure that they make use of this independence by solving some problems, as suggested below.

After demonstrating the fact that the falling time is independent of the horizontal velocity, the equations describing each component of the motion (as on page 328) can be developed in class. Discussion can be based on the motion shown in Figures 21-4 and 21-5, (pages 327-8). (A description of the trajectory or the derivation of an equation for y in terms of x should be deferred temporarily.)

Ask the students for an equation to describe the y coordinate of a freely falling body dropped vertically. After writing the answer, $y = -1/2\,gt^2 = -4.9t^2$ (or $y = -5t^2$), on the board, ask for the equation for y if $v_{xo} = 10$ m/sec. Most students will realize quickly that $y = -4.9\,t^2$ applies, independent of v_x; but this should be covered in class so that it is emphasized for all students. Ask for an equation for the horizontal motion and write the answer, $x = v_{ox}t$, on the board. (You may want to use some simple value of v_{ox} such as 2 m/sec, 10 m/sec, etc.)



Give the class some simple practice problems which are designed to have easy solutions if the components of the motion are separated. A suggested sequence of problem types is given below. In each case an object leaves a table with a horizontal velocity, v_{ox}, at $t = 0$. In early problems the initial vertical velocity should be taken as zero.

Making the numbers simple so that the arithmetic is easy will help students concentrate on the nature of the motion. (Using $y = -5t^2$ is most helpful for this simplification.)

1. The very simplest problems are one-step problems involving the elapsed time, t.

(a) Give two of the quantities in $x = v_{ox}t$ and ask for the third.

(b) Give one of the unknowns in $y = -4.9\,t^2$ and ask for the other.

2. Give two of the three quantities v_{ox}, x, and y, and ask for the third.

(a) Begin by asking first for t, and the other unknown next. For example:

If $x = 20$ m, and $v_{ox} = 2$ m/sec, what is t? What is y?

If $y = -4.9$ m and $x = 10$ m, what is t? What is v_{ox}?

If $y = -4.9$ m and $v_{ox} = 3$ m/sec, what is t? What is x?

(b) Later you can give two of the three quantities, v_{ox}, x, and y, and ask for the third without explicitly asking for t. However, students should get into the habit of finding t as an intermediate step. If y is given, t can be found immediately. If x and v_{ox} are given, t can be found.

3. You may want to extend these problems to include an initial vertical velocity.

In choosing problems it should be remembered that the most important aspect of projectile motion is the independence of the components. The best way to learn this independence is to use it often in solving problems. This independence deserves much more attention than does the trajectory of the motion.

Problems such as Problem 15 in HDL can be done after this type of introductory exercise.

DEMONSTRATIONS 1. Independence of Vertical and Horizontal Motion. It will be helpful to use one or more demonstrations of the fact that two identical objects will fall together independent of their horizontal motion. Begin by asking the students what they expect. Many students expect intuitively that the object with horizontal velocity takes longer to fall because "it has farther to go". The equal fall times can be demonstrated easily and effectively; such demonstrations can be used to introduce a detailed development of the analytic consequence of the vector nature of Newton's law of motion.

Problem 12 suggests that the students try to observe this with marbles. It should be assigned at some stage. Almost any pair of objects can be used. Start by using common objects rather than a piece of "demonstration equipment". The film "Falling Bodies" and the solution to Problem 15 in the green pages show how coins can be used. More elaborate apparatus demonstrating independence of fall time can be used after you have shown the simpler example. The PSSC film "Falling Bodies" includes some good examples.

2. Trajectories. As mentioned above, independence of the motion of the components rather than the trajectories should be stressed. For this reason, a graphical demonstration of trajectories rather than an analytic treatment is useful. This particular demonstration further stresses the independence of the components of the motion. The equipment is simple and is described in the Guide for Section 2 of this chapter.

a. Initial Horizontal Motion.

(1) Hold a rod similar to the one on which the strings are attached and ask a student to show where an object moving with uniform horizontal velocity (no vertical motion) would be at different times. Mark equal distance intervals which correspond to equal time intervals.

(2) For a few of these distances ask the student to indicate what the y position would be if the object were in free fall.

(3) Call the students' attention to the rod with the suspended strings. Make the point that if gravity were not present all the strings would have zero length. Suspending the strings is, in a sense, like turning on the effect of gravity.

b. Initial Motion at an Angle.

(1) What would the path of a particle be if it were shot at some angle (if there were no gravity) with the same speed as used above? This could be shown merely by pointing the

marked rod (without strings) in the appropriate direction.

(2) What would the path be if gravity is present? Students should realize that the strings take free fall into account (for the particular initial velocity along the rod). This is true whether the rod is horizontal, at an angle, or even vertical.

c. Solving Problems. Once students realize that a line connecting the weights at the ends of the strings would represent the trajectory, you can use the rod and suspended strings to do problems. The number of strings is a measure of the time. The slope of the trajectory (as in Problems 13 and 16) gives the ratio of the velocities. You will find this rod and its strings particularly useful for showing qualitatively how the trajectory changes as the initial angle changes. In particular, it will be easy to see that the maximum range is achieved at 45°.

SUPPLEMENTARY INFORMATION 1. Trajectory as a function of firing angle, θ. You will not want to develop the equations appropriate to different initial angles θ in class even though these are not very complicated. However, you may want the following formulas to help answer quickly questions which may arise.

In terms of θ, $v_{ox} = v \cos \theta$, $v_{oy} = v \sin \theta$. Then, $x = v (\cos \theta)t$, $y = v (\sin \theta) t - \frac{1}{2} gt^2$. In order to find the distance traveled in a horizontal direction when the object returns to the ground, we find the time, T, at which y is 0. Factoring the equation for y we have

$$y = t\ [v \sin \theta - \frac{1}{2} gt].$$

Hence $y = 0$ for $t = 0$, and for $t = T$ which is given by:

$$v \sin \theta - \frac{1}{2} gT = 0, \text{ or } T = \frac{2v \sin \theta}{g}.$$

At this time the distance X traveled is:

$$X = v (\cos \theta) T = \frac{2v^2 (\sin \theta) (\cos \theta) T}{g}.$$

This expression can be put in a simpler form by remembering that $\sin 2\theta = 2 \sin \theta \cos \theta$. Using this,

$$X = \frac{v^2}{g} \sin 2\theta.$$

To see how X varies for a fixed v, we need only examine the values of $\sin 2\theta$. This has a maximum value of 1 for $2\theta = 90°$ or $\theta = 45°$.

2. Complications in Treating Air Resistance. Equations of motion become excessively complicated if air resistance is taken into account; in fact the problems must be solved numerically rather than by using simple formulas. (For this reason, one finds ballistics tables rather than simple formulas.)

An idea of the complication can be obtained by considering the simplifying approximation that the force due to air resistance is proportional to the square of the total velocity. (This is a rather good approximation for ping pong balls or bullets.)

The total air resistance force can then be written as $F_a = -kv^2$ where k is a constant. This force is directed in the $-\vec{v}$ direction.

The horizontal and vertical forces are then given by:

$$F_{ax} = F_a \frac{v_x}{v} = -kv^2 \frac{v_x}{v} = -kv\ v_x, \text{ and}$$

$$F_{ay} = F_a \frac{v_y}{v} = kv^2 \frac{v_y}{v} = -kv\, v_y .$$

Since $v = \sqrt{v_x^2 + v_y^2}$, F_x is no longer independent of v_y, and F_y is no longer independent of v_x. Therefore, the acceleration in the x direction depends on the motion in the y direction, and the acceleration in the y direction depends on the motion in the x direction.

Section 5 - Deflecting Forces and Circular Motion
Section 6 - Earth Satellites
Section 7 - The Moon's Motion

PURPOSE To apply Newton's law to the case in which the force is constant in magnitude, but acts in a direction which is always perpendicular to the instantaneous motion, thus giving rise to uniform circular motion.

CONTENT a. A force perpendicular to the motion leads to a perpendicular acceleration which changes the direction of the velocity without changing its magnitude.

b. A force of constant magnitude, acting in a direction perpendicular to the motion, produces uniform circular motion.

c. The kinematic equations for uniform circular motion are:

$$v = \frac{2\pi R}{T};$$

$$a = \frac{2\pi v}{T} = \frac{4\pi^2 R}{T^2} = \frac{v^2}{R}.$$

d. Centripetal force, F, is

$$F = ma = \frac{m4\pi^2 R}{T^2} = \frac{mv^2}{R}.$$

e. An application of the above relations to calculating the period and velocity of an artificial satellite, and the centripetal acceleration of the moon.

EMPHASIS Traditionally this material is very difficult to master. Be prepared to spend about three class periods developing the ideas that are involved.

LABORATORY Experiment III-6 should be done before the derivation of the formulas relating to centripetal acceleration and force.

FILM Class discussion of these sections can be organized around the film "Deflecting Forces",

DEVELOPMENT There are three important stages in the understanding of this material; they should be developed independently even though the first two are closely related.

1. The first stage involves an appreciation of the qualitative idea that an acceleration which occurs in the direction perpendicular to the instantaneous velocity does not change the magnitude of the velocity. This idea is frequently a difficult one for students and some detailed discussion or review is usually in order. The development rests on the idea that vector acceleration is defined as rate of change of vector velocity. The magnitude of the velocity vector may remain unchanged.

2. The second stage involves detailed development (or review) of the kinematics of circular motion. Graphical work on velocity and acceleration will be needed to give students a basic understanding of the direction and magnitude of centripetal acceleration. The

final formulas that are derived are important, but formulas alone are not enough; their origin should be understood.

3. Only at the third stage of understanding do we pass from kinematics to dynamics. Understanding the acceleration, and knowing Newton's law, we are prepared to investigate the forces that are involved in circular motion.

* * *

The development of kinematics for circular motion requires graphical work by the students. Some of this can be done in class, but some should be assigned for the student to work out by himself at home. Suggestions for the treatment of the kinematic parts of this development are to be found in Appendix 1 of the Guide for Part 1. Further suggestions will be found in Appendix 1 of this Guide for Part III of the text.

* * *

The two similar equations for circular motion should be stressed:

$$v = \frac{2\pi R}{T}, \text{ and}$$

$$a = \frac{2\pi v}{T}.$$

This will remind the students of the derivation of the expression for a. Initially, it may be wise to use $a = \frac{2\pi v}{T}$ as the first step in doing a problem, rederiving $a = \left(\frac{2\pi}{T}\right)^2 R$ and $a = \frac{v^2}{R}$ when you need them. After a while, the students will remember $a = \frac{v^2}{R}$ and use it directly. The form $a = \left(\frac{2\pi}{T}\right)^2 R$ is particularly useful in simple harmonic motion. [Note that the vectors $\vec{a}$ and $\vec{R}$ point in opposite directions. Thus if we write the last equation in vector form, a minus sign must be used:

$$\vec{a} = -\left(\frac{2\pi}{T}\right)^2 \vec{R}.]$$

* * *

Newton's law implies that whenever an object moves in a curved path there must be a component of force perpendicular to the motion. To emphasize this ask the students to identify the centripetal force in common situations.

For example, what force makes a car go around a curve? The students should realize that this must be an external force exerted on the car. Hence it must be the force of the road on the tires; this is a frictional force on a flat road (and may be a normal force on a banked road).

What forces keep the passengers going in the curve along with the car? These forces are external to the passenger (but, of course, come from parts of the car). They include the forces exerted on the feet by the floor, on the body by the seat, and often on the body by the door or adjacent passengers.

* * *

Motion without centripetal force. Many students must be reminded that when the centripetal force is removed, as in Figure 21-11, the subsequent motion is linear and tangent to the circle. Intuition sometimes fools one here because one may suppose that if the force pulling toward the center is removed, the object will move radially outward. Ask students about this, preferably in conjunction with a demonstration.

* * *

Section 8 – Simple Harmonic Motion

PURPOSE To use Newton's Law to predict the motion when a slightly more complicated force is acting: a force which is always directed along the same straight line but varies in magnitude and sense.

CONTENT a. When an object is moved slightly from its equilibrium position, there is often a restoring force proportional to the displacement: $F = -kx$.

b. The component of the centripetal force along any diameter satisfies the same equation $F_x = ma_x = -m\left(\frac{2\pi}{T}\right)^2 x$, where x is the distance along the diameter from the center.

c. Because of the vector nature of Newton's law, the motion produced by $F = -kx$ must be the same as the projection of circular motion along the diameter.

d. From these equations, $k = m\left(\frac{2\pi}{T}\right)^2$, or $T = 2\pi\sqrt{\frac{m}{k}}$.

e. If a pendulum is displaced through a small angle, it oscillates with simple harmonic motion, and $T = 2\pi\sqrt{\frac{\ell}{g}}$.

EMPHASIS Unless you intend to use more that 9 weeks for Part III, you will not be able to treat simple harmonic motion in detail. With a limited time you might use only the film, "Periodic Motion", or only demonstrations without going over the arguments in the text to show the relationship between circular and simple harmonic motion. You could also use a demonstration to verify the prediction of the formulas that T is independent of the amplitude. With such demonstrations, solving one or two sample problems could complete a minimal treatment of this topic.

It would be worthwhile to mention the fact that Newton's law holds for all forces and that this particular force is chosen because it is a common case applying to small vibrations in general.

If you are crowded for time you can omit this section completely without sacrifice of background for the work which follows.

FILM "Periodic Motion" is a valuable resource in teaching simple harmonic motion, for either a minimal or normal development.

COMMENTS Appendix 2 to Part 3 of the Guide presents a detailed stepwise arithmetic treatment of a problem of simple harmonic motion.

Appendix 3 gives an exact treatment of the problem via the solution of the differential equation of motion.

DEMONSTRATION You can show that the projection of circular motion is the same as simple harmonic motion. Mounting a turntable in the vertical rather than the horizontal plane enables a comparison of the projection of circular motion with a mass oscillating vertically on a spring. If you have a relatively frictionless setup, you might be able to use a mass oscillating horizontally. This is comparable to the demonstrations in the film "Periodic Motion".

If you do not have a standard demonstration of this type, you may want to use the type of apparatus described below.

Simple harmonic motion can be demonstrated by observing the shadow of an object moving in a circle at constant speed. The shadow will move with simple harmonic motion back and forth on a screen. A comparison of the motion of a pendulum with the motion of the shadow of the object moving in a circle shows that a pendulum with small amplitudes also performs simple harmonic motion. The apparatus in Figure (a) can be used to demonstrate this.

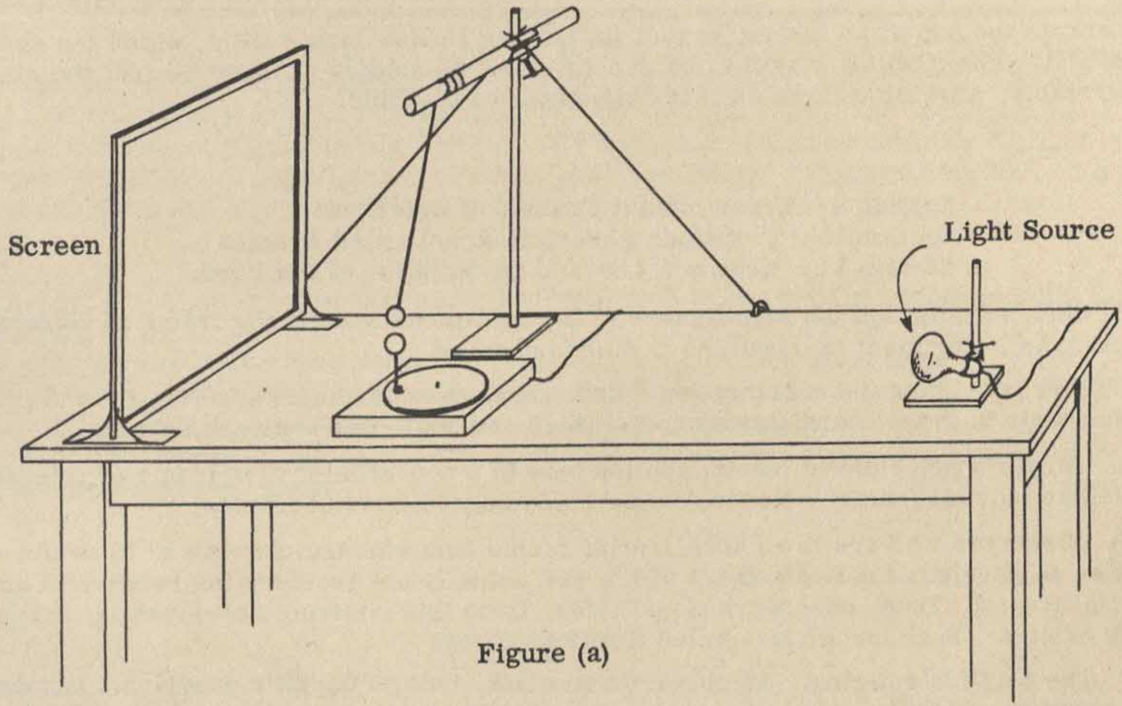


Figure (a)

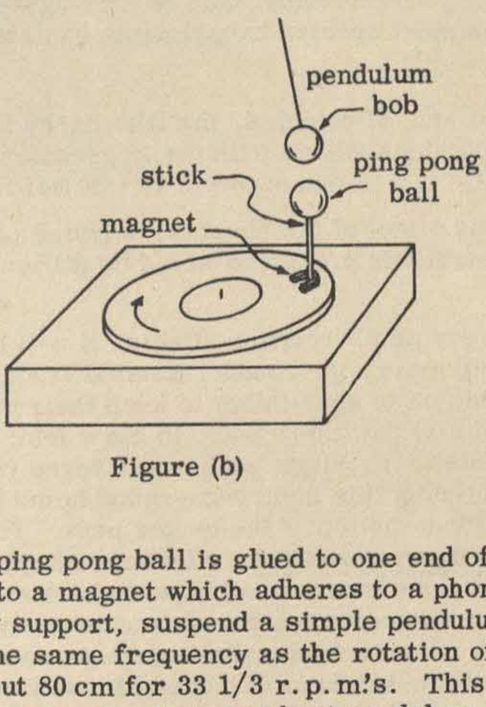


Figure (b)

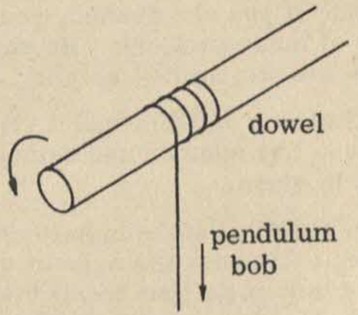


Figure (c)

A ping pong ball is glued to one end of a thin stick while the other end of the stick is taped to a magnet which adheres to a phonograph turntable (Figure b). From a tall, sturdy support, suspend a simple pendulum with its length adjusted so that it will swing with the same frequency as the rotation of the turntable. The length of the pendulum should be about 80 cm for 33 1/3 r.p.m.'s. This turntable frequency is best since a higher frequency necessitates a short pendulum which swings through too large an angle. Hang the bob above the turntable so that the bob is slightly higher than the ping pong ball (as shown in Figure b).

A dowel or rod clamped to a ringstand provides the pivot for the pendulum. This pivot can be rotated (Figure c) to adjust the length of the pendulum until the period is the same as that of the turntable.

The light source should be as far from the pendulum and turntable as possible to reduce the effects of parallax due to the changing distance between the light and the rotating ball.

Initially, the amplitude of the pendulum should be greater than the radius of the turntable so the amplitudes will be the same after the bob and the ball have been synchronized.

Release the bob at an instant so that its moving shadow lags a little behind the shadow of the ball. The motions may then be synchronized by holding a finger against the rim of the turntable, slowing it down until the two shadows coincide.

Section 9 – Experimental Frames of Reference
Section 10 – Fictitious Forces in Accelerated Frames
Section 11 – Newton's Law and the Rotation of the Earth

PURPOSE To point out the significance of the coordinate system (or frame of reference) in which an experiment in dynamics is being observed.

CONTENT a. Since the acceleration $\vec{a}$ depends on the coordinate system, $\vec{F} = m\vec{a}$ is correct only in those coordinate systems which are not themselves accelerating.

b. Galileo's principle of inertia applies only in a non-accelerating frame of reference (called an inertial frame). Newton's Law holds only in inertial frames.

c. Observers who are in an accelerating frame (and who are unaware of it) would ascribe an acceleration to an object which was actually not accelerating relative to an inertial frame. These observers would infer, from this apparent acceleration, that a force existed. Such forces are called fictitious forces.

d. The earth is rotating. An observer on earth, using Foucalt's pendulum, can detect that rotation. Objects and observers on the earth's surface must then be moving with a (very small) centripetal acceleration. All but the most precise experiments can safely ignore this acceleration.

EMPHASIS If you are rushed, you can let the text and, if possible, the film carry the message of these sections. Be careful not to leave the students with the impression that Newton's law has limited validity; it is always valid if $\vec{a}$ is measured in an inertial frame.

FILM "Frames of Reference" vividly displays the effect of the observer's frame of reference. The number and nature of the demonstrations presented would be difficult to duplicate in class.

DEVELOPMENT Before mentioning the earth's very small rotation effects, it will be instructive to discuss the case of a rapidly rotating merry-go-round. Remind students of the way they must lean toward the center (or hold on to something) to keep their balance. A man who lived on a giant merry-go-round would feel that there was, in his world, a strong force field pulling things away from the center. He might call such a force (as we sometimes do) a centrifugal force. A man, entering this merry-go-round home from the outside world might hang a Foucalt pendulum from the top of the center post. The plane of the pendulum's swing would rotate on the merry-go-round. (It would remain fixed in an inertial system.) The merry-go-round could now be concluded to be a rotating system. Viewed from an inertial frame, a man on the merry-go-round is constantly subject to a centripetal acceleration. His feet are in contact with the merry-go-round, and friction transmits the requisite centripetal force to his feet. To keep his body over his feet, the man must arrange for a centripetal force to act there, too. He can do this by holding one of the uprights or by leaning inward. As seen from this point of view, there is no outward force acting. From this view the apparent outward force observed by the man on the merry-go-round is a fictitious force.

COMMENT The perception that we sometimes invoke "fictitious" forces to explain familiar phenomena may lead some students to ask penetrating questions about the fundamental nature of force. After all, we cannot really "see" gravity or electric and magnetic forces, only their effects. Might not these, or some of these, forces also be fictitious in some frame we have not yet thought of? As yet we do not know enough about the causes of gravitational or electrical forces to know whether these "action-at-a-distance" forces are real or "fictitious". Einstein, in his theory of general relativity (see the supplement for the next chapter, Appendix 4, at the back of this volume), has formally accounted for the force of gravity in terms of the properties of space and time, but as of now we do not have a "unified field theory" to account similarly for electric and magnetic effects. Thus, at least at the present stage of development in physics, we treat these action-at-a-distance effects as real forces. We simply do not know of other frames of reference which we can apply (as we can with the "centrifugal" force) which will make electric and magnetic forces vanish.

* * *

Although for many experiments and calculations, it is convenient and acceptable to consider the earth to be an inertial frame, we can demonstrate experimentally that this is not the case. The fixed stars, in turn, although we take them to be a "good" reference frame, are not really fixed relative to one another, but are moving at enormous speeds. Imaginary lines between them may rotate in a true Newtonian frame. However, their distance from the earth is so vast, that on the time scale of a human life, they can be considered "fixed".

Chapter 21 – Motion at the Earth's Surface

For Home, Desk and Lab – Answers to Problems

The analysis and solution of problems in class is always helpful. It will be especially useful in connection with Chapter 21. Many students will need help in grasping and analyzing the problem in terms that can lead to its solution. Students should be encouraged to make sketches and graphs of the force relationships and motion involved, even though the problem statement may not call for them.

Problems 7, 8, and 9 are straightforward dynamics problems, but if your class has not reviewed kinematics while working on Chapters 20 and 21, you may want to assign (or review) some simpler free fall problems before working on 7, 8, or 9. Problem 7 deals with distance intervals requiring the use of $d = v_1 t + \frac{1}{2}at^2$. A direct solution of Problem 8 requires using a sign convention to distinguish downward from upward motion. Problem 9 involves general formulas and graphs.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 21-18 to 21-41.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1*, 2			1*, 2	
2	3	4, 5, 8*, 7, 10*, 17, 18	6, 9, 11	4, 5, 6, 7, 8*, 9, 10*, 11, 17, 18	5
3, 4	12*, 15*	13, 14, 16		12*, 13, 14, 15*	12
5, 6, 7	20, 21*, 22*	19, 23, 24*, 25, 26, 27*		19, 21*, 22*, 23, 24*, 25, 26, 27*	
8	31	28*, 29, 30		29, 30	28
9, 10, 11			32	32	

SHORT ANSWERS

1. (a) 1 nt.
   (b) 4.9 kg. } using 10 nt/kg for g.

2. (a) 120 nt; 700 nt.
   (b) 70 kg; 70 kg. } using g = 10 nt/kg.

3. $5\ \text{m/sec}^2$.

4. No.

5. (a) Yes.
   (b) Zero.
   (c) Slows down until it reaches its terminal velocity.

6. Stone would accelerate as it moves toward center of earth, and would then slow down after it passed. The acceleration would decrease as the center is approached.

7. (a) 1 second.
   (b) $v = -9.8$ m/sec.
   (c) $\Delta t = 0.27$ sec.
   (d) $v = -12.4$ m/sec.
   (e) $\Delta d = -24.5$ meters.

8. (a) $v = 3.2$ m/sec.
   (b) $d = 10.9$ meters.
   (c) $v = -7.5$ m/sec.
   (d) $d = 8.6$ meters.
   (e) $a = -9.8\ \text{m/sec}^2$.

9. (a) $t = \frac{h}{v_1}$ provided $v_1 \geq (4.9\ h)^{1/2}$.

(b) The ball is rising if $v_i > (9.8\ h)^{1/2}$; it is falling if $v_i < (9.8\ h)^{1/2}$; The collision occurs when the ball is at its peak when $v_i = (9.8\ h)^{1/2}$.

10. (a) 9.8 nt.
    (b) 9.8 m/sec^2 ≈ 10 m/sec^2.
    (c) 4.9 m/sec^2 ≈ 5 m/sec^2.

11. (a) F = 1/2 mg = 5m newtons. (or 4.9m nt) along plane.
    (b) a = 1/2 g = 5 m/sec^2 (or 4.9 m/sec^2) down along plane.

12. The two marbles reach the ground together.

13. (a) See graph on page 21-27.
    (b) 0.64 seconds.
    (c) 31 m/sec, 12° below horizontal.

14. Both bullets drop together. For effects of air resistance during later stage of motion see discussion on page 21-28.

15. 11.0 meters.

16. (a) 2.55 meters.
    (b) 10.2 m.
    (c) 10 m/sec.

17. 19.4 meters.

18. 12.0 m/sec, down (or 11.5 m/sec, down).

19. $\dfrac{v^2}{R} = \left[\dfrac{(L/T)^2}{L}\right] = \left[\dfrac{L}{T^2}\right] = a.$

20. (a) 3.5×10^{-3} m/sec, down.
    (b) 6.1×10^{-6} m/sec^2, toward center.

21. (a) 1.8×10^2 m/sec^2.
    (b) 1.3×10^3 nt.

22. 99 m/sec.

23. (a) Not according to definition.
    (b) These refer to total supporting force rather than that part of supporting force needed when there is no acceleration and no other force.

24. T = weight + 0.2 nt = 4.1 nt.

25. (a) String will break; it must supply additional centripetal force.
    (b) String must be twice as strong. Diameter must increase by a factor $\sqrt{2}$; 2.8 mm.

26. 7.1×10^4 m/sec.

27. (a) The particle spirals in.
    (b) Its speed does not change.

28. Change the period by changing the length.

29. (a) See graph on page 21-38.
    (b) y = -0.10m.
    (c) It oscillates about y = -0.10 with simple harmonic motion; it reaches the extreme, y = -0.20m, in 0.317 seconds.

30. (a) T = 0.628 sec.
    (b) a = +9.8 m/sec^2.
    (c) 19.6m nt upward.

31. (a) T = 3.1 seconds.
    (b) g would be 10.5 nt/kg.

32. Bus is going with uniform velocity to give straight line motion of marble and bus is decelerating to give parabolic path.

COMMENTS AND SOLUTIONS

PROBLEM 1

> (a) Approximately what is the gravitational force on a mass of 100 gm at the earth's surface?
> (b) What is the approximate mass of a weight of 49.0 newtons?

This problem illustrates the important distinction between weight and mass. It should be supplemented with examples involving objects familiar to the students so that they get a physical feeling for weight expressed in newtons.

Note: A number of teachers find that the approximation that g = 10 newtons per kilogram is extremely useful in eliminating complicated numbers. Without this simplification many students tend to speak of ‘‘the weight of a 22 kg mass’’ or ‘‘a weight of 22 kg’’ because they do not want to bother to multiply 22 by 9.8. However, through Problem 1 and extensions of it, you can easily get students to convert automatically from 22 kg to 220 newtons, or from 39 newtons to 3.9 kg.) Sacrificing only 2% in accuracy often produces a remarkable improvement in understanding.

a) Since g is approximately 10 nt/kg, the force on 100 gm = 0.1 kg is

$$0.1 \text{ kg} \times 10 \text{ nt/kg} = \underline{1 \text{ nt}}.$$

b) 49 nt is approximately $\frac{49.0 \text{ nt}}{10 \text{ nt/kg}} = \underline{4.9 \text{ kg}}$. (If one bothers to calculate using 9.8 nt/kg, the answer would be 5.0 kg.)

PROBLEM 2

The gravitational field at the surface of the moon is one-sixth of that at the surface of the earth.

(a) How much would a 70-kg man weigh on the moon? On the earth?

(b) What would be his mass on the earth? On the moon?

This problem brings out the distinction between mass and weight.

a) A 70 kg man would have a weight of 700 newtons on earth (70 kg $\times \frac{10 \text{ nt}}{\text{kg}} = 700$ nt).

On the moon, his weight would be 1/6 of this or $\frac{700 \text{ nt}}{6} \approx \underline{120 \text{ newtons}}$.

b) His mass is 70 kg on earth, on the moon, or anywhere.

Note: While you will not want to take much time for it, the above data can be used as a base for an interesting class discussion on the characteristics of everyday life for earth people living on the moon.

The forces a man can exert would be the same. Those activities in which these forces are used to overcome gravity will now be considerably altered: a man will be able to jump six times as high from the moon's surface as he can from the earth's. He can lift bodies whose masses are six times those he could lift on earth. On the other hand, processes in which gravity does not enter (for example, in the acceleration of a body in a horizontal direction by pushing on it) would not be affected. An interesting example of an act involving both types of process is throwing a baseball. The maximum velocity that a man can impart to a baseball (a process unaffected by gravity, and governed only by the masses of the baseball and the man's arm, and his muscles) will be the same as on earth. The distance the ball will travel after it leaves his hand before hitting the moon's surface does depend on g, however. Thus for a given throwing angle, the ball can be thrown six times farther on the moon than on earth. A pitcher's "fast ball" will be no faster than on earth, but a fly ball to deep center field in Yankee Stadium would travel nearly half a mile on the moon! Imagine a lunar baseball game!

You might ask students to think about driving a conventional car on the moon. Could you turn as sharply at 50 mph as on earth? Should the turns in the roadway be banked more, less, or the same as on earth?

Students will undoubtedly be able to think of other examples. Discussions should bring out clearly that all characteristics relating only to mass are unchanged; inertia, chemical reactions, heat content, collisions, etc. are examples of characteristics that would not change.

PROBLEM 3

The earth's gravitational field at a certain point out in space accelerates a 1-kg mass at 5 m/sec^2. How much will it accelerate a 3-kg mass?

This is a simple problem which stresses the relationship between inertial and gravitational mass.

From F = ma, $a = \frac{F}{m}$. Thus acceleration decreases with an increase in mass, and increases with an increase in force. If the accelerating force were constant, the larger mass would accelerate more slowly. But, we find experimentally that the gravitational force is proportional to the mass. Hence if the mass increases by a factor of 3, so does the gravitational force, and the acceleration due to gravity stays constant — it is independent of changes in mass. At a point in space where the earth's gravitational field accelerates a 1 kg mass at 5 m/sec^2, a 3 kg mass will be accelerated at 5 m/sec^2.

PROBLEM 4

If different objects had different ratios of their inertial and gravitational masses, would they all accelerate at the same rate in the gravitational field of the earth? Explain.

This problem calls attention to the simplest implication of the relation between gravitational and inertial mass.

The most direct answer can be given in terms of the equations on page 325 of the text. Since $\vec{a} = \frac{m_g}{m_i}\vec{g}$, a variable ratio m_g/m_i implies a variable acceleration. Thus, for instance, if objects that are made of different materials had different ratios, $\frac{m_g}{m_i}$, they would accelerate differently in the earth's gravitational field.

Note that the constancy of m_g/m_i is not a derived fact; it is based on experiments which demonstrate the universality of the acceleration due to gravity. If m_g/m_i were not constant, the only changes to be expected are in those phenomena directly related to the force of gravity. Other aspects of dynamics would be quite unchanged.

Inasmuch as the constancy of m_g/m_i is founded directly on experiment, an experimental physicist might well design experiments to test this constancy more carefully. Direct acceleration experiments cannot be done with extreme precision or with a large range of masses, but every such experiment that has been performed has been found to be consistent with a constant m_g/m_i to within the limits of accuracy of the experiment. The most precise experiments yet performed essentially involve the comparison of the periods of pendulums made of different materials.

These experiments show that, to about 1 part in 10^8, the ratio, m_g/m_i, is the same independent of whether the pendulum is made of a light material (such as aluminum) or a heavy material (such as lead). Artificial earth satellite experiments, currently being planned, will extend the precision of related experiments to about 1 part in 10^{10}.

PROBLEM 5

Throw a baseball as hard as you can (outdoors) and note how fast it goes. Then throw a crumpled-up piece of paper equally hard.

(a) Does the paper seem to slow down appreciably?

(b) What is the terminal velocity in a horizontal direction for any object thrown through the air?

(c) What do you expect to happen to any object thrown downward at a speed greater than its terminal velocity?

a) The paper slows down noticeably, usually almost stopping within 10 to 15 feet.

b) The terminal velocity for any object thrown in a horizontal direction is zero. Since there is only the decelerating force due to air resistance, the horizontal velocity will eventually go to zero.

c) For an object thrown downward with a velocity greater than its terminal velocity, initially the upward resistive force will be greater than the gravitational force, and the object will decelerate to its terminal velocity. At this velocity the force of gravity is balanced by the resistive force of the air.

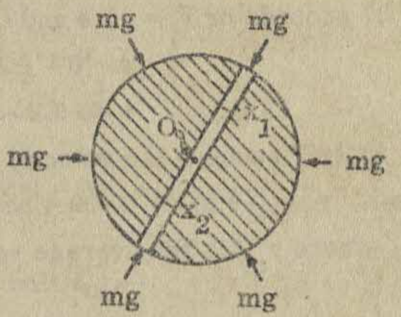
PROBLEM 6

What would you expect to happen if a stone were dropped into a hole that extends through the center of the earth from the North Pole to the South Pole? Would you expect the stone to undergo constant acceleration all the way to the center? Can you reason out crudely how the *g* field changes inside the hole as you approach the center of the earth? Be prepared to discuss this in class.

This problem should be treated qualitatively. A student attempt to handle it quantitatively will lead to frustration.

A stone dropped from the North Pole would accelerate until it reached the center of the earth; then it would decelerate. The force acting on the stone, and thus the acceleration, would decrease gradually because that part of the mass of the earth which the stone has already passed by on its trip down the hole would be pulling it away from, rather than toward, the center of the earth.

It is hard to decide just how the g field varies with position in the hole without using calculus or its equivalent. However it is possible to describe the motion in some detail even if the exact force is not known. Consider the symmetry about the center of the earth. Due to this symmetry the magnitude of the force depends only on the distance from the center. The direction of the force is toward the earth's center. Thus the stone will accelerate continuously (neglecting air resistance etc.) and will begin to decelerate as soon as it passes the earth's center (since there the direction of the net force reverses). Consider any two points in the hole, x_1 and x_2, which are equidistant from the earth's center, O. The stone will be accelerated by a varying force as it travels from x_1 to O, and it will be decelerated by an opposite but equal force as it travels from O to x_2. Due to the symmetry then, the stone's speed goes through a maximum at the center and then retraces its pattern of speeds as a function of distance from the center. It is was dropped (rather than pushed) from the North Pole, it will reach the South Pole with zero speed and will then fall back toward the center. The motion will be oscillatory.



* * *

Note: The question may arise: Why choose the North Pole–South Pole axis? It is hardly practical to dig a hole through the earth, but if it were, there would be a very good reason for using this axis, since it is the axis of rotation of the earth. If the hole were in any other direction (but still running through the earth's center) the stone would not fall straight down, but would veer to one side. This is because the frame of reference which is fixed relative to the hole (this is the one in which we calculate) would now rotate relative to the frame which is fixed relative to the earth's axis. In the rotating frame there would be "fictitious" forces (the centrifugal force is one) which would not be parallel to the hole, and which would move the stone toward the side of the hole. No fictitious forces would enter if we adopt a frame of reference fixed relative to the inertial frame of the fixed stars, and we could explain the same result by saying that the earth rotated while the stone was falling, so that the stone, which keeps the traverse velocity it had when at the earth's surface, catches up with the side of the hole and hits it.

PROBLEM 7

A stone drops from the edge of a roof.
(a) How long does it take to fall 4.9 meters?
(b) How fast is it going at the end of that fall?
(c) How much longer does it take to fall an *additional* 3 meters?
(d) How fast is it going at the end of this 7.9-meter fall?
(e) Through what distance does it move during the third second?

a) $d = -\frac{1}{2}at^2 = -\frac{1}{2}(9.8)\,t^2 = -4.9\,t^2$, where the negative value of a indicates that the acceleration is down, and the negative value of d indicates a fall.

But $d = -4.9$ m. Consequently, $-4.9\text{ m} = -4.9\text{ m/sec} \times t^2$, and $\underline{t = 1 \text{ second}}$.

b) After 1 second the speed is $v = at = -9.8 \times 1 = \underline{-9.8\text{ m/sec}}$.

c) The most direct approach is to find out how long a 7.9 m drop would take.

$$-7.9\text{ m} = -4.9\text{ m/sec}^2 \times t^2,\ \text{or } t = \left(\frac{7.9}{4.9}\right)^{1/2},\ \text{and } t = 1.27\text{ sec.}$$

The additional three meter drop, therefore, required $1.27 - 1.00 = \underline{0.27\text{ seconds}}$.

d) The final speed can be found from $v = v_i + at$. One can use either $v_i = 0$ and $t = 1.27$ seconds or $v_i = -9.8$ and $t = 0.27$ seconds.

$$v = 0 - 9.8\,(1.27) = \underline{-12.4\text{ m/sec}},\ \text{or}$$

$$v = -9.8 - 9.8\,(0.27) = \underline{-12.4\text{ m/sec}}.$$

e) After 2 seconds the velocity is $v_2 = (-9.8)(2) = -19.6$ m/sec. Similarly, after 3 seconds, $v_3 = (-9.8)(3) = -29.4$ sec.

Then, where v_{av} is the average velocity between $t = 2$ and $t = 3$ sec,

$$v_{av} = \frac{-19.6 - 29.4}{2} = -24.5\text{ m/sec.}$$

The distance is $d = v_{av}t = (-24.5)(1\text{ sec}) = \underline{-24.5\text{ meters}}$.

PROBLEM 8

A ball is thrown straight up with a velocity of 15 m/sec.
(a) How fast will it be going after 1.2 sec?
(b) How far above the ground will it be at that time?
(c) How fast will it be going after 2.3 sec?
(d) How far above the ground will it be at that time?
(e) What is the ball's acceleration at the top of its rise?

This problem can be done using the two equations $v_f = v_i + at$ and $d = v_{av}t$.

a) The ball's speed upward is 15 m/sec. Therefore $v_i = +15$ m/sec; $a = -9.8\text{ m/sec}^2$, due to gravity.

$$v_f = v_i + at$$

$$v_{1.2} = 15\text{ m/sec} + (-9.8\text{ m/sec}^2)\,(1.2\text{ sec}) = \underline{+3.2\text{ m/sec}}.$$

b) $d = v_{av}t$

$$d_{1.2} = v_{av}\,(1.2\text{ sec}) = \left(\frac{15 + 3.2}{2}\text{ m/sec}\right) \times 1.2\text{ sec} = \underline{+10.9\text{ meters}}.$$

c) $v_f = v_i + at$

$$v_{2.3} = 15\ \text{m/sec} + (-9.8\ \text{m/sec}^2) \times 2.3\ \text{sec} = \underline{-7.5\ \text{m/sec}}.$$

d) $d = v_{av}t$

$$d_{2.3} = v_{av}(2.3\ \text{sec}) = \left(\frac{15 - 7.5}{2}\ \text{m/sec}\right) \times 2.3\ \text{sec} = \underline{+8.6\ \text{meters}}.$$

e) The ball's acceleration at the top of its rise, and at all other times in its flight, is $\underline{-9.8\ \text{m/sec}^2}$.

PROBLEM 9 A stone is dropped off a cliff *h* meters high. At the same instant a ball is thrown straight up from the base of the cliff with initial velocity of *v* m/sec.

(a) Assuming the ball is thrown hard enough, at what time *t* will stone and ball meet?

(b) When they meet, is the ball still rising?

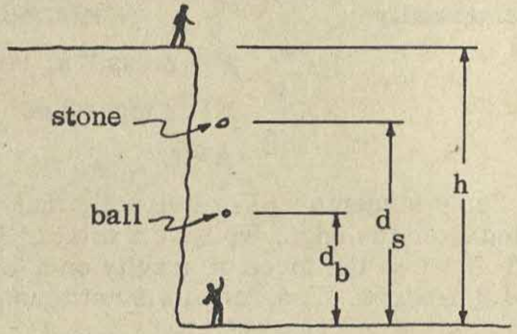
Although this problem is rather difficult it has two particularly good features. First, students must think about the physical problem to set up the correct equations. Then, they must re-examine the physical situation to interpret their results.

a) The key to setting up this problem properly is to measure distance in a consistant way. One approach is to measure both d_s and d_b from the base of the cliff. Then

$$d_s = h - \frac{1}{2}gt^2$$

$$d_b = v_it - \frac{1}{2}gt^2.$$

$\therefore$ When $d_s = d_b$, $h = v_it$ and $t = \frac{h}{v_i}$.



Actually $t = h/v_i$ does not hold unless the ball meets the stone above the ground. If v_i is too small, the ball and stone do not meet before the ball has fallen back to the ground. This can be seen by realizing that t must be less than the time t_h which the stone takes to drop to the ground. This time is given by $h = \frac{1}{2}(9.8)\,t_h^2$ or $t_h = \left(\frac{h}{4.9}\right)^{1/2}$.

b) When the ball meets the stone, the ball can be either rising, falling, or at rest. We can tell which it is by finding its velocity v_m at t, the time the ball and the stone meet. This is

$$v_m = v_i - gt.$$

But we know already that $t = h/v_i$; so

$$v_m = v_i - gh/v_i.$$

If the ball and stone meet when the ball is at rest, then $v_m = 0$, and

$$v_i = gh/v_i \text{ or}$$

$$v_i^2 = gh.$$

That is: If the ball is thrown up at a speed exactly equal to $\sqrt{gh}$ it will be stationary when it meets the stone. If thrown at a greater speed it will still be rising; at a lower speed, descending.

PROBLEM 10

Two 0.5-kg Dry Ice discs on a flat table are tied together by a string.
(a) What is the total gravitational force on these discs?
(b) At what rate will they speed up if pulled across the table by a horizontal force equal to the gravitational force found in (a)?
(c) If one disc remains on the table and the other hangs over the edge, what will be the acceleration of the discs? Neglect friction.

This problem deals with free fall, a special case of Newton's law. It also introduces a new kind of problem in which gravitational force on part of a system can accelerate the entire system.

a) The total gravitational force is approximately 10 nt, because each 0.5 kg mass experiences about a 5 nt force. (Using 10 nt/kg for g.) Actually, $F = 2 \times 0.5 \times 9.8 = \underline{9.8 \text{ nt}}$.

(Note that since the discs will not accelerate in this case, the table must push up on each disc with a force equal to the disc's weight.)

b) If a single object is pulled with a net force equal to the gravitational force, it must have the same acceleration it would in free fall, $\underline{9.8 \text{ m/sec}^2}$ (or approximately 10 m/sec^2). Mathematically:

$$F = \text{mass} \times a, \text{ but } F = 2mg \text{ and mass} = 2m$$

$$F = 2\,mg = 2\,ma$$

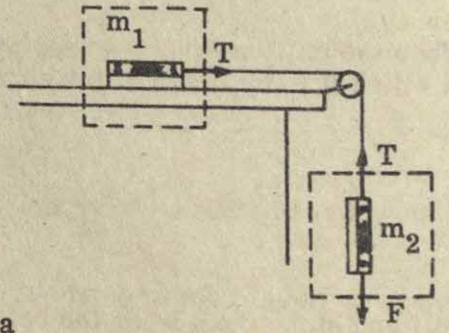
$$\underline{a = g.}$$

c) Some students will see directly that, with one 0.5-kg mass on the table, and the other hanging over the edge, we have a total of 1 kg of mass and that the only accelerating (unbalanced) force is the force of gravity on a 0.5-kg mass. This force is 0.5 kg × 9.8 newtons/kg = 4.9 newtons. A force of 4.9 newtons acting on 1 kg of mass results in an acceleration of 4.9 m/sec^2. A bit more formally:

$$a = \frac{F}{m} = \frac{F}{m_1 + m_2} = \frac{m_2 g}{m_1 + m_2} = \frac{0.5 \text{ kg} \times g}{1 \text{ kg}} = \frac{1}{2}\, g = \underline{4.9 \text{ m/sec}^2}.$$

Other students may think they "see", but not be sure, and require a more detailed approach. There are several ways this can be done. One alternative is to ask students to consider each disk as a separate system (using a dashed box to emphasize the boundary of the system and the external forces acting), and remind them that:

1. The tension in the string is T at all points, and
2. Since the disks move together, the magnitude of $\vec{a}_1$ equals the magnitude of $\vec{a}_2$ even though their directions are different. Call this magnitude a.



$$\text{For } m_1: \quad T = m_1 a$$

$$\text{For } m_2: \quad F - T = m_2 a$$

$$\therefore \quad F - m_1a = m_2a.$$

But $F = m_2g$. Substituting,

$$m_2g - m_1a = m_2a$$

$$a(m_1 + m_2) = m_2g$$

$$a = \left(\frac{m_2}{m_1 + m_2}\right) g = \frac{1}{2} g = 4.9 \text{ m/sec}^2.$$

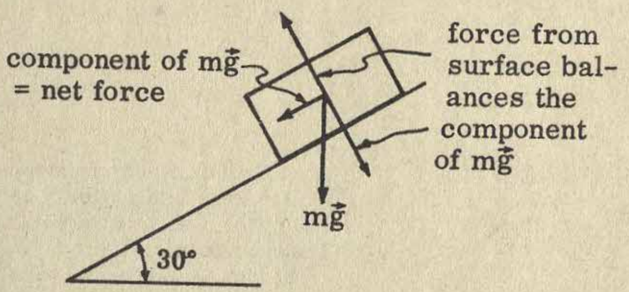
PROBLEM 11

An object of mass *m* is moving on a frictionless inclined plane that makes an angle of 30 degrees with the horizontal.

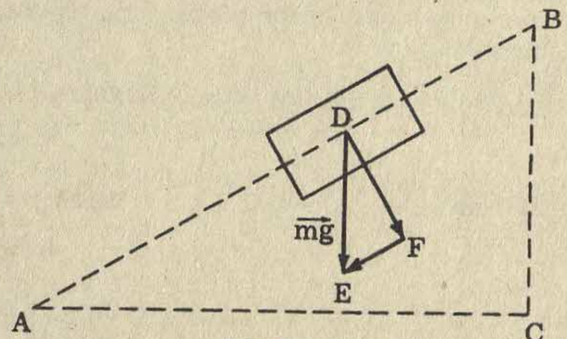
(a) What is the net force on the object?
(b) What is its acceleration down the plane?

This problem on gravitational attraction requires a resolution of the weight, mg, into two components of which one is directed along an inclined plane. The solution requires either trigonometry or a scale drawing.

a) We can resolve the force $m\vec{g}$ into two components, one along AB and one perpendicular to it. Since the object doesn't move into the surface, there is no net force component perpendicular to the surface. The perpendicular component of $m\vec{g}$ is balanced by the force from the surface.


component of $m\vec{g}$ = net force
force from surface balances the component of $m\vec{g}$
$m\vec{g}$
30°

Since there is no friction, the force from the surface of the plane can have no component parallel to the plane. The only force component which is parallel to the plane is the parallel component of $m\vec{g}$. This parallel component of $m\vec{g}$ is the net force. To find it, construct DE with a magnitude equal to $m\vec{g}$ on the scale chosen. Construct DF perpendicular to AB passing through D, and EF parallel to AB passing through E.



Force diagram

For clarity, instead of drawing the forces on the sketch of the inclined plane, it may be best to draw a force diagram in which the directions, AB, BC, and CA are shown dotted. Lengths on this new diagram clearly represent forces and are less likely to be confused with distances. Using geometry, the students should be able to show that triangle EDF is similar to triangle BAC and that ∠EDF = ∠BAC = 30°. Therefore $\overline{FE}$, which represents the component of the force parallel to the plane, is given by

$$\sin 30° = \frac{\overline{FE}}{mg}, \text{ or } \overline{FE} = mg \sin 30° = \frac{1}{2} mg.$$

The normal force $\overline{DF}$ is exactly balanced by the reaction force of the plane surface; each has a magnitude mg cos 30°, and their vector sum is zero.

Therefore, net force = $\frac{1}{2}$ mg along the slope of the plane.

b) The acceleration down the plane is given by

$$F = ma = \frac{1}{2} mg, \text{ and}$$

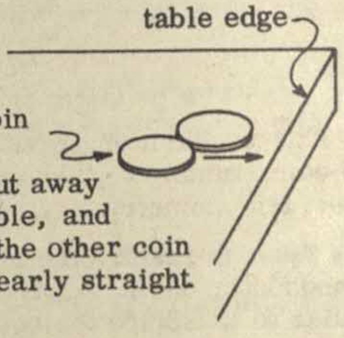
$$\underline{a = \frac{1}{2} g, \text{ down along plane.}}$$

PROBLEM 12

As an easy project, try rolling one marble slowly off the edge of a table at the same instant that you snap another marble off the table horizontally with your finger. Do they hit the floor at the same instant? Explain.

While this project could be done as a class demonstration, it is so easy and so instructive that it is worth having students do it themselves.

If the experiment is done over a hard floor the times of impact can be clearly detected from the sound. One suitable "launching" technique for coins is described in the film "Falling Bodies." The same technique can be used with marbles or other objects.



Snap this coin with finger driving it out away from the table, and "nudging" the other coin off to fall nearly straight

PROBLEM 13

A baseball thrown horizontally starts at a height of 2.0 meters with a velocity of 30 m/sec.

(a) Plot the trajectory of the ball, using 1/10-sec intervals.

(b) How long before the ball strikes the ground?

(c) Determine both the magnitude and direction of the velocity of the ball at the instant it strikes the ground.

To choose a scale for plotting the curve, it will be helpful to know how long the whole motion will take.

We can solve for the time in flight using

$$y = h - \frac{1}{2} gt^2.$$

The ball hits the ground at $y = 0$. When $y = 0$,

$$h = \frac{1}{2} gt^2, \text{ or}$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2.0 \text{ m}}{9.8 \text{ m/sec}^2}} = 0.64 \text{ sec.}$$

We can find the horizontal distance the ball travels before it hits the ground from

$$x = vt = 30 \text{ m/sec} \times 0.64 \text{ sec} = 19.2 \text{ m.}$$

From these values, our graph needs to be "2 m" high by "20 m" wide.

a) If the horizontal and vertical components, relative to the point where the ball starts, are x and y, then as functions of t,

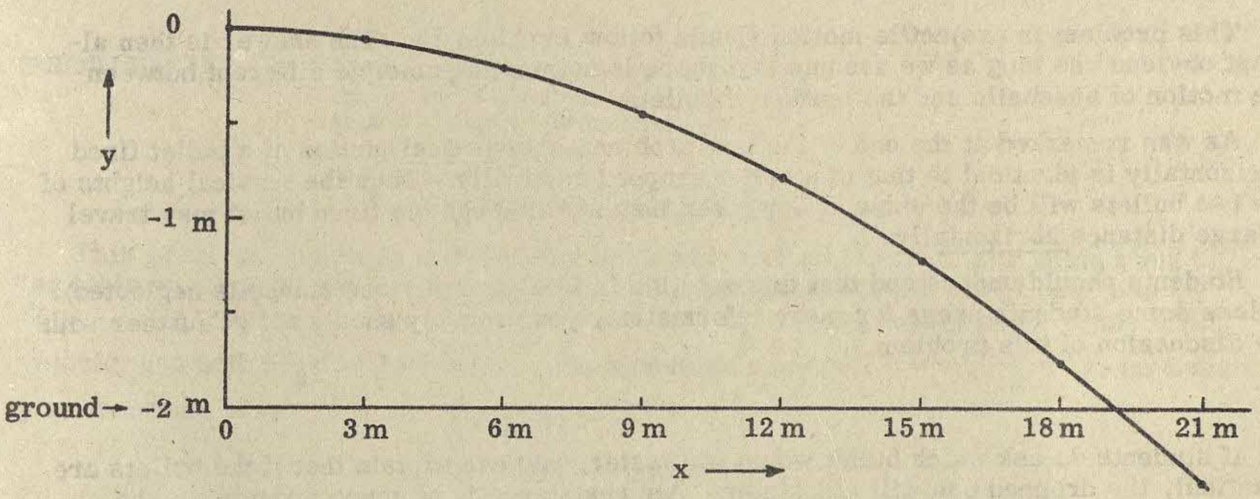
$$x = vt = 30\,t$$

$$y = h - 1/2\,gt^2 = -4.9\,t^2$$

where y is positive upward, and so is negative here. Equal intervals of t give equal intervals of x. We obtain the following table:

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
x	0	3.0	6.0	9.0	12.0	15.0	18.0	21.0
y	0	-0.05	-0.20	-0.44	-0.78	-1.22	-1.76	-2.40

We give y to two decimal places because we shall need a larger scale in y than in x. We also continue it to t = 0.7 (i.e., below ground) to be able to answer (c) more accurately.



b) From the graph, the time the ball takes to strike the ground is between 0.6 and 0.7 sec. A more careful interpolation in x (since x is proportional to t) gives results as accurate (in principle) as we like. t ≈ 0.65 sec. (Actually, = 0.64 sec. as we saw above.)

c) Remember that the slope of this graph is not a velocity. It is the ratio of vertical to horizontal velocity. Since we know that the horizontal velocity is constant, and equal to 30 m/sec, we can then get the vertical velocity from the measured ratio.

A crude interpolation (between t = 0.6 and t = 0.7 from the table) gives this ratio as

$$\frac{v_y}{v_x} = \frac{-2.40 - (-1.76)}{21.0 - 18.0} = -0.21$$

$$\text{i.e., } v_y = -.21 \times 30 = -6.3\,\text{m/sec.}$$

We could obtain a more accurate result by drawing a careful curve, and measuring the slope at the point where it crosses the line y = 2. Or, from $v_y = -gt$, we also find that $v_y = -6.3$ m/sec.

From the vertical and horizontal velocity components, the actual velocity at the time of impact can be computed using the Pythagorean theorem:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{30^2 + 6.3^2} = \underline{31\,\text{m/sec.}}$$

The vertical and horizontal velocity components also give the slope at the time of impact, which enables determining the ball's direction.

$$\tan\theta = \frac{v_y}{v_x} = \frac{6.3}{30} = 0.21.$$

$$\theta = \underline{12^\circ, \text{ below horizontal}}.$$

In anticipation of Problem 14, it should be pointed out once again that the vertical component of the motion is exactly the same as that of a body falling from 2 meters with no horizontal motion.

PROBLEM 14 A bullet is dropped from the same height and at exactly the same moment that another bullet is fired horizontally. Discuss the motion of each of the bullets.

This problem in projectile motion should follow Problem 13. The answer is then almost obvious, so long as we assume that there is nothing in principle different between the motion of baseballs and the motion of bullets.

As was remarked at the end of the last problem, the vertical motion of a bullet fired horizontally is identical to that of a bullet dropped vertically. Thus the vertical heights of the two bullets will be the same at any given instant, although the fired bullet may travel a large distance horizontally.

Students should understand that this solution is idealized (air resistance is neglected). Unless some students press for more information, you probably should not go further with the discussion of this problem.

* * *

If students do ask which bullet would fall faster, you can explain that if the bullets are identical, the dropped one will fall faster. Air resistance is so much greater for the bullet that is fired, that the upward component of the air resistance on the fired bullet retards its fall more.

PROBLEM 15 A baseball is thrown from center field to home plate. If the ball is in the air 3.00 seconds and we neglect air resistance, to what vertical height must it have risen?

This simple and interesting projectile problem usually generates a good class discussion.

At first it might seem as though too little information is given. But the solution can be found if one makes only the additional assumption that the ball is caught at the same height from which it was thrown. In this case, the symmetry of the trajectory makes it clear that the ball rises for 1.50 seconds and falls for 1.50 seconds.

Consider the vertical motion. For it

$$v_y = v_{yo} - gt.$$

We know that at $t = 1.5$ sec, $v_y = 0$.

$$\therefore 0 = v_{yo} - g(1.5), \text{ or } v_{yo} = 14.7 \text{ m/sec.}$$

The height the ball will reach can be found either from $h = \frac{1}{2}gt^2$, or from the average vertical velocity × time:

$$h = v_{ave}t = \frac{14.7 + 0}{2} \times 1.5 = \underline{11.0 \text{ meters}}.$$

* * *

This problem will probably bring up additional questions from your baseball enthusiasts; you may want to exploit their interest by calculating the initial speed of the throw. For this, you will have to assume some horizontal distance. A reasonable number might be 30 meters. This implies an initial horizontal velocity of $v_x = \frac{d}{t} = \frac{90}{3} = 30$ m/sec. The total velocity, v, is then given by

$$v^2 = v_{xo}^2 + v_{yo}^2 = 900 + 216$$

$$v = 33 \text{ m/sec.}$$

This velocity is quite high for a baseball throw; it is perhaps realistic for a professional player but much too high for most students.

PROBLEM 16

Plot the trajectory of an object that is thrown into the air with an initial velocity of 10 m/sec at an angle of 45 degrees with the horizontal.

(a) How high does it rise?
(b) How far does it go?
(c) At what velocity does it strike the ground?

This problem in plotting a trajectory is similar to Problem 13. The calculation of the table of x and y, the horizontal and vertical coordinates of the object, can now be done a little more elegantly. Since the horizontal and vertical components of the initial velocity are both $\frac{10\sqrt{2}}{2} = 7.07$ m/sec, the kinematic equation, $d = v_i t + \frac{1}{2}at^2$ gives x and y as functions of t:

$$x = 7.07t$$

$$y = 7.07t - 4.9t^2.$$

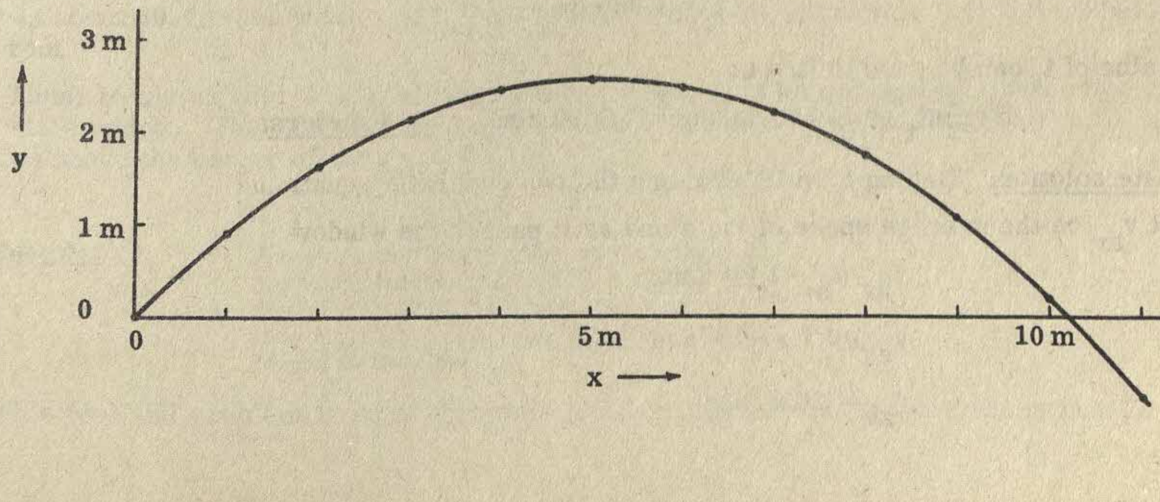
We could, as in Problem 13 calculate x and y for various values of t. It is a little simpler, however, to eliminate t from the above equations, and to obtain one equation relating x and y.

$$t = x/7.07$$

$$y = 7.07(x/7.07) - 4.9(x/7.07)^2 = x - 0.098x^2.$$

We can now plot y against x. The curve is a parabola, and we can calculate its salient features. The curve crosses the x axis both at x = 0 and at x = 1/.098 = 10.2 m. The maximum value of y is halfway between at x = 5.1; here y = 2.55 m. In tabular form:

x	0	1	2	3	4	5	6	7	8	9	10	11
y	0	0.90	1.61	2.12	2.43	2.55	2.48	2.20	1.73	1.06	0.20	-0.86



a) $\underline{y_{max} = 2.55\text{ m}}$; See above

b) $\underline{10.2\text{ m}}$; See above

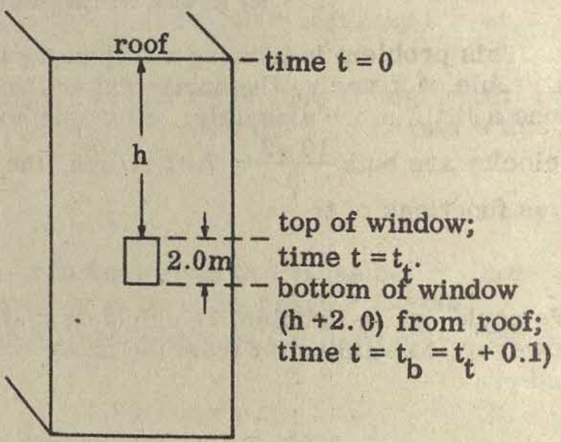
These results can be obtained in several other ways, such as working with the original equations involving t. For example, $y = 0$ for $t = 0$ or for $t = \frac{7.07}{4.9}$. y_{max} corresponds to $t = \frac{7.07}{2(4.9)}$. The total distance $x = 7.07t = 7.07\left(\frac{7.07}{4.9}\right) = 10.2\text{ m}$.

c) When the object hits the ground again its vertical velocity is equal in magnitude (but opposite in direction) to the initial vertical velocity; $v_{fy} = -7.07\text{ m/sec}$. The x velocity is unchanged; $v_x = 7.07\text{ m/sec}$. The magnitude of the velocity is therefore $\underline{10\text{ m/sec}}$ as before. This can be seen from the symmetry of the trajectory that has been plotted. But remember that velocity is a vector; although the speed is unchanged the direction is ↘ instead of ↗ .

PROBLEM 17 A stone drops from the edge of a roof and is observed to pass by a window which is 2.0 meters high in 0.10 second. How far is the roof above the top of the window?

The unknown distance, h, from the top of the window to the roof is related to the time, t_t, required to fall by:

$$h = \frac{1}{2}gt_t^2 .$$



Similarly, for the fall to the bottom of the window:

$$h + 2.0 = \frac{1}{2}gt_b^2 = \frac{1}{2}g(t_t + 0.1)^2 .$$

Subtract the first equation from the second:

$$2.0 = \frac{1}{2}g(t_t + 0.1)^2 - \frac{1}{2}gt_t^2$$

$$2.0 = \frac{1}{2}g(t_t^2 + 0.2t_t + 0.01) - \frac{1}{2}gt_t^2$$

$$4.0 = 0.2\,gt_t + 0.01\,g.$$

But $g = 9.8$:

$$4.0 - 0.098 = 1.96t_t$$

$$3.90 = 1.96t_t$$

$$t_t = 1.99\text{ sec.}$$

This value of t_t can be used to find h:

$$h = \frac{1}{2}gt_t^2 = \frac{1}{2} \times 9.8\text{ m/sec}^2 \times (1.99\text{ sec})^2 = \underline{19.4\text{ meters}}.$$

Alternate solution. Finding t_t while avoiding the two quadratic equations:

Let v_{av} be the average speed of the stone as it passes the window.

$$v_{av}(t_b - t_t) = 2\text{ m}$$

$$v_{av}(0.1\text{ sec}) = 2\text{ m}$$

$$v_{av} = 20\text{ m/sec}$$

But $v_{top} = gt_t$; $v_{bottom} = gt_b = g(t_t + 0.1)$

$$v_{av} = 1/2\,(v_{top} + v_{bottom}) = 1/2\,g(2t_t + 0.1)$$
$$v_{av} = gt_t + 0.05\,g$$
$$20 = 9.8\,t_t + 0.49$$
$$9.8\,t_t = 19.51$$
$$t_t = 1.99\text{ sec.}$$

You can also find h without using a quadratic equation. Let v_{av} be the average velocity during the fall through h:

$$h = v_{av}\,t_t.$$

$$\text{But } v_{av} = 1/2\,(v_{roof} + v_{top}) = 1/2\,(0 + 9.8\,t_t)$$
$$v_{av} = 1/2\,(9.8)(1.99) = 9.75\text{ m/sec.}$$
$$\text{Then, } h = v_{av}t_t = 9.75\text{ m/sec} \times 1.99\text{ sec} = 19.4\text{ meters.}$$

* * *

Alternate Solution. (Approximate).

This problem can be done somewhat more easily, to a very good approximation, as follows:

When the stone is in view, its average velocity, v_{av}, is:

$$v_{av} = \frac{d}{t} = \frac{2.0\text{ m}}{0.1\text{ sec}} = 20\text{ m/sec.}$$

It would actually reach a speed of 20 m/sec at a time, t_c, midway between t_t and t_b. From the time it fell from the roof, the time it took to get a speed of 20 m/sec was:

$$t_c = \frac{v}{g} = \frac{20\text{ m/sec}}{9.8\text{ m/sec}^2} = 2.04\text{ seconds.}$$

During that entire time, it had an average velocity, $v_{av} = \frac{1}{2}(0 + 20\text{ m/sec}) = 10\text{ m/sec}$. Therefore, during t_c it fell a distance, h′, given by

$$h' = v_{av}\,t_c = 10\text{ m/sec} \times 2.04\text{ sec} = 20.4\text{ m}$$

Therefore, the roof is 20.4 m above the stone when the stone has a speed of $v = 20\text{ m/sec} = v_{av}$. But since this occurs midway in time between the top and the bottom of the window, the stone is very slightly above the center of the window. Thus, the midpoint of the window is about 20.4 m below the roof; therefore the top of the window is 19.4 meters below the roof.

Thus, to the number of significant figures we use, this approximation gives exactly the same result. The exact position of the stone at the middle of the time interval is only 0.01 m above the center of the window!

PROBLEM 18 A boy on a cliff 49 meters high drops a stone and one second later throws a second stone after the first. They both hit the ground at the same time. How fast did the boy throw the second stone? Neglect air resistance.

This free fall problem is somewhat more difficult than average; the arithmetic will take a little longer to do than usual.

First find the time it takes for the dropped stone to hit the ground. From $d = \frac{1}{2}gt^2$,

$$t = \sqrt{2d/g} = \sqrt{\frac{2 \times 49\,\mathrm{m}}{9.8\,\mathrm{m/sec}^2}} = \sqrt{10} = 3.16 \text{ sec (or 3.2 sec).}$$

Since the second stone was dropped one second after the first, it takes t = 2.16 seconds (or 2.2 seconds) to drop 49 meters. Its average velocity is therefore

$$v_{av} = \frac{49}{2.16} = -22.7 \text{ m/sec.}$$

$$\text{(or } v_{av} = \frac{4.9}{2.2} = -22.3 \text{ m/sec).}$$

But $v_{av} = \frac{v_i + v_f}{2} = \frac{v_i + (v_i + at)}{2} = v_i + \frac{1}{2}at$

Using t = 2.16, and $v_{av} = -22.7$, $v_i = -22.7 + (4.9 \times 2.16) = \underline{-12.0 \text{ m/sec}}$.

(Using t= 2.2, and $v_{av} = -22.3$, $v_i = -22.3 + (4.9 \times 2.2) = \underline{-11.5 \text{ m/sec.}}$)

This problem could also be done by solving for v_i in $d = v_it + \frac{1}{2}at^2$ once t = 2.16 (or 2.2) is known. The arithmetic would then be:

$$v_i = \frac{d - 1/2\,at^2}{t} = \frac{-49 + 4.9 \times 2.16^2}{2.16} = \underline{-12.0 \text{ m/sec}}.$$

PROBLEM 19 Show that the expression $\frac{v^2}{R}$ for centripetal acceleration has the units of acceleration.

An exercise in dimensional analysis related to circular motion.

The units of velocity are (length/time), so the units of v^2/R are $(\text{length/time})^2 \times \frac{1}{\text{length}} = \frac{\text{length}}{\text{time}^2}$ the same as those of acceleration.

* * *

For more complicated situations, it is useful to have a notation for the calculation of dimensions. We frequently denote the dimension of a quantity by enclosing it in square brackets. The dimensions of all quantities we have met so far can be expressed in terms of length, time and mass, which we can denote by L, T, and M. Thus, $[v] = LT^{-1}$, so that

$$\left[\frac{v^2}{R}\right] = L^2T^{-2} \times L^{-1} = LT^{-2} = [\text{acceleration}].$$

Dimensional analysis is extremely useful as a check on derived formulas, to ensure that no factors have been lost. You may want to illustrate by considering additional examples. For instance, does the equation $d = v_it + \frac{1}{2}at^2$ check dimensionally?

PROBLEM 20 The minute hand of a large tower clock is 2.0 m long.

(a) At a quarter past the hour, what are the direction and magnitude of the velocity of the end of the minute hand?

(b) What is its acceleration?

a) To find the direction of $\vec{v}$, imagine drawing a tangent to the circle on which the end of the minute hand moves. At quarter past the hour the direction of $\vec{v}$ is down.

The magnitude of $\vec{v}$ can be found from the fact that the tip of the minute hand travels a circular path equal to $2\pi R$ in one hour.

$$v = \frac{2\pi R}{T} = 12.6 \text{ meters/hr} = 12.6\,\frac{\text{m}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = \underline{3.5 \times 10^{-3} \text{ m/sec}}.$$

b) The acceleration in uniform circular motion is always perpendicular to the velocity vector. It is always directed toward the center of the circle. In this case it is directed from the three toward the nine on the clock. Its magnitude is:

$$a = \frac{v^2}{R} = \frac{(3.5 \times 10^{-3} \text{ m/sec})^2}{2 \text{ m}} = \underline{6.1 \times 10^{-6} \text{ m/sec}^2}.$$

PROBLEM 21

(a) What is the centripetal acceleration of a hammer thrower's hammer if his axis of rotation is 2.0 m from the hammer and he turns around once every $\frac{2}{3}$ seconds?

(b) If the hammer weighs 70 newtons, what force must the hammer thrower exert to maintain this centripetal acceleration?

This is an easy problem but it excites some students because it gives typical numbers for a well-known but poorly understood sport. Possible extensions of the problem are given.

a) Since the athlete turns once every 2/3 seconds, and the head of the hammer goes $2\pi R = 2\pi \times 2.0 = 4.0\pi$ meters per turn, the velocity of the hammer is

$$v = \frac{4.0\pi}{2/3} = 6.0\,\pi \text{ m/sec} \approx 19 \text{ m/sec}.$$

The acceleration is then

$$a = \left(\frac{2\pi}{T}\right) v = \frac{2\pi}{2/3}(6\pi) = (3\pi)(6\pi) = 18\pi^2 = \underline{1.8 \times 10^2 \text{ m/sec}^2}.$$

b) To find the force required to produce this acceleration we must first find the mass, m, from the given weight. A 70 newton hammer corresponds to a 7 kg mass (7.14 kg if higher precision were warranted). The required centripetal force is therefore

$$F = ma = 7 \times 1.8 \times 10^2 = \underline{1.3 \times 10^3 \text{ nt}}.$$

* * *

Possible Extension:

A. How far would the hammer go if it were released when it made an angle of 45° with the horizontal?

The initial vertical velocity would be $v_{iy} = 6\pi \sin 45° = 6\pi \times 0.707 = 13.3$ m/sec. The time for the hammer to reach its highest point is then found from $v_y = v_{iy} - gt$.

$t = \frac{v_{iy} - v_y}{g} = \frac{13.3}{9.8} = 1.35$ seconds. The total flight time is then 2.70 seconds. The horizontal distance traveled is then $d = v_{ix}t = 13.3 \text{ m/sec} \times 2.7 \text{ sec} = 45.9$ meters.

B. Students may be interested in the world's record for hammer throwing. The 7.3 kg hammer used has been thrown 70 meters. To get a 70 meter throw, the initial velocity must be increased. The distance of the throw is proportional to v^2:

$$v_{ix} = v_{iy} = 0.707\,v, \text{ because } \theta = 45°.$$

For the x motion, $x = 0.707\,vT$. T can be obtained from the y motion because it takes T/2 sec to reach the greatest height (at which $v_y = 0$).

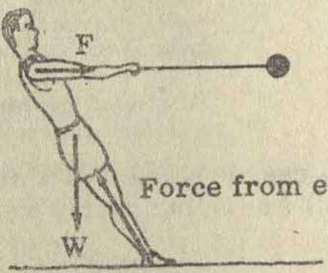
Since $v_y = v_{iy} - gt$, $T = \frac{v_{iy} - v_y}{g} = \frac{0.707\,v}{9.8}$.

Since $T = \frac{t}{2}$, $T = \frac{0.707\,v}{4.9}$. Therefore $x = 0.707\,v \times \frac{0.707\,v}{4.9} = \frac{v^2}{9.8}$.

If $x = 70$ m, $v^2 = 9.8\ \text{m/sec}^2 \times 70$ m, or $v = 26.2$ m/sec. If $R = 2$ meters, $a = \frac{v^2}{R} = 343\ \text{m/sec}^2$. The required centripetal force for a 7.3 kg hammer, thrown 70 m, is $7.3\ \text{kg} \times 343\ \text{m/sec}^2 = 2.5 \times 10^3$ nt (550 lbs!)

C. You will probably not want to discuss the position of the hammer thrower in class. But in case some students should express interest, the following explanation may help you.

In what position does the thrower release the hammer? The hammer must be released when the velocity is in the required direction, i.e., when its handle and the thrower's arms are perpendicular to the desired direction.



The hammer exerts a horizontal force on the thrower. Why doesn't he fall over? Where does the counter acting horizontal force come from?

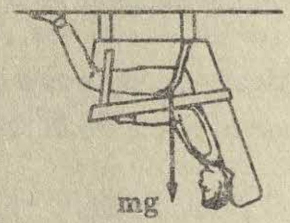
The answer is illustrated in the figure. The thrower leans back until the moment about the pivot (his feet) of the force due to the hammer is balanced by the moment of his own weight. Although at this stage students have not been formally introduced to the concept of moments, the treatment of this point can be convincing although rather sketchy.

PROBLEM 22

> How fast must a plane fly in a loop-the-loop of radius 1.00 km if the pilot experiences no force from either the seat or the safety belt when he is at the top of the loop? In such circumstances the pilot is often said to be "weightless."

This is an interesting problem in which centripetal acceleration must be equated to weight.

Since the pilot cannot "turn off" gravity we know that, whatever other forces may or may not be acting, he is always subject to the force of gravity, mg. This force acting alone results in an acceleration downward as though in free fall, $a = -9.8\ \text{m/sec}^2$. If there is no apparent force from seat or belt opposing this motion, they too must be accelerating with $a = -9.8\ \text{m/sec}^2$. This condition could occur if the pilot, seat and belt were all falling freely. It also occurs when all three are moving in a circular path with a centripetal acceleration of $9.8\ \text{m/sec}^2$ downward.



The speed that would provide this centripetal acceleration can be found from $a = \frac{v^2}{R}$.

$$v = \sqrt{aR} = \sqrt{9.8\ \text{m/sec}^2 \times 10^3\ \text{m}} = \underline{99\ \text{m/sec}}\ \text{(about 220 miles/hour)}.$$

The pilot is said to be weightless because he does not experience any support for his weight such as normally would be supplied by the floor under a standing man.

From $a = \frac{v^2}{R}$ we can see that an increase in velocity (and/or a decrease in radius) will increase the acceleration. Hence if the velocity of a plane in a loop of radius 1 km is greater than 99 m/sec, the centripetal acceleration will be greater than 9.8 m/sec^2 and the pilot will be pushed down by the seat when he is at the top of the loop. A lesser velocity will result in less acceleration and the pilot will hang from his seat belt.

PROBLEM 23

Some people speak of a pilot coming out of a power dive in a jet airplane as weighing several times his own weight. We sometimes say that the same pilot, falling free before opening his parachute is weightless.

(a) Do these statements make sense in the light of our definition of weight as the force of gravity on an object?

(b) What is meant by "several times his own weight" and "weightless" above? Be prepared to discuss these questions in class.

This problem calls for a qualitative discussion which is related to the quantitative calculation of apparent weightlessness in Problem 22, and of apparent extra weight in Problems 24 and 25.

a) If weight is used in its technical sense as the force of gravity on an object, statements referring to weightlessness or increased weight of a particular man in a particular gravitational field do not make sense. Otherwise they do; they mean mg is 0 or is increased, etc. The weight of an object may depend on its position (insofar as the gravitational field varies with position), but it does not depend on the motion.

b) "Several times his own weight" and "weightless" refer to the supporting force rather than merely to that part of the force used to overcome the gravitational force. For example, consider a man standing in a closed elevator. He judges his own weight by the upward force exerted by the floor; this force would be equal to his weight if the elevator were not accelerating. This force can be judged by the pressure on the feet; it could also be measured by a scale if the man stood on a scale in the elevator.

Now imagine the elevator accelerating upward. The floor of the elevator must push up with a bigger force than before in order to accelerate the man. This force is quite real and would be measured accurately either by the scale or by a man whose feet were quantitatively sensitive to pressure. Thus, if the man did not realize he was accelerating he might think that he did weigh more. This same effect occurs when a pilot is coming out of a power dive or when an object swings from a string as in Problems 24 and 25.

If the elevator accelerates downward, there is less upward force exerted by the elevator. (The force of gravity is always the same, of course, but it is not measured directly; the net force is what is sensed by a scale or by the pressure on the feet.) If the elevator accelerates downward at 9.8 m/sec^2, there is no upward force exerted by the elevator and the man's feet feel no pressure. This effect can also be achieved at the top of a circular path as in Problem 22.

In summary, the apparent changes in weight are due to changes in supporting force. Actual weight changes can be judged from supporting forces only if there is no acceleration.

PROBLEM 24

A pendulum bob with a mass of 0.40 kg swings through the bottom part of its arc with a velocity of 0.70 m/sec. The distance from the point of suspension to the bob is 1.0 m. What is the tension on the string at this lowest point?

This problem requires the calculation of the force needed to produce a certain centripetal acceleration.

We know that $\vec{F} = m\vec{a}$ where $\vec{F}$ is the net force. In this case $\vec{F} = T - m\vec{g} = T - 3.9\,\text{nt}$.

The centripetal acceleration is

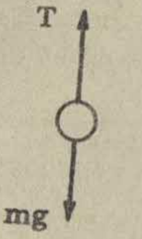
$$a = \frac{v^2}{R} = \frac{(0.70\ \text{m/sec})^2}{1.0\ \text{m}} = 0.49\ \text{m/sec}^2.$$

Substituting in $\vec{F} = m\vec{a}$,

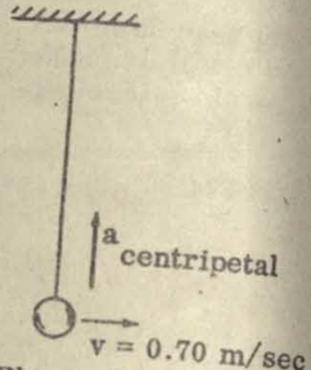
$$(T - 3.9)\ \text{nt} = 0.4\ \text{kg}\ (0.49\ \text{m/sec}^2)$$

$$T = 3.9\ \text{nt} + 0.2\ \text{nt}$$

$$T = \underline{4.1\ \text{nt.}}$$



Force diagram



Physical picture

PROBLEM 25

A string 5.0 m long of diameter 2.0 mm just supports a hanging ball without breaking.

(a) If the ball is set to swinging, the string will break. Why?

(b) What diameter string of the same material should be used if the ball travels 7.0 m/sec at the bottom of its swing?

This problem concerns the net force required to produce centripetal acceleration. It is similar to Problem 24. If the class has done Problem 24, part (a) of this problem will be easy. Part (b) is related to the strength of materials discussed in Chapter 4, Section 4 on scaling.

a) The string will break because it will have to exert a force in excess of the weight to produce the required upward (centripetal) acceleration.

b) The required tension is given by:

$$F = ma,$$

$$T - mg = ma,\ \text{and}\ T = mg + ma = m(g + a).$$

We can find $\vec{a}$ from v and R:

$$a = \frac{v^2}{R} = \frac{(7.0)^2}{5} = \frac{49}{5} = 9.8\ \text{m/sec}^2.$$

Since $T = m(g + a)$, and in this case $g = a$, $T = 2\,mg$.

Thus, the string tension for the swinging pendulum is twice the string tension for the pendulum at rest. The string for the swinging pendulum should be twice as strong.

In order to provide double the strength, the string must have twice the cross sectional area (see Chapter 4, page 46).

$$A_{\text{new}} = 2A_{\text{old}},\ \text{or}\ \frac{\pi d^2_{\text{new}}}{4} = 2 \times \frac{\pi d^2_{\text{old}}}{4},\ \text{or}\ d^2_{\text{new}} = 2 \times d^2_{\text{old}}.$$

$$d_{\text{new}} = \sqrt{2} \times d_{\text{old}} = \sqrt{2} \times 2.0\ \text{mm} = \underline{2.8\ \text{mm}}.$$

PROBLEM 26

An electron (mass = 0.90×10^{-30} kg) under the action of a magnetic force moves in a circle of 2.0 cm radius at a speed of 3.0×10^{6} m/sec. At what speed will a proton (mass = 1.6×10^{-27} kg) move in a circle of the same radius if it is acted upon by the same force?

We are given the mass, speed and radius of an electron moving in a circle. From this we can compute the centripetal force required to produce this motion. We are then given the mass of a proton and asked to find its speed when moving in a circle of the same radius under the same force.

From $F = \frac{mv^2}{R}$ we could compute the centripetal force on the electron. Then knowing this force and the mass of the proton, we could solve for the velocity of the proton when traveling in the same circle as the electron.

However, the step by step computation is not necessary:

For the electron, $F_e = \frac{m_e v_e^2}{R}$. For the proton, $F_p = \frac{m_p v_p^2}{R}$. We are given that $F_e = F_p$.

Hence, $\frac{m_e v_e^2}{R} = \frac{m_p v_p^2}{R}$, or $m_e v_e^2 = m_p v_p^2$, and $v_p^2 = \frac{m_e v_e^2}{m_p}$.

$$v_p^2 = \frac{m_e v_e^2}{m_p} = \frac{0.90 \times 10^{-30}}{1.6 \times 10^{-27}} \times (3.0 \times 10^6)^2 = \frac{9.0}{1.6} \times 10^{-4} \times (3.0 \times 10^6)^2.$$

$$v_p = \frac{3.0}{1.26} \times 10^{-2} \times 3.0 \times 10^6 = \underline{7.1 \times 10^4 \text{ m/sec.}}$$

PROBLEM 27

A body moving with a speed *v* is acted upon by a force that always acts perpendicular to the motion of the body but increases steadily in magnitude.

(a) Draw a sketch of the trajectory.

(b) Does the speed of the object increase, decrease, or remain unchanged?

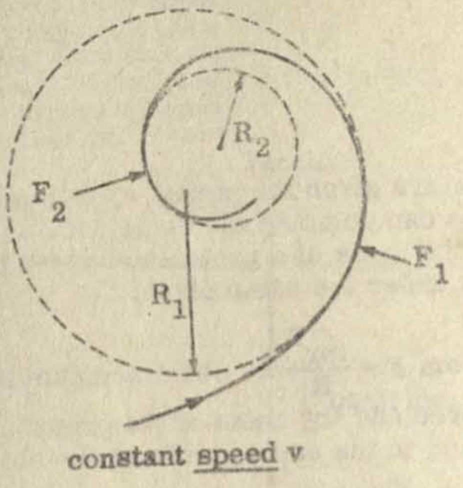
This problem on circular motion should be handled qualitatively. A quantitative treatment will be beyond most high school students (or anyone else).

a) We know that if the force were constant and perpendicular to the velocity, only the direction of v would change, making the body travel in a circle. If we consider what happens with a constantly increasing force, we can say that during a short interval of time the perpendicular force is approximately constant, and equal to, say, F_1, so the path during this time is approximately part of a circle of radius R_1, where

At any time later, the same argument can be made about a segment of the path, except that now the force is greater, (F_2, say), so the path at this later time is approximately part of a circle of radius R_2, where

$$F_2 = \frac{mv^2}{R_2}.$$

Since F_2 is greater than F_1, R_2 will be less than R_1. In mathematical terminology, the radius of curvature of the path is decreasing with time. In the figure, we have sketched the path, and also some of the circles of which the path is a part at different times.



The exact solution of this motion shows that the path of the particle, for a force increasing linearly with time, is the "Cornu spiral", which is well-known in optics. The exact solution is much too complicated to be undertaken with a high school class, so do not encourage students to do more than describe the qualitative characteristics of the motion.

b) The force acting on the body is always perpendicular to the velocity. Thus the acceleration produced by this force is perpendicular to the velocity at all points. The magnitude of the velocity therefore does not change, but its direction does change.

PROBLEM 28 As a home project, carry out the experiment indicated in Fig. 21-14. You may want to use a pendulum in place of the mass on the end of a spring. Make your own pendulum. If the period of your pendulum does not match the period of rotation of the phonograph turntable, what do you do?

If the periods of the turntable and pendulum do not match, the pendulum can be lengthened or shortened, altering its period to match that of the turntable. A few modern turntables have continuously variable speed controls. [The period of oscillation of a pendulum is: $T = 2\pi\,(\ell/g)^{1/2}$.]

PROBLEM 29 A spring of force constant k = 196 newtons/m hangs vertically from a peg with its lower end at $y = 0$. Then a 2.00-kg mass is hung on the spring.

(a) Plot the net force on the mass as a function of the y coordinate of its position from $y = -.250$ meter to $y = +.250$ meter.

(b) At what position on the y axis should the mass be released from rest if you want it to stand still?

(c) Suppose the mass is released from rest at $y = 0$. Describe its subsequent motion. How far will it move before it again is momentarily at rest? How long a time does it take?

This problem and Problem 30 deal with vertical springs in simple harmonic motion. Either problem would be suitable for a class discussion. If you wish, you can use the even simpler problem of a mass oscillating horizontally under the influence of a spring for an earlier class discussion.

a) The net force on the mass is the vector sum of the varying force due to the spring and the constant weight = -19.6 nt (where the negative sign indicates the downward direction). To plot the net force, tabulate the spring force and the net force as follows:

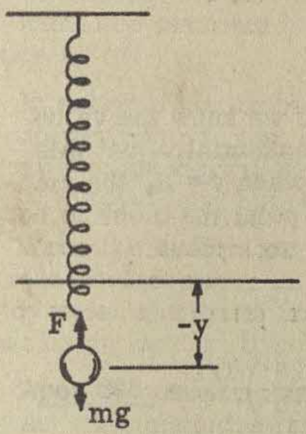
y (m)	-0.25	-0.20	-0.15	-0.10	-0.05	0	+0.05	+0.10	+0.15	+0.20	+0.25
F_{spring} (nt)	+49.0	+39.2	+29.4	+19.6	+9.8	0	-9.8	-19.6	-29.4	-39.2	-49.0
F_{net} (nt)	+29.4	+19.6	+9.8	0.0	-9.8	-19.6	-29.4	-39.2	-49.0	-58.8	68.6

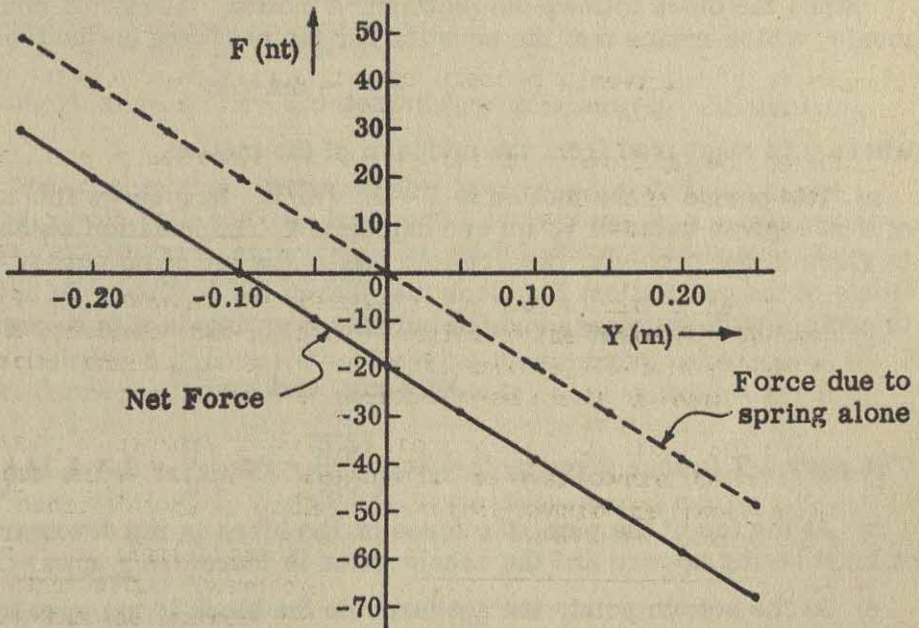
Alternatively, write the equation for the net force, F_{net}:

$$F_{net} = F_{spring} + \text{weight} = -ky - mg.$$

This is a straight line; therefore find F_{net} for the two extremes, $y = -0.25$ and $y = +0.25$, and use a ruler to get the line.

The graph then can be plotted:





Note: Since $F = kx$ is clearly a straight line, only two points on the table were really necessary.

b) The mass will remain at rest only at $F_{net} = 0$. From the graph this occurs at $y =$ -0.10 meters, i.e., 0.1 m below its old resting place. [Analytically, $F_{net} = -mg - ky = -19.6 - 196\,y = 0$. Thus, $y = -0.01$.]

c) At $y = 0$, the mass is subject to a net downward force of 19.6 nt; therefore, the 2.00-kg mass has a net downward acceleration of $9.8\,\text{m/sec}^2$. (This is just as in free fall because at $y = 0$, the spring exerts no force.) It will oscillate symmetrically about the point where the net force is zero, i.e. about $\underline{y = -0.10}$. It moves from $y = 0$ to $y = -0.20$ m, a distance of 0.2 meters. In moving from $y = 0$ to $y = -0.20$ m, the mass goes through half of a complete cycle. The time for a complete cycle, the period T, is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \left(\frac{2.00}{196}\right)^{1/2} = 0.634 \text{ sec.}$$

The time taken to reach the lowest point is half of this, i.e., 0.317 sec.

PROBLEM 30

A block of mass *m* rests on a horizontal platform. The platform is driven vertically in simple harmonic motion with an amplitude of 0.098 meter. When at the top of its path, the block just leaves the surface of the platform. (This means that at this point its acceleration is 9.8 m/sec² downward.)

(a) What is the period of the simple harmonic motion?

(b) When at the bottom point of its path, what is the acceleration of the block?

(c) What is the force exerted by the platform on the block at this bottom point?

Since the block follows the platform's motion, its motion must also be simple harmonic, which means that the equation for the net force on the block must be:

$$F_{net} - ma = ky$$

where y is measured from the midpoint of the motion.

a) The period of the motion is $T = 2\pi\sqrt{m/k}$. How do we find k? If we know the value of F at a given value of y, we can calculate k. Information which is essentially just this is given in the problem: we are told that at the top of its path (i.e., when y = R, the amplitude of the oscillation) the block just leaves the surface. So at this point the block is not in contact with the plane, and the only force acting on it is its weight mg. Thus

$$kR = mg, \text{ and } k = \frac{mg}{R}.$$

The period T is then given by $T = 2\pi\sqrt{\frac{mR}{mg}} = 2\pi\sqrt{\frac{R}{g}} = 2 \times 3.14 \left(\frac{0.098}{9.80}\right)^{1/2} = \underline{.628 \text{ sec}}.$

b) At the top of the path, the force on the block is mg downward, therefore at the bottom it must be mg upward and the acceleration is therefore g upward.

c) At the bottom point, the net force on the block is mg upward. This is the resultant of its weight mg downward, and the force exerted on it by the platform. The latter force must therefore be 2 mg upward.

PROBLEM 31

(a) What is the period of a pendulum consisting of a mass of 2.0 kg suspended on a light cord 2.4 meters long if *g* = 9.8 newtons/kg?

(b) Notice that we can use a pendulum to measure *g*. If the period of this pendulum is 3.0 sec, what is *g*?

This problem illustrates how a pendulum can be used to measure g quite accurately.

a) The period T, for a simple pendulum is given by the equation

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \times 3.14 \left(\frac{2.4}{9.8}\right)^{1/2} = 6.28 \times .495 = \underline{3.1 \text{ sec.}}$$

b) If ℓ remains 2.4 m, but T becomes 3.0 sec instead of 3.1 sec, what would g have to be? The above relation gives:

$$\left(\frac{T}{2\pi}\right)^2 = \frac{\ell}{g}, \text{ and } g = \ell\left(\frac{2\pi}{T}\right)^2 = 2.4\left(\frac{6.28}{3.00}\right)^2 = \underline{10.5 \text{ nt/kg}}.$$

Be sure that students realize how easy it would be to find this difference. If the swings of the pendulum are counted for six minutes, for example, there would be 116 swings for part a) and 120 swings for part b). It is easy to set up a simple pendulum as a class demonstration. Accurate values of the period can be obtained by timing a large number of swings. You can verify that the period varies as $\sqrt{\ell}$, and also obtain a surprisingly accurate value for g.

The formula we have used was derived under the assumption that the angle of swing is "small". If the angle of swing is kept less than about 30° the error will not be appreciable.

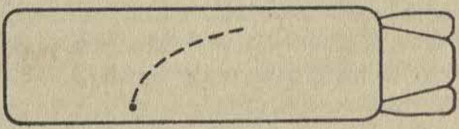
PROBLEM 32 While a bus is in motion along a level, straight road, we roll a marble from one side to the other across the floor of the bus. Its path is a straight line relative to the bus. Later we roll it again, and this time the path is a parabola which bends toward the front of the bus (concave toward the front of the bus). Describe the motion of the bus in each case. Be prepared to discuss in class.

This is a problem involving relative motion and the effect of a moving or accelerating frame of reference.

In solving this problem, one must realize (idealize) that the marble is not subject to any force from the bus. Thus, while it is rolling, the marble must move in a straight line relative to an inertial system such as the fixed stars. The marble can, of course, have any initial velocity but that velocity then cannot change.

When the marble moves in a straight line relative to the bus, a man in the bus sees no marble acceleration, so he concludes that no force is acting on the marble. We have already noted that in an inertial frame, no force acts on the marble, so in this case the bus must be an inertial frame; i.e., it is moving at constant speed (or is at rest).

When the marble moves in a parabola toward the front of the bus, the observer in the bus sees a constant apparent acceleration, $\vec{a}$, (relative to the bus) toward the front of the bus. He concludes that a force, $F = ma$ is pulling the marble toward the front of the bus.



However, we know that in an inertial frame the marble has no force acting on it. Therefore we can conclude that rather than the marble accelerating forward, the bus must be accelerating backward in the inertial frame.

This is a deceleration, and it must have the value, $\vec{a}$, derived above by the rider. The force deduced by the rider is then what we call a fictitious force. In the non-inertial frame it must be assumed to be acting in order to understand the observed motion, but in an inertial frame it does not exist.

Chapter 22 – Universal Gravitation and the Solar System

PURPOSE To introduce Newton's law of universal gravitation in its historical context, and to present an account of man's continuing effort to understand the workings of the universe.

CHAPTER SUMMARY

Chapter 22 presents a historical account of the attempts to construct a kinematic and dynamical model for the solar system, culminating in the law of universal gravitation.

Sections 1 through 5 present a bit of the history of the centuries of speculation about the nature of the solar system, beginning with the ideas of the ancient Greeks. The sections briefly discuss how Kepler built on the work of Copernicus and Brahe, in arriving at a simple but satisfactorily precise kinematic description.

Section 6. From the viewpoint of kinematics, the choice of a frame of reference is arbitrary. However, when the dynamics of a situation are considered, one frame may become highly favored as a logical necessity.

Sections 7 through 11 discuss some of Newton's many contributions, stressing the source and use of his law of universal gravitation. In closing, it is noted that even this useful law has limitations, and does not precisely explain all observations.

EMPHASIS

Chapters 20, 21, and 22 form a natural unit in which the ideas of Chapters 20 and 21 are brought to bear on planetary dynamics and universal gravitation in Chapter 22. Thus Chapter 22, as well as introducing some new material, provides a substantial review of the earlier work. If you are pressed for time, Chapter 22 can be handled largely as a reading assignment with a bare minimum of class discussion and problem solving, without loss in continuity in progressing through the volume.

However, an overly rapid survey of this chapter will miss some of the most exciting parts of physics. A study of science cannot be divorced completely from a study of the history of science, nor can an appreciation of the contribution of great scientific figures be achieved without a knowledge of the facts of science. The professional scientist's enthusiasm for his work can be traced in part to his appreciation for the fact that he is a member of a team – a team that began many centuries ago and which will continue indefinitely. Just as the historical figures were able to contribute to the understanding of problems that disturbed them, so does the modern scientist, drawing on their successes and failures, hope to contribute to the clarification of problems of contemporary interest. Modern physics derives from the work of men like Galileo and Newton. Mechanics, viewed from the scientific environment of its great innovators, can be vital and exciting. Hopefully, it is this flavor which your students can derive from the present chapter. If your students can find the time for it, a little outside reading at this point can contribute hugely to their perception not only of scientific principles, but of the human nature of science, and the simply tremendous intellectual accomplishments of men like Newton, Kepler and Galileo. Further, planetary physics is not simply a matter of historical interest. A realistic perception of the problems of space travel is possible only through a grasp of the dynamics involved. In this course, the ideas return in the Rutherford model of the atom.

SCHEDULING CHAPTER 22

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part III. Sections which are enclosed by brackets [ ] can be deemphasized or skipped in class discussion without loss of continuity.

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. [1], [2], [3], [4], 5	1	0	-	$\frac{1}{2}$	0	-
Sec. 6	1	1	III-7	$\frac{1}{2}$	0	-
Secs. 7, 8, 9, 10, [11]	3	0	-	2	0	-

SUPPLEMENTARY MATERIAL FOR CHAPTER 22

Laboratory. Experiment III-7, Law of Equal Areas, uses a simple pendulum, whose bob distributes sand as it moves in an ellipse, to investigate the law of equal areas in equal times. The bob and sand not only mark out an ellipse, but serve as a timing device. The mass of sand deposited on a given arc is proportional to the time required by the pendulum to swing through that arc. This experiment can be done about the middle of Chapter 22. See the yellow pages for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk(*). Answers to problems are given in the green pages.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1 - 4		3*, 4*		3*, 4*	1, 2
5	5, 6	7		5, 7	
6		8*		8*	
7		9*			
8	10*, 14, 15 16*, 17*	11*, 12* 18*	13	10*, 11*, 12*, 13	
general	19*	20*, 21, 22	23*	19*, 20*, 21, 22, 23*	

Films. "Solar System Kinematics" is in preparation. When complete it can be used with the first few sections of this chapter.

"Universal Gravitation" by J. N. P. Hume and D. G. Ivey of the University of Toronto reconstructs the logic behind the law of universal gravitation through imagining how the law might have been discovered on "Planet X" in another solar system which has only the one planet. The film points out how the frame of reference adopted affects a kinematic description. Observation of the orbits of artificial satellites enables determination of the inverse square law of force, the constancy of R^3/T^2 and eventually leads to a proper kinematic description of Planet X's solar system, and to the law of universal gravitation. This film is closely related to Sections 6, 7, and 8.

"Elliptic Orbits" by A. V. Baez. Beginning with an elliptic orbit and Kepler's law of areas, gravitational force is demonstrated to obey an inverse square relation. The derivation is almost entirely geometric. This film can be used in connection with discussion of Sections 7 and 8. Running time: 18 minutes.

Supplementary Reading. In addition to the references at the end of Chapter 22, the following Science Study Series books will interest many students:

The Birth of a New Physics, by I. Bernard Cohen;
The Universe at Large, by Herman Bondi;
The Watershed, by Arthur Koestler.

Other titles relating to Part III of the course will be published in this series.

Section 1 – Early Planetary Models
Section 2 – Copernicus' Planetary System
Section 3 – Objections to Copernicus' Theory
Section 4 – Tycho Brahe

PURPOSE To present the historical background for the work of Kepler and Newton.

EMPHASIS This material can be handled almost entirely as a single reading assignment. It is not necessary to belabor the details of geocentric models, but how different frames of reference lead to different interpretations should be pointed out.

FILM If "Frames of Reference" has not been shown previously, it can be used with the material of these sections.

COMMENTS It is impossible to separate a discussion of modern science from its historical roots. Present day thinking is as much conditioned by the questions raised by the ancient Greeks as by the work of this century. While some of the answers proposed by the ancients may seem humorously wrong in the light of modern theory, no scientist can help but marvel at the enormous insight apparent in their questions, nor wonder where we might stand today if their original efforts had been steadily maintained instead of stagnating for almost two thousand years.

The weakness in the approach of the Greeks to a study of astronomy was their reliance on intuition rather than on direct observation in their search for reasonable models. An example was their insistence on the use of circular orbits for celestial motion, simply because the circle was considered "perfect". No one bothered to study the motion in sufficient detail, as did Brahe centuries later, to show that motion was not quite circular. Because of this strong reliance on instinct, it is difficult to separate ancient science from its contemporary philosophy. Even today, basic philosophical questions are frequently as disturbing to scientists as direct observations for which they have no ready explanation.

The notion of a universe composed of bodies moving in "perfect circular" orbits did not originate with Plato. These ideas go back at least as far as the Pythagorean school (580 to 500 B.C.) more than a century earlier. At least two members of the Pythagorean school, Hicetas and Ecphantus, proposed a fair model of what we generally call the Copernican view of the solar system, with an earth which rotates daily on its axis and an immobile sphere of fixed stars. However, in this model, the sun and moon were assumed to move in circular orbits about the earth. Plato believed firmly that the earth stood still, but even in his day, Heraclides taught that the earth rotates on its axis once a day, and even suggested that Venus and Mercury moved in circular orbits about the sun while the latter revolved about the earth. The accepted "Copernican" model was probabl first stated by Aristarchus of Samos (ca. 310 to 230 B.C.), who purportedly published the following hypothesis: "The fixed stars and the sun remain unmoved, but the earth revolves about the sun in a circumference of a circle, the sun lying in the middle of the orbit." The basic suggestion for the Ptolemaic system undoubtedly arose from the work or the great ancient astronomer, Hipparchus, who died about 125 B.C. Hipparchus, unlike many of his predecessors and successors, was a great observer and made very careful measurements of celestial motion. He detected small variations in the plane of the ecliptic* due to precession of the earth's axis, deriving a figure which agrees, within one percent, with its modern accepted value. This motion is so small and slow that it takes about 26,000 years for its complete cycle.

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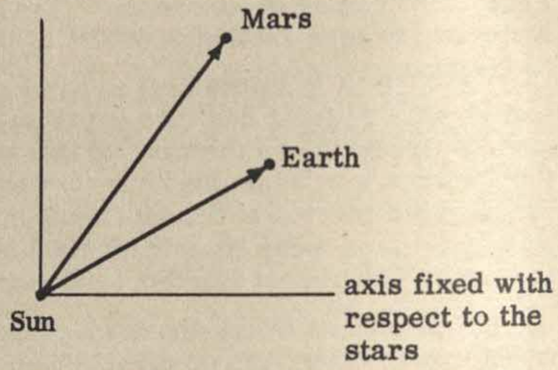
*The ecliptic plane is the plane of the earth's orbit.

The heritage of Greek science was advanced by the Arabs, after their conquest of the Egyptian city of Alexandria which had become the principle center of Greek learning by the third century B.C. During the middle ages, the Ptolemaic system returned to Europe via translations from the Arabic, together with star catalogues compiled by Arab astronomers

As pointed out in Chapter 3, parallax should be noticeable when observing any system from one which is in relative motion. Copernicus was concerned about the fact that he could not observe stellar parallax, particularly since, in his notion, the sphere of fixed stars was fairly near the earth. He had no idea of the enormous distances actually involved, and the fact that stellar parallax is far too small to measure by simple means. It was not until the year 1838 that stellar parallax was first noted by the German astronomer, F. W. Bessel. Remembering that the largest parallactic angle ever observed is 0.756 seconds of arc (for the nearest star, alpha Centauri), it is easy to see why the effect was so long unobserved. The correct notion of the almost infinite distance between the earth and the stars was probably first proposed by Giordano Bruno in the 15th century. (See Figure 3-5.)

* * *

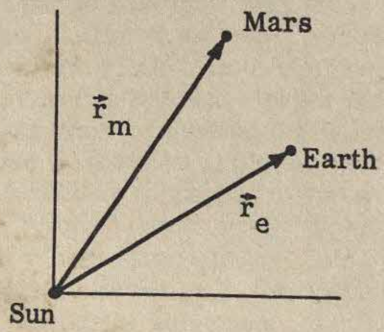
The description of planetary motion and the equivalence of the different descriptions (geocentric, heliocentric etc.) is greatly aided by the use of vectors. The figure at the right shows the radius vectors from the sun to the earth and from the sun to Mars. The axes are fixed relative to the stars.

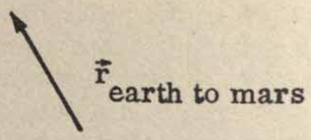


It is a good approximation to say the planets move in circles about the sun at constant (but different) speeds. Then in our diagram the two radius vectors from the sun, $\vec{r}_e$ to the earth and $\vec{r}_m$ to Mars, simply turn about the sun with unchanging length. This is Copernican view.

To locate a planet such as Mars relative to the earth we simply note that

$$\vec{r}_m = \vec{r}_e + \vec{r}_{\text{earth to mars}}; \text{ or conversely, } \vec{r}_{\text{earth to mars}} = \vec{r}_m - \vec{r}_e.$$





The vector $\vec{r}_{\text{earth to mars}}$ changes as both planets move just as if the earth and Mars were joined by a rotatable joint at the sun. A model of such a mechanical linkage is easy to make with two sticks and a nail. Note that the vector from sun to earth must make one revolution each year relative to the reference axes, and that from sun to Mars one revolution in a Martian year (687 days).

* * *

In evaluating the work of Copernicus, it is important to remember that, prior to Brahe, there was no really precise data on the motions of celestial bodies. Brahe, who was no believer in the Copernican theory, was probably motivated as much by a determination to disprove Copernicus' model as by anything else. To him, one of his great achievements was the failure to detect stellar parallax, even with his highly accurate observations, thereby "disproving" the Copernican model.

At the same time, Brahe was not willing to accept the enormous complexities of the Ptolemaic model and developed a geocentric system of his own. In Tycho's system Mercury and Venus were assumed to revolve around the sun, while the sun and other planets revolved about the earth (similar to the model of Heraclides).

* * *

Most of the reaction against the Copernican system arose not on publication of Copernicus theory, but much later, about the time that Galileo confirmed the Copernican hypothesis. Before Galileo, the Copernican system apparently was dismissed simply as just one more hypothesis with slightly more geometric simplicity than that proposed by Ptolemy. The real concern arose when Galileo, with his telescope, presented hard facts which destroyed the long held myth of "perfection" in the heavens.

Galileo found four moons revolving around the planet Jupiter, thus providing an example of rotation about an object other than the earth. Furthermore, he was able to observe the phases of Venus, and in so doing, he discredited the Ptolemaic model (Figure 22-4) which placed Venus always between the earth and sun. In the latter case, Venus would never progress beyond the crescent phase. He also observed the motion of sun spots which suggested that the sun was rotating about its axis, thus lending credence to the proposal that the earth itself might be spinning.

Section 5 — Kepler
Section 6 — Kinematic Description and the Dynamical Problem

PURPOSE To introduce Kepler's laws of planetary motion in their historical context, and to indicate their significance in leading to a simple, unified, dynamical explanation of motion.

EMPHASIS Some class time, perhaps one half a period, should be spent on discussion and problems. Students should "know" Kepler's three laws and understand how they contributed to the final solution of the dynamical problem.

FILM When the film "Solar System Kinematics" is completed, it will be appropriate for use with these sections.

LABORATORY Experiment III-7, Law of Equal Areas, can be done in connection with work on these sections.

DEVELOPMENT The important points to be brought out in discussing these sections are:

1. Brahe's painstaking observations provided the data for the problem in hand. An analogy could be drawn between Brahe's records and the lists of angles of incidence and refraction that Snell must have had before he was able to discern the order represented by Snell's law.

2. Kepler's work was analogous to the enunciation of Snell's law. However, in the case of the solar system problem, the data were much more complex and the accomplishment of seeing order among them was that much the greater. Kepler stated the kinematical problem in analytic terms.

3. A properly conceived geocentric system will do just as good a job of describing

the motions of the solar system as does the heliocentric system. However, when one looks for a solution to the dynamical problem he must propose a force to account for every observed acceleration. In a geocentric system, the accelerations are so complex that the gravitational force might easily get lost in the maze of other, fictitious, forces. In a heliocentric system, the central force stands out clearly. Thus from such an approach Newton was able to analyze the problem in a productive way.

QUIZ QUESTIONS 1. An earth satellite, S_1, moves in a circular orbit with radius four times the earth's radius. Compare its period of revolution with that of a second satellite, S_2, which is very close to the earth's surface. What is the ratio of their speeds?

($\frac{R^3}{T^2} = K$. Therefore, $\frac{R_2^3}{T_2^2} = \frac{(4R_2)^3}{T_1^2}$, and $\frac{T_1^2}{T_2^2} = \frac{64R_2^3}{R_2^3}$. Compared to satellite S_2, the satellite S_1, which orbits at 4R, will take 64 times as long to complete one circuit around the earth. The ratio of speeds is given by:

$$\frac{v_2}{v_1} = \frac{2\pi R_2}{T_2} \times \frac{64T_2}{2\pi 4R_2} = 16.$$ Hence, $v_2 = 16\, v_1$.)

2. Can two different frames of reference give equally valid kinematic descriptions of the motion of an object? (Yes, but one may be much more complicated than the other.)

3. How does Newton's law influence the choice of a frame of reference? (The kinematical description should lead to the simplest and easiest force analysis.)

4. Describe the motion of an artillery shell fired horizontally from the viewpoint of
   (a) an observer moving along in a jet plane which has a horizontal velocity equal to the horizontal component of velocity of the shell;
   (b) an observer in free fall (no air resistance).

(a. The shell drops straight down with acceleration = g. b. The shell appears to be going away in straight line horizontal travel with constant speed.)

Section 7 — Newton
Section 8 — Universal Gravitation

PURPOSE To present a simplified version of Newton's analysis of the dynamical problem couched in Kepler's kinematical laws.

EMPHASIS These two sections are the heart of Chapter 22. Students will profit from a classroom discussion following home study of the sections. Discussion should be directed first toward consolidation of their understanding of Newton's treatment of Kepler's laws. When they have this well in hand, discussion should be extended to the relevant exercises in HDL.

LOGICAL DEVELOPMENT 1. Approximate the planetary orbits as circles.

2. Any object in uniform circular motion moves with a centripetal acceleration, $a = 4\pi^2 R/T^2$.

3. Kepler noted that for all planets (sun satellites) $R^3/T^2 = K_s$. Substituting, $a = 4\pi^2 K_s/R^2$; and applying Newton's law, $F = 4\pi^2 K_s m/R^2$.

4. Assume (as we now know!) that for earth satellites there is also a relationship $R^3/T^2 = K_e$. Then for any object in the earth's gravitational field, the force which acts would be: $F = 4\pi^2 K_e m/R^2$.

5. At the earth's surface, $F_{surface} = mg$, therefore $g = 4\pi^2 K_e/R^2_{surface}$, and knowing g and $R_{surface}$, K_e can be found.

6. Having established a value for K_e and knowing the radius, R_m, of the moon's orbit around the earth, its acceleration, a_m (or its period, T_m) can be calculated and compared to the observed values. The agreement is found to be well within the errors inherent in the observations.

7. It remains to establish a connection between K_s, K_e and similar constants that would apply to other sources of gravitational attraction. Symmetry suggests the trial, $K_s = Gm$, $K_e = Gm_e$, etc., where G is a universal constant which describes the strength of the gravitational interaction. This gives for the gravitational force, $F = Gm_1 m_2/R^2$, a law that has now been verified by a host of observations.

FILMS "Universal Gravitation" deals directly with the principal ideas leading to the concept of universal gravitation. "Elliptic Orbits" gives an interesting demonstration of how the inverse square law of force can be deduced from an elliptic orbit.

COMMENTS It may be useful for you to have clearly in mind the role played by each of Kepler's laws. Their significance can be described as follows:

1) The observation that equal areas are swept out in equal times by a radial line to the planet from a fixed point in space can be interpreted as proof that the force which is acting is a "central force" – i. e., a force whose origin is a fixed point in space and whose direction of action is toward or away from that point. (Note that this is equally true for planetary motion under an inverse square law and for the pendulum motion of Experiment III-7, where the motion is simple harmonic and the force is proportional to the first power of the displacement.)

2) Analysis of the observation that any single orbit is elliptic in shape with the force center at one of the foci of the ellipse provides proof that the force which is acting has an inverse square dependence on the distance. (Note that in Experiment III-7 the center of force is at the center of the ellipse, not at one of its foci. This is consistent with the direct first power and not the inverse square force law.) A demonstration that the inverse square force law follows from an elliptic orbit is shown in the film "Elliptic Orbits".

Here it should be noted that for circular orbit, the center of the orbit and the foci of the degenerate ellipse are at one and the same point. Therefore no conclusion can be drawn about the force law in this case. Another way of seeing this is to note that a circular orbit samples only one radial distance. Therefore it cannot be expected to give information about the distance dependence of the force in question.

3) Kepler's third observation, summarized in his third law, was that for every planet the ratio R^3/T^2 had the same value. This observation can be used in various ways depending on the structure of logic that is being developed.

At the high school level it is not possible to analyze elliptic orbits. Therefore the text takes note of the fact that the ellipticity is usually small and then goes on to approximate each orbit as a circle, with radius equal to the mean radius at the true ellipse. In this approximation as noted above, it is impossible, from the orbit of any single planet, to deduce anything about the distance dependence of the force which is acting. However, under the assumption that the planets are all attracted to the sun by forces which are in direct proportion to their masses (the equivalence of gravitational and inertial mass) the inverse square law can be deduced from the constancy of R^3/T^2. In this development, other evidence is accepted as proof of the equivalence of inertial and gravitational mass. This equivalence serves, in effect, to normalize the behavior of the various planets to fit one single force field. Then $R^3/T^2 = K$ identifies the distance dependence of this force field.

On the other hand, if the elliptic orbits are accurately observed and analyzed, any one of them is sufficient to provide proof for the inverse square law over the range of distance which is sampled by that orbit. If this evidence is accepted with complete generality, as proof of the inverse square law, then the $R^3/T^2 = K$ data serve as evidence that the ratio of inertial to gravitational mass is the same for all the planets.

Section 9 – Some of Newton's Later Accomplishments
Section 10 – Laboratory Tests of the Law of Universal Gravitation
Section 11 – A Small Discrepancy

PURPOSE To fill out the story of the law of universal gravitation.

EMPHASIS Very little discussion of these sections should be required. However, some closely related topics are presented in the following comments. If the film, "Forces", was not used earlier, the sequence showing the nature of the Cavendish experiment is relevant here.

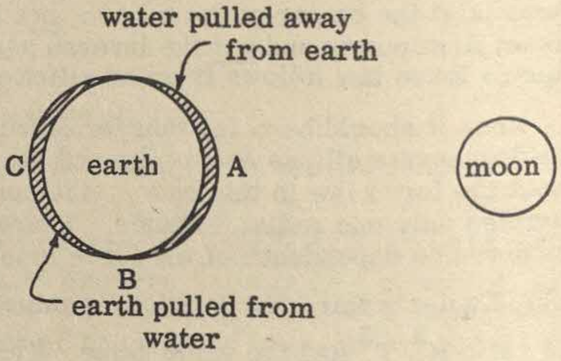
ADDENDA

General Relativity. Newton's work led to the recognition of the proportionality of gravitational and inertial mass. Einstein, in his General Theory of Relativity, takes this apparent coincidence and states it to be a fundamental and inherent property of matter. It is related to the indistinguishability of the effects of gravitation and acceleration. Appendix 4 at the end of this volume of the Guide is a supplement on General Relativity.

* * *

Tides. Students living in the middle of the continent may not be familiar with tides. Although the details of tidal motion are complicated by local variations in the shore line and ocean floor, the principal motions are caused by gravitational attraction of the moon for the earth. This phenomenon is illustrated below. (Scale enormously exaggerated.)

Now that students know that the force between the moon and earth results in an acceleration, they may be able to better understand the tides. Essentially, the moon and earth freely fall (accelerate) toward one another. The force per unit mass is the parameter which establishes the acceleration of a particle. Since the gravitational force field of the moon varies as the inverse square of the distance from the moon's center, the acceleration toward the moon of a particle at A tends to be greater than that of a particle at B, which in turn tends to be greater than that of a particle at C. If the earth were rigid, no tidal motion could result. The external force differences would be compensated by internal forces. When water is the substance at A or at C, since it is free to flow, changes in level will occur. Thus the moon's attraction produces two bulges in the ocean on opposite sides of the earth. Since the earth rotates once in 24 hours, there are two high tides per day at any ocean location, although they may not be of exactly equal height. While the influence of the sun must also be considered in calculating the exact size of the tides, its effect is considerably smaller than that of the moon.



The force exerted by the sun upon a unit mass anywhere on the earth is about 100 times greater than the corresponding force exerted by the moon. Nevertheless, the tidal effect of the moon is about twice that of the sun. This is because the difference between the force of the moon on a unit mass on the earth's surface at A and on another at D, is more than twice as great as the difference in the sun's force at these points (this is because the fractional change in the inverse square of the distance from the moon to these points is almost 400 times greater than in the fractional change in the inverse square of the distance from the sun to these points.

The highest tides occur about twice a month approximately with the new moon and full moon, when the sun, earth, and moon are all in positions along the same line. Tides may vary considerably in magnitude from place to place along a coast because of details of the local topography. The largest tides are most frequently seen in partially enclosed bays. In the Bay of Fundy, for instance, the difference between high and low tide is over 50 feet.

Fluctuations in Gravitational Force. To first approximation, the acceleration of gravity is constant over the survace of the earth. To second approximation, it varies from point to point on the surface because of variations in height of the terrain and the fact that the earth is not exactly spherical. To third approximation, the acceleration of gravity varies with the distribution of masses in the earth's crust at the particular location. This fact is of great importance to geologists, since hidden features of the earth's curst may be detected by accurate measurement of local variations in g. If g is measured above a large cavern which may be filled with oil, it will have a smaller value than if measured over a region of solid rock, and a still smaller value than if measured over a large deposit of lead or gold. The value of g also depends on the rotation of the earth, as discussed in Problem 21.

* * *

Composition of the Earth. Cavendish's experiment enabled him to find the mean density of the earth, and determined that this was about 5 1/2 times that of water. Since the surface rocks have a density of only 2 to 3 times that of water, the interior of the earth must obviously be made of a material of higher density than the surface crust. The molten basalt, or lava, which underlies much of the earth's crust is itself far too light to account for the measured density of the earth. A good deal of the earth's mass apparently resides in a relatively small heavy "core" at the center of the earth, of density about ten times that of water. This core is presumably composed of metals like iron and nickel rather than rocks. The core, as determined by seismic measurement, is essentially spherical in shape and about 2,000 miles in radius. The core itself is apparently not uniform in density but shows a distinct rise in density, increasing as the center of the earth is approached, starting at about 1,000 miles from the center. At the center of the earth the density is about 16 times that of water.

* * *

UNANSWERED QUESTIONS Students should not feel that gravitation is a completely settled issue. There are a number of unresolved basic questions about gravity, even years after Einstein's theory of general relativity: as the following:

1) Why does the gravitational field follow an inverse square law?

2) What is the speed of propagation of the gravitational field?

3) Does gravity theory hold for very large and very small distances?

4) Does the law of gravitation hold everywhere in the universe?

5) Is the gravitational "constant", G really constant, or does it change slowly with time over billions of years?

6) Is the gravitational force between two bodies independent of the distribution of matter in the rest of the universe?

We have no answers to these "simple" questions, or for many others of great fundamental interest. If gravity is a field of investigation which is currently somewhat static, it is not through exhaustion of problems, but of physicists. It is extremely difficult to conceive and execute experiments to test details of gravity theory on a terrestrial scale, and neither technology nor the economy is yet ready to support an intergalactic astrophysical year!

Chapter 22 — Universal Gravitation and the Solar System

For Home, Desk and Lab — Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1-4		3*,4*		3*,4*	1,2
5	5, 6	7		5,7	
6		8*		8*	
7		9*			
8	10*,14,15 16*,17*	11*,12* 18*	13	10*,11*,12* 13	
general	19*	20*,21,22	23*	19*,20*,21 22,23*	

SHORT ANSWERS

1. Home project.
2. Home project.
3. (a) About one revolution in 4 weeks (27.3 days).
   (b) Earth would appear fixed from the moon.
   (c) About one revolution in 4 weeks (29.5 days). Sun appears to rotate a little slower than the fixed stars.
4. (a) Once a month (29.5 days).
   (b) 8×10^{10} m.
   (c) Moon's path always concave toward sun. See sketches on page 22-16 and 22-17.
   (d) No.
5. Elliptical.
6. (a) $2.2 \times 10^{15}\ \mathrm{m^2/sec}$.
   (b) $2.1 \times 10^{11}\ \mathrm{m^2/sec}$.
7. 5.3×10^{12} m.
8. See discussion on pages 22-19 and 22-20.
9. (a) $v_1 = 2v_2$.
   (b) $F_1 = 2F_2$.
   (c) See discussion on page 22-20.
10. 2.6×10^{6} m.
11. (a) 3.6×10^{7} m.
    (b) $0.22\ \mathrm{m/sec^2}$.
    (c) $0.23\ \mathrm{m/sec^2}$.
12. 5.5×10^{24} kg.
13. (a) $\rho T^2 = \frac{3\pi}{G}$, a constant.
    (b) $1.41 \times 10^{11}\ \mathrm{kg\text{-}sec^2/m^3}$.
14. (a) 5.1×10^{3} sec.
    (b) 10^4 sec.
15. Mars: 360 newtons.
    Earth: 980 newtons.
    Jupiter: 2400 newtons.
16. 2.8×10^{-7} newtons.
17. $F \approx 1.9 \times 10^{-44}$ newtons.
18. (a) 1.5 greater than on earth.
    (b) Planet density = 3/4 earth density.
19. Tangential velocity is sufficient to result in a stable orbit.
20. (a) Same.
    (b) Negligibly different from present moon-earth path around sun.
    (c) 2.2.
    (d) See discussion on page 22-26.

21. (a) 3.4 newtons less.
    (b) One revolution in 1.4 hours.
    (c) 17 times greater.

22. (a) See sketch on page 22-28.
    (b) See sketch on page 22-29.
    (c) 1/1 and 1/2.

23. (a) See discussion on page 22-29.
    (b) No.

COMMENTS AND SOLUTIONS

PROBLEM 1

As a project, observe the position of one of the planets with respect to the fixed stars once or twice a week for a month or two. Begin by plotting a map of the stars surrounding the planet. Then mark the new position of the planet on this map at each observation.

Students who try this would be well advised to construct a crude sextant or, perhaps, use the parallax viewer from Experiment I-2 to obtain reasonably quantitative results. Otherwise, it would be best to start with a scaled "star map" of the region of the sky under study

PROBLEM 2

Another project: The four largest moons of Jupiter can be seen through reasonably good binoculars. One of the moons moves so fast that its motion relative to Jupiter can be detected by observations just a few hours apart. With your binoculars, observe the positions of the moons every clear night for about three weeks.

(a) Plot their positions relative to Jupiter for each observation.

(b) Can you determine the period of revolution for each moon?

(c) Which of these moons is nearest to Jupiter?

Reasonably good binoculars are required since inferior ones may have too much spherical and chromatic distortion to permit resolving the moons. The binoculars must be mounted on a rigid support for this observation. Remind students that Galileo made this measurement with optical equipment far less adequate than a pair of commercial binoculars!

Jupiter has eleven satellites. The four largest of these, Io (I in following table), Europa (II), Ganymede (III), and Callisto (IV) were discovered by Galileo in 1610. The smallest of the four, Europa, is about the size of our moon, while the two largest, Ganymede and Callisto, are larger than the planet Mercury. Pertinent information about the satellites is given in the following table:

The Satellites of Jupiter

Designation	Discovery	Distance from Jupiter (mi)	Diameter (mi)	Period of Rev.
I	Galileo, 1610	261,000	2318	1 d 18 h 28 min.
II	Galileo, 1610	416,600	1967	3 d 13 h 14 min.
III	Galileo, 1610	664,400	3200	7 d 3 h 42 min.
IV	Galileo, 1610	1,168,800	3218	16 d 16 h 32 min.
V	Barnard, 1892	112,600	100?	0 d 11 h 57 min.

VI	Perrine, 1904	7,114,700	80?	250 d 17 h
VII	Perrine, 1905	7,288,600	25?	260 d 1 h
VIII	Melutte, 1908	14,600,000	15?	739 d
IX	Nicholson, 1914	14,900,000	15?	745 d
X	Nicholson, 1938	7,300,000	15?	270 d
XI	Nicholson, 1938	14,000,000	15?	735 d

The seven small satellites are far too small to be observed except with the largest astronomical telescopes and under highly favorable conditions. The outer satellites are so small that they could not be seen from Jupiter without a telescope!

PROBLEM 3

> The fixed stars go around in about one day as seen by an observer on the earth. About how long do the following revolutions appear to take?
>
> (a) The sphere of the fixed stars as seen by someone on the moon.
>
> (b) The earth about the moon as seen from the moon.
>
> (c) The sun as seen from the moon. Does the sun appear to rotate as if it were one of the fixed stars? Does it appear to rotate faster or slower? Remember that the earth and moon go around the sun once every year.

This question can be answered at several different levels. At this stage a brief qualtative answer should be expected.

Answer #1 (brief):

a) We know that, because the earth rotates on its axis, the stars appear to go around the earth once a day. But what about the stars as seen from the moon? Does the moon rotate on its axis once a day as does the earth?

This last question can be answered from an examination of the solar system table on page 350 of the text. The period of the moon's rotation on its axis is the same as the period of the moon's revolution around the earth. This is 2.36×10^6 sec, or 27.3 days. Since, relative to the fixed stars, the moon rotates on its axis once in 27.3 days, the fixed stars would appear to a moon observer to rotate once in 27.3 days.

Some students should be able to arrive at an approximate answer without referring to the table. The key is in properly interpreting the fact that, from the earth, we always see the same "face" of the moon. As the moon travels through an angle θ in its revolution around the earth, the moon must also rotate on its axis through the same angle in order to keep the same "face" toward the earth. Hence the period of the moon's rotation on its axis is the same as the period of the moon's revolution around the earth. In earlier science work some students may have learned this period as 28 days (more accurately, 27.3 days).

b) Since the moon keeps the same "face" toward the earth, an observer on the moon would always see the earth in the same position in the sky. From the moon, the earth would not appear to be revolving around the moon.

c) Since from part (a) we found that the moon rotates on its axis about once in 4 weeks, this is the approximate time required for the sun to move from one "high noon" position to the next as seen from a particular point on the moon. If the moon were only rotating on its own axis, and not moving around the sun, then for a moon man the sun would appear to move with the fixed stars. However, because the moon is going around the sun as well as rotating on its own axis, the moon must make a little more than one revolution on its axis in order for the sun to reappear at a "high noon" position. Hence the sun will appear to rotate around the moon a little slower than the fixed stars. (The actual rate at which the sun will appear to rotate around the moon is once in 29.5 days – see Answer #2 below.)

The answer given above carries this problem about as far as is probably worthwhile for your class at this point. If you are pressed for additional details the approach given below may be helpful in discussions outside of class.

* * *

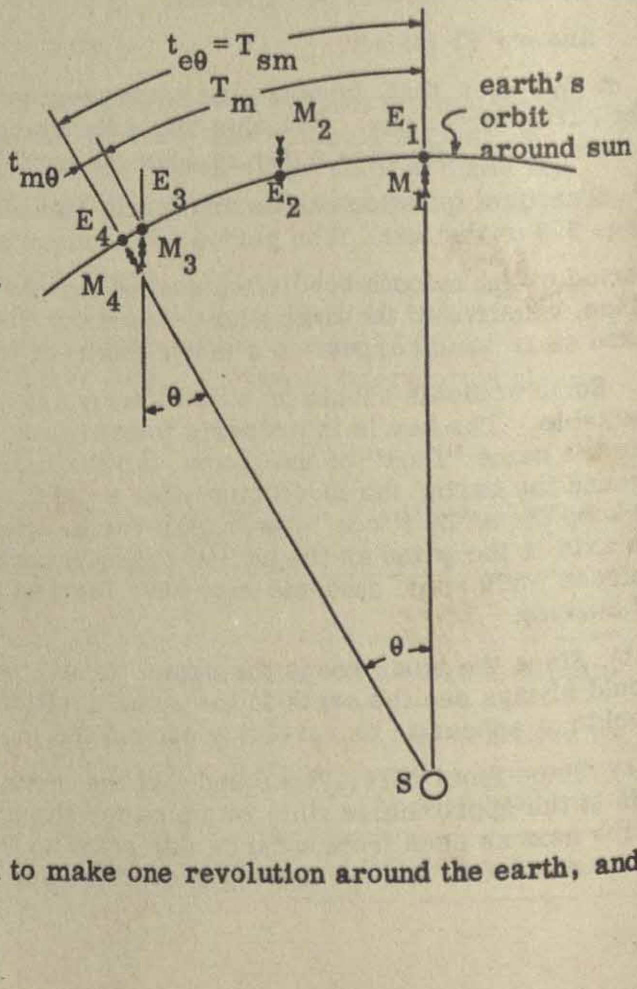
Answer #2:

The rotations of the earth on its axis, the earth in its orbit, the moon in its orbit about the earth, and the moon on its own axis are all in the same direction. Furthermore, the rate of revolution of the moon in its orbit and its rate of rotation on its own axis are identical. The period of these two motions is 27.3 days.

a) Imagine an arrow, planted extending through the moon, with its head pointing toward the earth. The head will always remain pointing toward the earth. As the moon turns on its axis, the angle between this arrow and an imaginary diameter of the fixed star sphere will change. The arrow rotates 360° relative to such a line in 27.3 days. Therefore, for an observer on the moon (to whom the arrow appears fixed) the diameter of the fixed star sphere (and thus the sphere itself) appears to rotate with a 27.3 day period.

b) The earth, however, remains fixed in the direction that the arrow points. Thus it neither rises nor sets for an observer on the moon. From the moon, the earth's apparent rate of revolution around the moon is zero.

c) The apparent rate of revolution of the sun is a more difficult problem. The figure shows various configurations of the earth-moon system. All motions are counter-clockwise. Position 1 is a new moon position for an earth observer. In Positions 1 and 3 the moon arrows are parallel. In moving from 1 to 3, the moon has rotated on its axis 360° relative to the fixed star sphere. (Position 2 is an intermediate position after 180° of rotation.) Note that at Position 1, the sun is at its zenith for an observer on the moon who is located at the tail of the moon arrow. However, at Position 3, it is not yet again high noon for this observer. Noon does not come until Position 4 is reached.



The angle along the earth's orbit, between Positions 1 and 4, is θ. It can be seen from the diagram that θ is also the angle through which the moon moves around the earth in going from Position 3 to Position 4. Thus one full apparent revolution of the sun around the moon requires a time, T,

$$(1)\ T_{sm} = T_m + t_{m\theta},$$

where T_m is the time required for the moon to make one revolution around the earth, and

$t_{m\theta}$ is the time required for the moon to move through the angle θ. The moon goes through an angle of 360° in the time T_m. Hence, the time required for the moon to move through an angle of θ degrees is

$$(2)\ t_{m\theta} = \frac{\theta}{360} T_m.$$

Similarly, the time required for the earth to move through the angle θ is

$$(3)\ t_{e\theta} = \frac{\theta}{360} T_e.$$

The diagram shows that the time required for the earth to move from Position 1 to Position 4 is the same as the time required for one apparent revolution of the sun around the moon. Hence,

$$(4)\ T_{sm} = t_{e\theta}.$$

Equations 2 and 3 can be combined, and (from equation 4) T_{sm} can be substituted for $t_{e\theta}$.

$$(2)\ \frac{t_{m\theta}}{T_m} = \frac{\theta}{360},\quad (3)\ \frac{t_{e\theta}}{T_e} = \frac{\theta}{360}$$

$$\frac{t_{m\theta}}{T_m} = \frac{t_{e\theta}}{T_e} = \frac{T_{sm}}{T_e}$$

$$(5)\ t_{m\theta} = \frac{T_{sm} T_m}{T_e}.$$

The expression for $t_{m\theta}$ in equation 5 can be substituted in equation 1.

$$T_{sm} = T_m + t_{m\theta} = T_m + \frac{T_{sm} T_m}{T_e} = \frac{T_m T_e}{T_e - T_m} = \frac{27.3 \times 365}{365 - 27.3} = \underline{29.5 \text{ days.}}$$

It is interesting to note that Positions 1 and 4 are both new moon phases for an earth observer. Hence 29.5 days is the period of our lunar month.

PROBLEM 4

The earth's orbit around the sun is nearly circular, and the moon moves in a nearly circular orbit around the earth. The radius of the earth's orbit is 1.5×10^{11} meters, and that of the moon's orbit is 4×10^{8} meters.

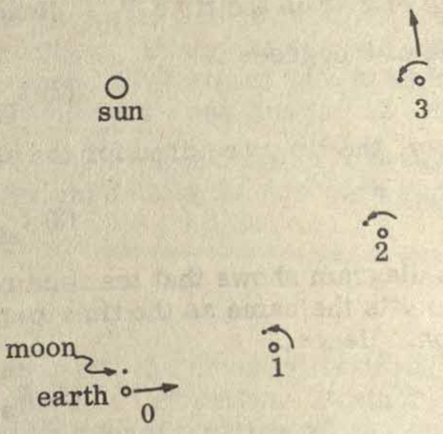
(a) How often is the moon between the earth and the sun?

(b) How far has the moon moved around the sun in the interval between two successive times when the moon is between the earth and the sun?

(c) Sketch the orbit of the earth going around the sun and on the same figure sketch the orbit of the moon going around the sun.

(d) Does the moon show retrograde motion as seen by an observer at the sun? (Retrograde motion is motion like that shown in Fig. 22-2.)

a) Since the moon takes about 4 weeks to go around the earth, and the earth takes 52 weeks to go around the sun, the moon revolves around the earth about 13 times in one year. (This can also be found from the periods for the sun and moon given in the solar system table on page 350. $\frac{T_s}{T_m} = \frac{3.16 \times 10^7}{2.36 \times 10^6} = 13.4$.) In these annual motions of the earth and moon, the moon will be between the earth and sun 12 times. This can be seen from the diagram which shows the positions of earth, sun and moon at one month intervals for 3 months or 1/4 of a year. It can be seen that during the time the earth has moved from 0 to 1, the moon must make a little more than one revolution of the earth to be between earth and sun. During the 1/4 year required for the earth to move from 0 to 3, the moon must make an extra quarter of a revolution around the earth in order again to be between the earth and sun. Therefore, a total of 3 1/4 revolutions per quarter year are required for the moon to appear between the earth and sun 3 times. Hence in one year (13 revolutions of the moon around the earth) the moon will be between the earth and sun 12 times or once a month. (As seen in Answer #2 for Problem 3, a more precise answer is once every 29.5 days.)

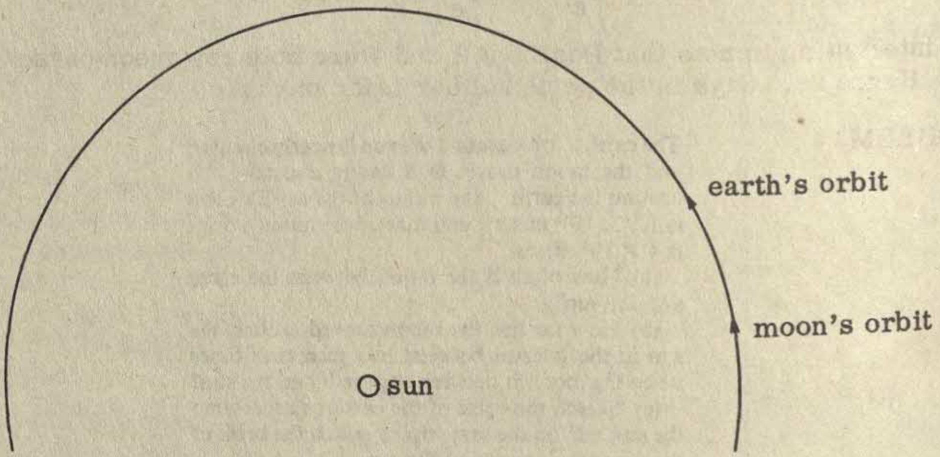


[A similar argument shows that the earth turns on its axis 367 times during leap year and 366 times during other years.]

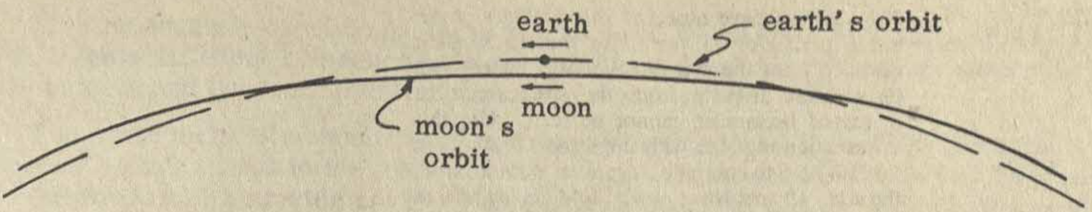
b) During one month, the moon (and earth) have moved 1/12 × circumference of the earth's orbit, or $1/12 \times 2\pi \times 1.5 \times 10^{11}\ \text{m} \approx \underline{8 \times 10^{10}\ \text{m}}$.

(more precisely, $\frac{29.5}{365} \times 2\pi \times 1.5 \times 10^{11}\ \text{m} = 7.6 \times 10^{10}\ \text{m}$.)

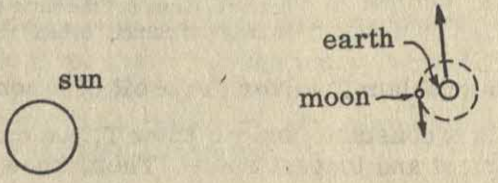
c) In the sketch below, the earth's orbit around the sun has a radius of 2 inches. The ratio of the radius of the moon's orbit around the earth to that of the earth around the sun is $\frac{3.8 \times 10^{8}}{1.5 \times 10^{11}} = \frac{1}{400}$; therefore the moon's orbit around the earth, on the same sketch, would have a radius of 2/400 inches. This would be less than the width of the line used to indicate the earth's orbit.



Frequently diagrams are drawn showing the moon weaving back and forth across the earth's orbit. These are seldom correct, since the moon's orbit around the sun is always concave toward the sun. This can be seen from the fact that the gravitational force between the sun and moon is about twice as great as the force between the earth and moon. Hence the moon is always accelerated toward the sun. The moon's acceleration toward the earth is $2.7 \times 10^{-3}\ \text{m/sec}^2$; its acceleration toward the sun is $5.9 \times 10^{-3}\ \text{m/sec}^2$. Thus even when the moon is between the earth and sun as shown in the diagram at the top of the next page, the net acceleration of the moon is toward the sun.



d) To an observer on the sun, the moon would not show retrograde (reverse) motion. If the moon were to show retrograde motion its speed "backward" at a point such as shown in the diagram would have to be greater than the "forward" speed of the earth-moon system. The speed of the earth-moon system in its orbit around the sun is:



$$\frac{2\pi \times 1.5 \times 10^{11}\ \text{m}}{365 \times 8.6 \times 10^{4}\ \text{sec}} \approx 30\ \text{km/sec}.$$

The speed of the moon relative to the earth is:

$$\frac{2\pi \times 3.8 \times 10^{8}\ \text{m}}{27 \times 8.6 \times 10^{4}\ \text{sec}} \approx 1\ \text{km/sec}.$$

Since the moon and earth travel around the sun at 30 km/sec, the moon's speed of 1 km/sec around the earth is too small to cause retrograde motion.

PROBLEM 5 In addition to planets, the solar system contains other "wanderers" called comets. Many comets reappear at regular intervals, apparently becoming brighter and larger in a few weeks as they approach the earth and then becoming dimmer and smaller until they are invisible for a period of years. What kind of orbit do you think they have?

While all orbits are elliptical, those of the reappearing comets are long and narrow – highly eccentric compared to the nearly circular orbits of the planets.

PROBLEM 6 How fast, in m²/sec, is area swept out by

(a) the radius from sun to earth?
(b) the radius from earth to moon?

This is a straightforward problem related to Kepler's second law. Using the data given in Problem 4:

a) Area swept out by radius from earth to sun per second = $\pi R^2/T$, where T is the time in seconds for one complete revolution. Since the earth revolves once in 365 days (3.2×10^7 sec), the area swept out by earth to sun radius per second =

$$\frac{\pi \times (1.5 \times 10^{11}\ \text{m})^2}{3.2 \times 10^{7}\ \text{sec}} = \underline{2.2 \times 10^{15}\ \text{m}^2/\text{sec}.}$$

b) For the radius from the earth to the moon, the period is 27.3 days (2.4×10^6 sec). Thus, area swept out by earth to moon radius per second =

$$\frac{\pi\ (4 \times 10^{8}\ \text{m})^2}{2.4 \times 10^{6}\ \text{sec}} = \underline{2.1 \times 10^{11}\ \text{m}^2/\text{sec}.}$$

PROBLEM 7

Astronomers have observed that Halley's comet has a period of 75 years and that its smallest distance from the sun is 8.9×10^{10} meters, but its greatest distance from the sun cannot be measured because it cannot be seen. Use this information together with the second footnote in Section 22-5 to compute its greatest distance from the sun. (It was Newton who told Halley how to compute the orbit of a comet. Halley found and calculated the orbit and period of the comet that bears his name in the course of a general analysis he made of comets' orbits.)

This problem involves the orbit of a comet, and Kepler's third law. Since we know R^3/T^2 is a constant, and we know T, we can compute R. R is defined as the average of the shortest and longest radii. Then, knowing the shortest radius, we can compute the longest.

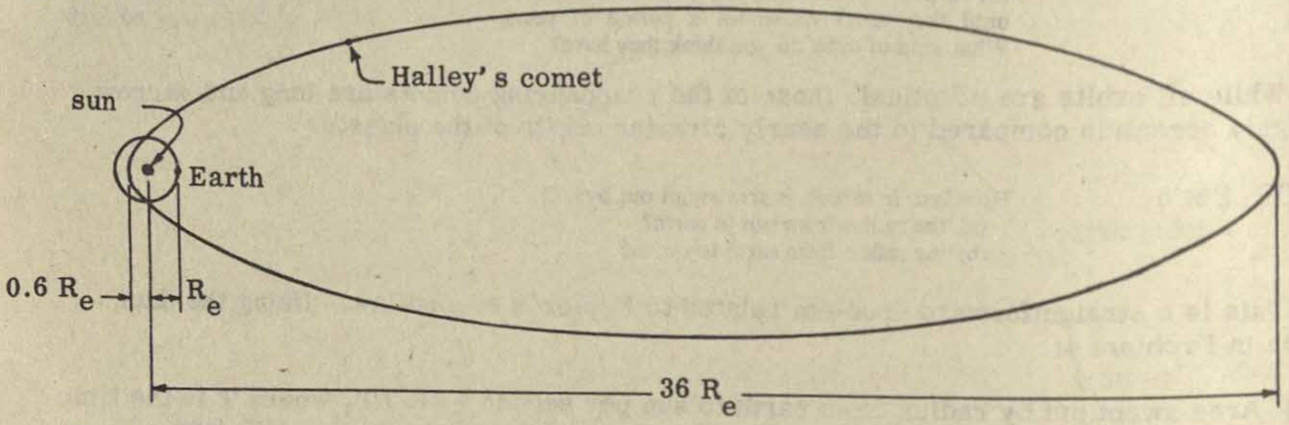
From the table for Kepler's third law (page 350), we take $\overline{R}^3/T^2 = 3.4 \times 10^{18}\ \mathrm{m}^3/\mathrm{sec}^2$. Since the period, T, of Halley's comet is 75 years $= 2.4 \times 10^9$ sec, $T^2 = 5.8 \times 10^{18}\ \mathrm{sec}^2$, and

$$\overline{R} = \left(3.4 \times 10^{18}\ \frac{\mathrm{m}^3}{\mathrm{sec}^2} \times 5.8 \times 10^{18}\ \mathrm{sec}^2\right)^{1/3} = 2.7 \times 10^{12}\ \mathrm{m}.$$

Since $\overline{R}$ is defined as $\frac{1}{2}$ the sum of the shortest and longest radii,

$$\overline{R} = \frac{R_{max} + R_{min}}{2}, \text{ and } R_{max} = 2\overline{R} - R_{min} = 2 \times 2.7 \times 10^{12}\ \mathrm{m} - 8.9 \times 10^{10}\ \mathrm{m} = \underline{5.3 \times 10^{12}\ \mathrm{m}}.$$

(The distance of closest approach, 8.9×10^{10} m is negligible compared to the long radius.) If this orbit is drawn to scale together with that of the earth, it is evidently highly elongated, as shown below.



PROBLEM 8

Three fireflies *X*, *Y*, and *Z* are on a moving bicycle at night. *X* is at the very center of one of the axles, which turns with the wheel. *Y* is on the rim of the wheel. *Z* is on the frame of the bike outside the circumference of the rim. Draw sketches and use a few words to describe the motions

(a) of *X* and *Y* as viewed by *Z*.
(b) of *Y* and *Z* as viewed by *X*.
(c) of *X* and *Z* as viewed by *Y*.
(d) of all three as seen by an observer standing near the bicycle.

This moderately difficult problem is best used for class discussion. It can be used to illustrate different reference frames – an example of the problems involved in relating heliocentric and geocentric descriptions of the universe.

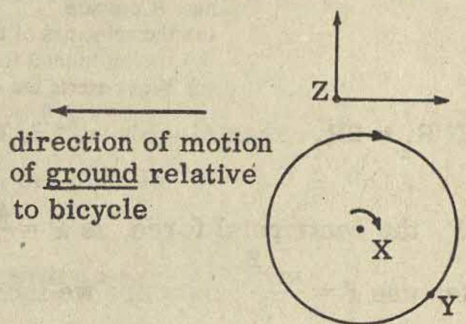
The basic difficulty encountered in this problem may be illustrated by an example. Suppose we ask "what is the motion of a man sitting on a bench as viewed by a man looking out the window of a moving car?" Two answers could be given. A perfectly sensible answer is that the man is not moving at all. Even though the car moves, we can see from the car that the man on the bench is sitting still, hence is "at rest". A second answer is that in a reference system fixed relative to the car, the man on the bench is moving backward at the speed of the car. Certainly the first answer is the more natural in terms of our experience. The motion of the car does not "fool" us. We intuitively choose to describe the position of the man relative to the ground. However, the second answer is completely satisfactory if our reference system is fixed to the car.

The lesson is simple: to describe motion of an object, we pick a reference system and describe the motion relative to the reference system. In the above example there are two possible choices (1) a system at rest on the ground and (2) a system at rest relative to the car. In giving our answer we must state clearly which reference system we choose.

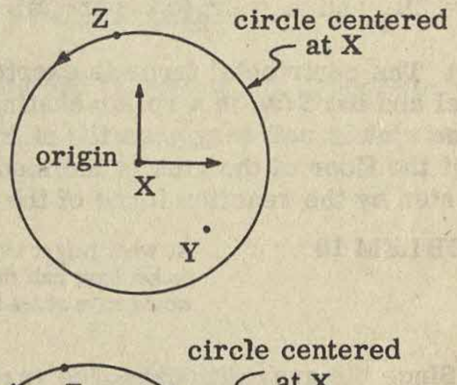
The man in the car can tell us what motion the bench sitter has relative to either system. For that matter, so could the man on the bench!

In Problem 8 we are given four reference systems. We can most easily think of them in terms of the origin and orientation of their coordinate axes. If we assume that Z, X, and the circle described by Y are all in the same plane, two coordinates are enough.

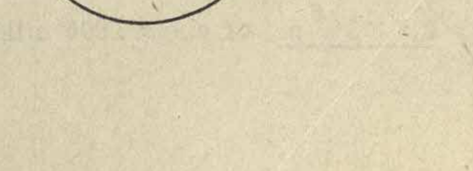
a) From Z's position on the frame of the bicycle, outside the wheel, X does not move, but spins at a fixed position in space. Y describes a circle around X, but since Z is in the plane of the circle, it will appear to Z that Y is in simple harmonic motion.



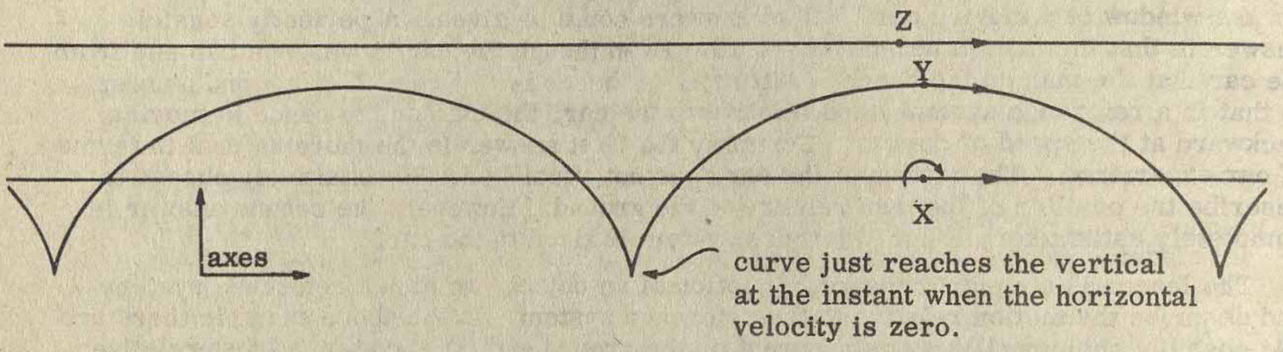
b) From X's point of view, spinning with the axle, it appears that Y, on the wheel rim, is at rest. Z on the frame, is at a fixed distance from X. Hence, as X rotates with the axle, Z seems to revolve around X in the opposite sense.



c) From Y's point of view on the rim, X will appear to be at rest, but Z moves in a circle centered at X.



d) For an observer standing near the bicycle, Z, on the bicycle frame, moves with uniform speed in a straight horizontal line. X, on the axle, also moves with uniform speed in a straight horizontal line, but is spinning. Y, on the wheel rim, moves in a cycloid. As the wheel rolls, Y's horizontal velocity is zero at the instant Y's part of the wheel is in contact with the ground. Thus we get:



curve just reaches the vertical at the instant when the horizontal velocity is zero.

The simplest way of describing the motion of Y is to say that Y moves in a circle relative to the axes which are fixed to the bicycle frame [part (a)]. and that these axes move horizontally at a constant speed without turning relative to the ground.

PROBLEM 9

Two skaters of equal mass on a circular rink go around the rink in the same time. One skater is twice as far from the center of the rink as the other. Compare

(a) the velocities of the skaters.
(b) the centripetal forces acting on them.
(c) What exerts the centripetal force on them?

a) If $R_1 = 2R_2$, then since $v = 2\pi R/T$, and T is the same for both skaters, $\underline{v_1 = 2v_2}$.

b) F, the centripetal force, is $F = \frac{4\pi^2 Rm}{T^2}$. Therefore, $\underline{F_1 = 2F_2}$ from $R_1 = 2R_2$. We can also use $F = \frac{mv^2}{R}$; but here we must remember that both v and R are different; namely $R_1 = 2R_2$ and $v_1 = 2v_2$ [c.f. part (a)]. Therefore $F_1 = \frac{2^2}{2} F_2 = 2F_2$, which is the same.

c) The centripetal force is exerted on the skates by the floor (or ice). If the floor is level and hard (as in a roller skating rink with a level surface) a frictional force is exerted. If the rink is soft (e.g., an ice rink, where the skate pushes a short distance into the ice), or if the floor of the rink is inclined ("banked" on the turns), the centripetal force is exerted by the reaction force of the floor on the skates.

PROBLEM 10

At what height above the earth's surface will a rocket have half the force of gravity on it that it would have at sea level?

Since the gravitational force varies as $1/R^2$, the force will be smaller by a factor of 2 when $1/R^2$ is smaller by a factor of 2 than $1/R_e^2$, where R_e = the radius of the earth. Hence, $\frac{1}{R_e^2} = 2 \times \frac{1}{R^2}$, and $R = \sqrt{2} \times R_e$. If $R_e = 6.4 \times 10^6$ m, $\sqrt{2} \times R_e = 9.0 \times 10^6$ m. The height above sea level at which the force of gravity would be reduced by one-half is thus $R - R_e = \underline{2.6 \times 10^6 \text{ m}}$, or about 1600 miles.

PROBLEM 11

(a) At what height will a satellite moving in the plane of the equator stay over one place on the equator of the earth? One way to get the answer is to compare this satellite with the moon, which is at 59.5 earth radii from the center of the earth and takes 27 days to go around the earth.

(b) How fast is the satellite accelerating toward the earth?

(c) Using the inverse-square law and *g* at the surface of the earth, find the gravitational field at the height of the satellite. Compare with your answer to (b).

a) The satellite will be stationary relative to a point on the earth if its period, T, is the same as that of the earth, very nearly one day (actually 365/366 days).

The radius of the satellite's orbit can be found from Kepler's third law and the knowledge that the moon is 59.5 earth radii from the center of the earth and revolves once every 27 (27.3) days. Let R_s and R_m be the radii of the satellite and moon orbits, and T_s and T_m be their periods.

$$\frac{R_s^3}{T_s^2} = \frac{R_m^3}{T_m^2}$$

$$R_s = \left(\frac{R_m^3 T_s^2}{T_m^2} \right)^{1/3} = \left[\frac{(59.5 \text{ earth radii})^3 \times (1 \text{day})^2}{(27 \text{ days})^2} \right]^{1/3} = 6.6 \text{ earth radii.}$$

Since the radius of the earth is 6.4×10^6 m, $R_s = 6.6 \times 6.4 \times 10^6 \text{ m} = 4.2 \times 10^7$ m. The height of the satellite above the earth's surface will be $4.2 \times 10^7 \text{ m} - 6.4 \times 10^6 \text{ m} =$ $\underline{3.6 \times 10^7 \text{ m}}$, or about 22,000 miles.

b) The satellite is accelerating toward the earth with an acceleration:

$$a = \left(\frac{2\pi}{T} \right)^2 R = \frac{4 \times \pi^2 \times 4.2 \times 10^7 \text{ m}}{(1 \times 24 \times 60 \times 60)^2 \text{ sec}^2} = \underline{0.22 \text{ m/sec}^2}.$$

c) The force due to gravity, and thus the acceleration due to gravity, is proportional to $1/R^2$. Thus the acceleration, g, at the height of the satellite and g at the earth's surface are related as follows:

$$g_s = \frac{K}{R_s^2}, \quad g_e = \frac{K}{R_e^2}$$

$$\therefore \; g_s = \frac{g_e R_e^2}{R_s^2} = 9.8 \text{ m/sec}^2 \times \left(\frac{6.4 \times 10^6 \text{ m}}{4.2 \times 10^7 \text{ m}} \right)^2 = \underline{0.23 \text{ m/sec}^2}.$$

Thus, we see in part (b) that the acceleration can be found from R and T without using an inverse square law of force, and in part (c) that it can be found from g at the earth's surface using the inverse square law. These are the two methods Newton used in calculating the acceleration of the moon (see text pages 354-356).

PROBLEM 12

A satellite circles the earth once every 98 minutes at a mean altitude of 500 km. Calculate the mass of the earth. The masses of planets are actually calculated from satellite motions, and one reason for establishing artificial earth satellites is to get a better value for the mass of the earth.

As stated in Section 22-10,

$$G = 0.667 \times 10^{-10}\,\frac{\text{m}^3}{\text{kg-sec}^2}$$

Kepler's third law states that R^3/T^2 is a constant for all satellite motion, but does not specify the nature of the constant.

Using the law of universal gravitation, we may equate the centripetal force with the gravitational force on the satellite (if the orbit is stable). Thus, if we designate the mass of the satellite by m_s, and that of the earth by m_e,

$$\frac{m_s 4\pi^2 R_s}{T_s^2} = G\frac{m_s m_e}{R_s^2}$$

$$\therefore m_e = \frac{4\pi^2 R_s^3}{G T_s^2}.$$

Substituting, $R = 6.37 \times 10^6\,\text{m} + 5.00 \times 10^5\,\text{m} = 6.87 \times 10^6\,\text{m}$, and

$$T = 98 \times 60\ \text{sec} = 5.88 \times 10^3\ \text{sec}.$$

$$m_e = \frac{4\pi^2 \times (6.87 \times 10^6\,\text{m})^3}{0.667 \times 10^{-10}\,\dfrac{\text{m}^3}{\text{kg-sec}^2} \times (5.88 \times 10^3\ \text{sec})^2} = \underline{5.5 \times 10^{24}\ \text{kg}}.$$

PROBLEM 13

(a) If T is the period of a satellite circling around just above the surface of a planet whose average density is ρ, show that ρT^2 is a universal constant.

(b) What is the value of the constant?

a) As in Problem 12, we may write:

$$\frac{m_s 4\pi^2 R}{T^2} = G\frac{m_p m_s}{R^2}, \text{ and}$$

$$m_p = \frac{4\pi^2 R^3}{GT^2}.$$

If $R \simeq R_p$, the radius of the planet, we may write:

$$m_p = \frac{4\pi^2 R_p^3}{GT^2}.$$

But $m_p = \frac{4}{3}\pi R_p^3 \rho$, where ρ = the density. Thus,

$$\frac{4\pi R_p^3 \rho}{3} = \frac{4\pi^2 R_p^3}{GT^2}.$$

Solving for ρT^2, we get:

$$\underline{\rho T^2 = \frac{3\pi}{G}},$$

which is a universal constant since G is a constant.

b) Evaluating, $\rho T^2 = \frac{3\pi}{0.667 \times 10^{-10}\ \mathrm{m^3/kg\text{-}sec^2}} = \underline{1.41 \times 10^{11}\ \mathrm{kg\text{-}sec^2/m^3}}.$

PROBLEM 14 Compute the periods of a satellite in an orbit close to the surface of

(a) the earth.

(b) the sun.

This problem follows easily after Problem 13, but it can also be done directly.

From Problem 13, we know that ρT^2 is a universal constant for a satellite moving close to the surface of any body. Hence,

$$\rho_s T_s^2 = \rho_e T_e^2 = 1.41 \times 10^{11}\ \mathrm{kg\text{-}sec^2/m^3}.$$

Since $\rho_e = 5.5 \times 10^3\ \mathrm{kg/m^3}$, $T_e = \left(\frac{1.41 \times 10^{11}\ \mathrm{kg\text{-}sec^2/m^3}}{5.5 \times 10^3\ \mathrm{kg/m^3}}\right)^{1/2} = \underline{5.1 \times 10^3\ \mathrm{sec}} = 85$ min.

Taking $\rho_s \approx 1.4 \times 10^3\ \mathrm{kg/m^3}$, $T_s = \left(\frac{1.41 \times 10^{11}\ \mathrm{kg\text{-}sec^2/m^3}}{1.4 \times 10^3\ \mathrm{kg/m^3}}\right)^{1/2} \approx \underline{10^4\ \mathrm{sec}} \approx 167$ min.

Working directly, the centripetal acceleration at the earth's surface is $9.8\ \mathrm{m/sec^2} = \frac{4\pi^2 R}{T^2}$. $R_e = 6.4 \times 10^6$ m, therefore:

$$T = \sqrt{\frac{4\pi^2 \times 6.4 \times 10^6}{9.8}} = 2\pi \times 0.81 \times 10^3 = \underline{5.1 \times 10^3\ \mathrm{sec}}.$$

Now in general,

$$\frac{4\pi^2 R}{T^2} = G\frac{m}{R^2}.$$

Therefore, $T^2 \propto R^3/m$. The sun's radius is $\frac{7.0 \times 10^8\ \mathrm{m}}{6.4 \times 10^6\ \mathrm{m}} = 1.1 \times 10^2$ times that of the earth.

The sun's mass is $\frac{2.0 \times 10^{30}\ \mathrm{kg}}{6.4 \times 10^6\ \mathrm{m}} = \frac{1}{3} \times 10^6$ times that of the earth. Consequently the period of the satellite's motion at the sun's surface is $\sqrt{\frac{(1.1)^3 \times 10^6}{1/3 \times 10^6}} \approx 2$ times its period at the earth's surface, i.e., $\underline{10^4\ \mathrm{sec}}$.

PROBLEM 15 Find the weight of a 100-kg man on planets with the following masses and radii.

	m	R
Mars	6.4×10^{23} kg	3.4×10^{6} m
Earth	6.0×10^{24} kg	6.4×10^{6} m
Jupiter	1.9×10^{27} kg	7.2×10^{7} m

Since the weight is simply the force of gravity on the 100 kg man, using

$$W = F = \frac{Gm_p m}{R_p^2},$$

by substituting the appropriate masses and radii for each of the three planets, we find the 100-kg man weighs 360 newtons on Mars, 980 newtons on Earth, and 2400 newtons on Jupiter.

PROBLEM 16 A 70-kg boy stands 1 meter away from a 60-kg girl. Calculate the force of attraction (gravitational) between them.

$$F = \frac{Gm_1 m_2}{R^2}$$

$$F = \frac{.667 \times 10^{-10} \frac{\text{nt m}^2}{\text{kg}^2} \times 60 \text{ kg} \times 70 \text{ kg}}{1 \text{ m}^2}$$

$$F = \underline{2.8 \times 10^{-7} \text{ newtons.}}$$

This force of attraction is only a small fraction of the girl's weight ($F/W = 4.8 \times 10^{-10}$). Obviously, it must be something other than gravity which attracts boys and girls!

PROBLEM 17 Find the gravitational attraction of the two atoms in a hydrogen molecule.

This easy problem, involving the law of universal gravitation, is intended to show the smallness of gravitational forces on an atomic scale.

While most students will probably remember that the mass of a hydrogen atom was found in Chapter 8 (1.67×10^{-27} kg), a good many students will be stumped when they try to find a value for R, the spacing between the two atoms in a hydrogen molecule. Those with sharp memories may recall from Chapter 7, Section 13 that the linear dimension of an atom is about 1 Angstrom, and from Chapter 8, Section 4 that the hydrogen molecule is about twice as long as it is wide. From this they could infer that the centers of the atoms are about 1 Angstrom apart. In assigning this problem, you may wish to tell students that clues to the spacing are in Chapter 7 and 8, or you may simply want to give them the number, $R \approx 10^{-10}$ m.

$$F = \frac{Gm_1 m_2}{R^2}$$

$$F = \frac{.667 \times 10^{-10} \times (1.67 \times 10^{-27})^2}{(10^{-10})^2} \approx \underline{1.9 \times 10^{-44} \text{ newtons.}}$$

By comparison, the electric force between a proton and an electron in a hydrogen atom is about 10^{-8} nt, about 10^{36} times larger! Gravitational forces are completely negligible in atomic problems.

PROBLEM 18 (a) If a planet of twice the diameter of the earth has a mass six times as great, how does the gravitational field at its surface compare with the gravitational field at the surface of the earth?

(b) How would the densities of the two planets compare?

This problem concerns the mass and distance dependence of the gravitation law. It provides practice in more abstract reasoning (i.e., an algebraic rather than numerical problem), and gives practice in scaling.

a) Let us take a test mass, m, at the surface of both the earth and the larger planet. Let M and R be the mass and radius of the earth.

On earth, $F_e = \frac{GmM}{R^2}$.

On the planet, $F_p = \frac{Gm6M}{(2R)^2} = \frac{6}{4} \times \frac{GmM}{R^2} = 1.5\ F_e$.

The gravitational field on the planet is 1.5 times greater than on the earth.

b) Since density $= \frac{\text{mass}}{\text{volume}}$ and volume is proportional to R^3, the density of the planet is $6/2^3$ or 3/4 that of the earth.

PROBLEM 19 The earth is acted upon by the gravitational attraction of the sun. Why doesn't the earth fall into the sun? Be prepared to discuss your answer.

This problem is intended primarily for class discussion to help clarify students' thoughts about planetary motion.

The class should understand that the earth is falling freely toward the sun. If it started from rest, it would fall on a straight line into the sun. The actual situation may be appreciated by thinking of the problem of bombing a target from a plane. The bomb is not released directly over the target. If it were, it would overshoot the mark. The faster the plane, the greater the overshot. An orbit results when the speed is within a range such that:

1. The speed is great enough that the overshot causes the bomb to miss the earth.
2. The speed is not great enough for the bomb to escape completely from the earth's gravitational field.

In the same manner, the earth does not fall into the sun simply because it is in a stable orbit; it has a tangential velocity such that it "misses" the sun as it falls toward the sun.

PROBLEM 20

(a) What is the speed of the moon around the sun compared to that of the earth around the sun?

(b) If the earth could be removed suddenly without disturbing the motion of the moon, what would be the subsequent path of the moon?

(c) Calculate the ratio of the force of attraction exerted by the sun on the moon to the force exerted by the earth on the moon.

(d) Why does the sun not capture the moon, taking it away from the earth?

This is a difficult problem for students to work through with any degree of rigor. The major point to bring out is that both earth and moon are attracted to the sun. (The earth and moon revolve about their common mass center which in turn goes about the sun. The center of mass of the earth-moon system is about 5/6 of the radius of the earth from the center of the earth.)

a) In a year, both the earth and the moon complete one trip around the sun. Their speeds around the sun are the same.

b) From Kepler's third law we know that, for all objects moving in a stable orbit around the sun, the value of R^3/T^2 will be a constant, independent of the objects' masses. Thus, since R and T are the same for both moon and earth (and the system of the two together), the present orbit would be an equilibrium orbit for either moon or earth alone as well as the two together.

Some students may point out that the moon's orbit around the sun would be dependent upon the exact point in the moon's orbit around the earth at which the earth "disappeared". This is because of small variations in the speed and direction of the moon's motion around the sun. At some points in the moon's orbit around the earth, the moon is moving faster than the earth in the moon-earth motion around the sun, at other points slower. The curvature of the moon's path around the sun also varies slightly with different positions of the moon relative to the earth and sun. However, compared to the moon-earth motion around the sun, the moon's motion around the earth is inconsequential. The speed of the earth-moon system around the sun is $\frac{2\pi \times 1.5 \times 10^{11}\ \text{m}}{365 \times 8.6 \times 10^{4}\ \text{sec}} = 3 \times 10^{4}$ m/sec. The speed of the moon around the earth is $\frac{2\pi \times 3.8 \times 10^{8}\ \text{m}}{27 \times 8.6 \times 10^{4}\ \text{sec}} = 10^{3}$ m/sec. Hence the moon's speed around the sun is 30 times greater than its speed around the earth. Annihilating the earth would not substantially change the length of a year for a moon resident!

c) There are two alternatives:

(1) We can write

$$\frac{F_{\text{sun on moon}}}{F_{\text{earth on moon}}} = \frac{m_{\text{sun}}}{m_{\text{earth}}} \times \frac{R^2_{\text{moon - earth}}}{R^2_{\text{moon - sun}}}.$$

Or, (2), we can write the force between the moon and sun, F_{ms}, as:

$$F_{ms} = \frac{4\pi^2 M_m R_{ms}}{T^2_{ms}}, \text{ and}$$

$$F_{me} = \frac{4\pi^2 M_m R_{me}}{T^2_{me}}.$$

Therefore, $$\frac{F_{ms}}{F_{me}} = \frac{R_{ms}}{T_{ms}^2} \times \frac{T_{me}^2}{R_{me}} = \frac{1.5 \times 10^{11}\,\text{m}}{3.8 \times 10^{8}\,\text{m}} \left(\frac{27.3\ \text{days}}{365\ \text{days}}\right)^2 = \underline{2.2}.$$

d) The sun exerts forces on both the moon and the earth. If we could maintain the sun's attraction for earth and moon, but "turn off" the attraction of the earth and moon for each other, they would both continue to orbit around the sun at the same radius and at the same speed. When we add the attraction between the earth and the moon, unless they go about each other they would fall together. Thus we see there is no capture even without an attraction between moon and earth. This equation is sometimes put in a slightly different form, "When the moon is toward the sun from the earth, why doesn't the sun steal the moon?" A good answer is, "It already has it."

PROBLEM 21

Assume the earth is perfectly round and has a radius of 6400 km.

(a) How much less does a man with a mass of 100 kg apparently weigh at the equator than at the poles because of the rotation of the earth?

(b) How fast would the earth have to spin in order that he would exert no force on a scale at the equator?

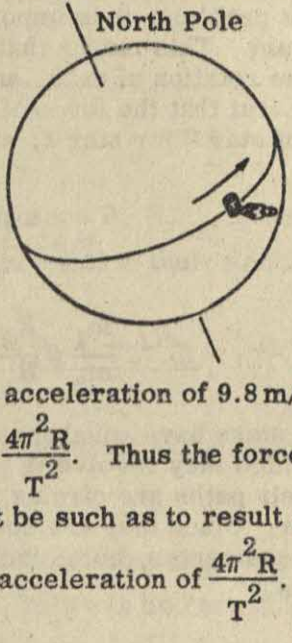
(c) How many times larger is the speed of rotation in (b) than the actual speed?

This is a moderately hard problem aimed at understanding the consequences of the earth's spin. We are asked about the apparent weight of a man — the reading of a scale on which he weighs himself.

a) Suppose a man stands on a scale, first at the pole, then at the equator.

At the pole, the man will turn with the earth, once each 24 hours, but the rotation of the man does not result in an acceleration toward the center of the earth. He is in static equilibrium. The force of the scale upward, F_s, balances the force of gravity, F_g, downward, giving $F_g = F_s$. The gravitational force, F_g,= mg = 100 kg × 9.8 newtons/kg = 980 nt. The scale will read 980 newtons.

Now let us look from the North Pole at the man on the scale at the equator. The forces acting on the man are F_g, the force of gravity toward the earth's center, and F_s, the force exerted by the scale away from the earth's center. But in this case, $F_g > F_s$. This is because the man, moving in uniform circular motion, is accelerating toward the earth's center. The force required to produce this acceleration is supplied by the gravitational force F_g. If this force, F_g, were not opposed by the force of the scale, it would result in an acceleration of 9.8 m/sec^2.



But it is opposed by the scale, limiting the acceleration to $\frac{4\pi^2 R}{T^2}$. Thus the force of the scale upward, F_s, which is opposed to F_g downward, must be such as to result in the net force downward being of a magnitude that will result in an acceleration of $\frac{4\pi^2 R}{T^2}$. The force that will produce this acceleration is $\frac{m4\pi^2 R}{T^2}$. Hence:

$$F_g - F_s = \frac{4\pi^2 Rm}{T^2} = \frac{4\pi^2 \times (6.4 \times 10^6\ \text{m}) \times 100\ \text{kg}}{24^2 \times 60^4\ \text{sec}^2} = \underline{3.4\ \text{newtons}}.$$

The man's apparent weight at the equator, as read by the scale, would be 3.4 newtons less than the 980 newtons read at the pole. This would be a "loss" of .35%.

b) If a man were to exert no force on a scale at the equator, it would be necessary that $\frac{4\pi^2 R}{T^2} = \frac{v^2}{R} = g$. Then, $v^2 = Rg = 6.4 \times 10^6\ \text{m} \times 9.8\ \text{m/sec}^2$, and $v = 7.9 \times 10^3\ \text{m/sec}$.

The period, T′, required for this speed would be:

$$T' = \frac{2\pi R}{v} = \frac{2\pi \times 6.4 \times 10^6\ \text{m}}{7.9 \times 10^3\ \text{m/sec}} = 5.1 \times 10^3\ \text{sec} = \underline{1.4\ \text{hours}}.$$

c) The speed found inpart (b) and the actual speed will be inversely proportional to the respective periods:

$$\frac{v'}{v_{\text{actual}}} = \frac{T}{T'} = \frac{24}{1.4} = \underline{17}.$$

PROBLEM 22

The two stars of a double star, far away from any other large masses, revolve in circular orbits, always remaining the same distance apart. Sketch their orbits if

(a) they have equal masses,
(b) one has twice the mass of the other.
(c) What is the ratio of the radii of the two orbits in each case?

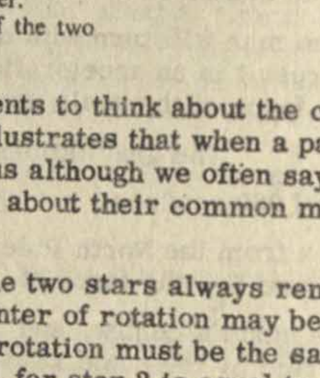
This moderately hard problem helps start students to think about the center of mass – one of the subjects of the next chapter. It also illustrates that when a pair of bodies revolve about each other, both actually move. Thus although we often say the moon goes about the earth once a month, actually both revolve about their common mass center once a month.

For this problem, it is important to note that the two stars always remain the same distance apart. This means that wherever their center of rotation may be, it must be the same for the rotation of each, and their periods of rotation must be the same. Now, we know in general that the force of attraction of star 1 for star 2 is equal to the force of attraction of star 2 for star 1, and it is the cause of the centripetal acceleration of each of them:

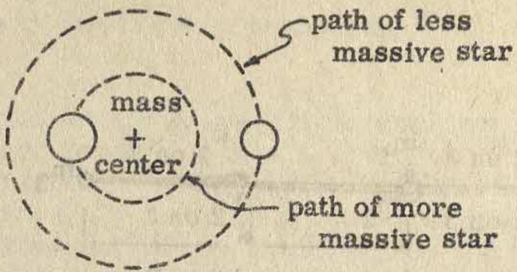
$$F = \frac{m_1 4\pi^2 R_1}{T^2} = \frac{m_2 4\pi^2 R_2}{T^2}$$

$$\therefore \frac{m_1}{m_2} = \frac{R_2}{R_1}.$$

a) If the stars have equal masses, the center about which they revolve is half-way between. Their paths are circles, about the mass center. Since they are equally distant from the mass center, both circles have the same radius.



b) If one star has twice the mass of the other star, the less massive star is twice as far from the center of rotation as the more massive.



c) The ratio of the radii of the two orbits in the case of stars of equal mass is 1/1. In the case of stars whose mass ratio is 2/1, the ratio of the radii is 1/2. The ratio of the masses is the inverse of the ratio of the radii.

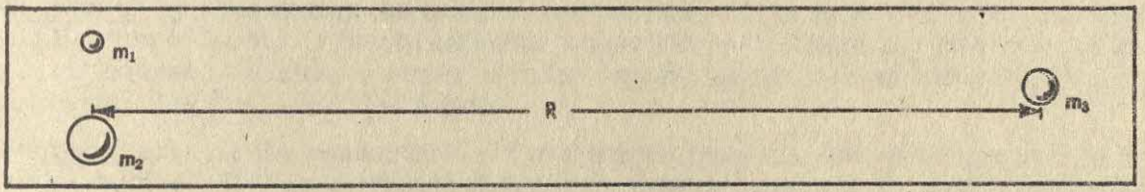
PROBLEM 23

Assume that the gravitational attraction exerted by m_1 on m_3 in Fig. 22-19 is not influenced by the presence of m_2, and the attraction of m_2 on m_3 is not changed by the presence of m_1.

(a) Using the universal law of gravitation, show that the force of attraction exerted on m_3 by the single "body" made of m_1 and m_2 together is proportional to its mass.

(b) If m_1 and m_2 were far apart compared to their distance from m_3, would they act like a single body?

22-19. For Problem 23.



a) Call $F_{1\text{ on }3}$ the force of mass 1 on mass 3, and $F_{2\text{ on }3}$ the force of mass 2 on mass 3. If the masses are close together, as shown in Figure 22-19, R is the same for both forces. The magnitudes of these separate forces will be given by:

$$F_{1\text{ on }3} = \frac{G m_1 m_3}{R^2}$$

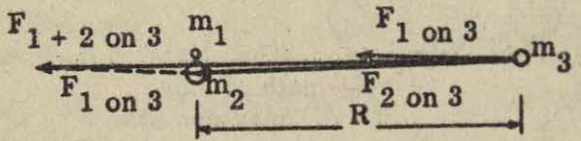
$$F_{2\text{ on }3} = \frac{G m_2 m_3}{R^2}.$$

_If_ masses 1 and 2 are close together compared to the distance R, $F_{1\text{ on }3}$ and $F_{2\text{ on }3}$ are in essentially the same direction and their vector sum is equal to their arithmetical sum. Hence,

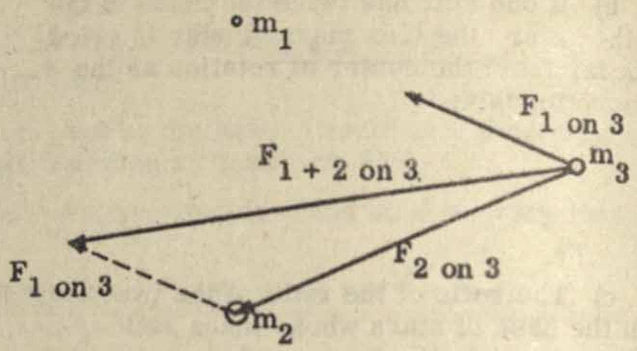
$$F_{1+2\text{ on }3} = F_{1\text{ on }3} + F_{2\text{ on }3} = \frac{G m_1 m_3}{R^2} + \frac{G m_2 m_3}{R^2} = \frac{G(m_1 + m_2) m_3}{R^2}.$$

Thus, if m_1 and m_2 are close together, the force on m_3 is equivalent to the force resulting from a single body of mass $m_1 + m_2$.

b) If m_1 and m_2 were far apart compared to R, their distances from m_3, they will not act as a single body equivalent to $m_1 + m_2$ at a distance R. This can be seen from a vector diagram. Suppose we assume $m_2 = 2m_1$, then $F_{2\text{ on }3} = 2F_{1\text{ on }3}$.



(1) m_1 and m_2 close compared to R.



(2) m_1 and m_2 far apart compared to R.

Clearly the magnitude of $F_{1+2 \text{ on } 3}$ in Figure (2) above is not as great as in Figure (1). Hence when m_1 and m_2 are far apart they do not act as a single body equivalent to $m_1 + m_2$ at a distance R.

Chapter 23 – Momentum and the Conservation of Momentum

CHAPTER SUMMARY

Chapter 23 introduces the concept of momentum and shows that the total momentum of an isolated system remains constant. Aside from the practical utility of momentum conservation as a tool in solving specific problems of mechanics, this idea takes its place among the conservation laws as one more entity that remains constant despite the whirling and dancing of submicroscopic particles and the often complicated motions and interactions of bulk matter – one of the fixed intellectual vantage points from which man organizes his attack on the problems of extending his control of the physical world.

Sections 1 and 2 introduce the idea of momentum through analyzing Newton's law in the form of $\vec{F}\Delta t = m\Delta\vec{v}$. In this form, Newton's law indicates that the quantity of motion is measured either by impulse, $\vec{F}\Delta t$, or momentum, $m\Delta\vec{v}$. By itself, the concept of impulse is not particularly important except as it leads to the definition of momentum.

Sections 3 and 4. Section 3 discusses in detail the momenta of two interacting bodies. This discussion leads to the law of momentum conservation in Section 4: for an isolated two-body system, the total momentum never changes, i. e., momentum is conserved. These two sections present the central ideas of the chapter.

Section 5 is an application of momentum conservation to rocket motion. Naturally, this subject will be of great interest to many students. It is, however, not vital to the sequence of ideas in the course.

Section 6, by considering the location and behavior of the center of mass, clarifies the application of Newton's law to extended rather than point objects. However, in the logic of this course, the idea of center of mass may be lightly treated with no great loss in understanding the topics that follow.

Section 7 extends the conservation of momentum from two bodies to any number of interacting bodies. It is important that students know that momentum conservation applies to systems of more than two objects. However, little further argument is required if they have really understood Section 4.

Section 8 discusses Newton's third law: If B exerts a force $\vec{F}$ on A, then A must exert a force $-\vec{F}$ on B. In the development of this course, this law follows easily from momemtum conservation.

SCHEDULING CHAPTER 23

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part III. Sections which are enclosed with brackets [] can be deemphasized or skipped in class discussion without loss in continuity. Even though these sections may be deemphasized in class, they should be assigned for home reading.

INSTITUTE OF EDUCATION FOR WOMEN
Dept. of Extension SERVICE.
CALCUTTA-27

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2	2	1	III-8	2	1	III-8
Secs. 3, 4	3	2*	III-9 III-10	2	1*	III-9
Sec. [5]	2	0		2**	0	
Sec. [6]	2	0				
Sec. 7	1	0		1	0	
Sec. 8	1	0		1	0	

* This laboratory work can be scheduled at this point or later in the chapter.
** Treat either Section 5 or 6.

RELATED MATERIALS FOR CHAPTER 23

Laboratory. Experiment III-8, Momentum Changes in an Explosion, can be used, prior to class discussion, as an introduction to the concept of momentum changes in two-body systems, or can be done right after the first class work on Section 3.

Experiment III-9, The Cart and the Brick, shows that momentum in conserved in a two-body collision even when the initial momentum is not zero. The experiment can be done during or after Section 4.

Experiment III-10, A Collision in Two Dimensions, emphasizes the vector nature of momentum conservation and that it is the vector sum of the individual momenta that is conserved. This experiment can be done during or after Section 4, or can be saved to serve as a base for summarizing your work on momentum and as a lead toward the conservation of energy.

Home, Desk and Lab. Even if you do not emphasize the content of Sections 5 through 8 in your classwork, the problems for these sections may be assigned. The techniques developed in Sections 1 through 4 plus the vocabulary of later sections are sufficient.

Problems 11 and 12 constitute a pair which illustrates that, in some cases, either conservation of momentum or $F = ma$ may be equally well applied.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Section	Easy	Medium	Hard	Class Discussion
1, 2	1, 2, 3, 5	4*, 6*, 7	8*	6*
3, 4	10, 11*, 12, 13, 14, 18*	9*, 15*, 17, 19	16	10, 13, 16
5		20*		20
6	21, 23*		22*	22
7		24*	25, 26*	24*, 25, 26*

Section 1 — Impulse
Section 2 — Momentum

PURPOSE To introduce the idea of impulse and use it to define momentum; to show that momentum is a dynamical, as distinguished from a kinematical, quantity.

CONTENT a. The product $\vec{F}\Delta t$ is defined as the impulse.

b. Impulse is a vector.

c. It is often possible to measure the product $\vec{F}\Delta t$ even though we know neither $\vec{F}$ nor Δt separately.

d. Since the impulse, $\vec{F}\Delta t$, is measured by a change in the quantity $m\vec{v}$, it is convenient to treat $m\vec{v}$ as a specific concept, $\vec{p}$, the momentum.

e. Momentum is a vector.

f. Force is equal to the rate of change of the momentum $F = \Delta p/\Delta t$.

EMPHASIS Momentum is an extremely important physical tool. It will not be wise, however, to dwell too long on these sections before covering Section 3. The usefulness of the idea of momentum begins to become apparent in Section 3, so it is best to go ahead through that material and then return to these sections if necessary.

LABORATORY Experiment III-8, Momentum Changes in an Explosion, should be performed as soon as students have been exposed to the concepts of momentum and impulse.

DEVELOPMENT 1) In introducing the concept of impulse and showing that the impulse gives rise to a change in mv, i.e.,

$$\vec{F}\Delta t = m\Delta\vec{v},$$

students should be reminded that they are dealing with nothing new. Refer them to Sections 20-8 through 20-10. Impulse = change in momentum is simply a statement of Newton's law. $\vec{F} = m\vec{a}$ is the form which applies to infinitesimal time intervals, $\vec{F}\Delta t = m\Delta\vec{v}$ applies to finite increments of time and velocity. You can emphasize this by asking for an acceleration. For instance Problem 5 of HDL might be asked:

> A skier is moving at 10 m/sec and is stopped in 1.5 seconds. What is his average acceleration? ($10/1.5 = 6.6\ \text{m/sec}^2$.) If the skier has a mass of 75 kg what average force was exerted on him? ($F = ma = 75 \times 1.0/1.5 = 5.00$ newtons.) What impulse did this force provide? (Impulse $= F\Delta t = 500 \times 1.5 = 750$ newton-sec.)

But the impulse can be found directly, since $m\Delta v = 75 \times 10 = 750$ newton-sec or kg-m/sec. Thus, saying "impulse = change of mv" is just a new way of looking at $F = ma$ — a simpler way for certain applications.

2) Perhaps some practice on this point will be helpful. You could give the students three of the four quantities, F, Δt, m, and $\Delta\vec{v}$; and ask for $\vec{a}$ so that they use either $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ or $\vec{a} = \frac{\vec{F}}{m}$; finally ask for the fourth quantity. Make your questions simple numerically and tabulate the four values and the two products $\vec{F}\Delta t$ and $m\Delta\vec{v}$.

For example, suppose you want to give a 1-kg object a speed of 12 m/sec in 3 seconds. What uniform acceleration and what force could do this?

$$a = \frac{12}{3} = 4\ \text{m/sec}^2$$

$$F = 1 \times 4 = 4\ \text{newtons}$$

$$m\Delta v = 12\ \text{kg m/sec};$$

$$F\Delta t = 4 \times 3 = 12\ \text{newton-seconds}.$$

What would the corresponding answers be if you were to reach this speed

in 1 second?	$a = 12\ \text{m/sec}^2$	$F = 12$ nt
in 2 seconds?	$a = 6\ \text{m/sec}^2$	$F = 6$ nt
in 4 seconds?	$a = 3\ \text{m/sec}^2$	$F = 3$ nt
in 6 seconds?	$a = 2\ \text{m/sec}^2$	$F = 2$ nt
in 12 seconds?	$a = 1\ \text{m/sec}^2$	$F = 1$ nt.

Of course, $m\Delta v$ has remained at 12 kg-m/sec; but notice that $F\Delta t$ has also remained at 12 nt sec. After you have given this rapid set of values, you can ask for the force directly. What force is needed if $t = 1/2$ sec? (24 nt.) What force is needed if $t = 20$ sec? (0.6 nt.)

You can easily make up similar problems. Keep the pattern of asking for $\vec{a}$ for a few changes in conditions, so that the numerical value of $\vec{F}\Delta t$ or $m\Delta\vec{v}$ is clear; then ask for the unknown without bothering with $\vec{a}$.

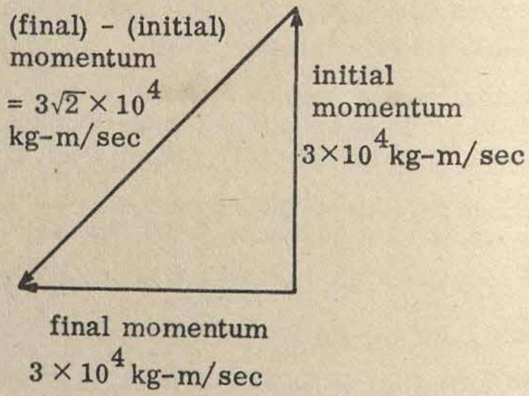
3) Once students appreciate that $m\Delta\vec{v}$ determines $\vec{F}\Delta t$, and vice versa, you might ask them whether (in practice!) the individual values of $\vec{F}$ and Δt (or m and $\Delta\vec{v}$) make any difference. For example, return to Problem 5 in which the 75-kg skier moving at 10 m/sec was stopped in a snow bank in 1.5 sec. Since the impulse needed to stop the skier was 750 newton-sec, 500 newtons were exerted by the snow bank. Would it make any difference in the impulse if the snow bank were denser and stopped the skier in 1/2 second? (No) Would it make any difference to the skier? (Yes! The force would be much greater, possibly enough to break a bone.)

4) Be sure to discuss at least one situation like that in Problem 7 where the impulse is provided by a relatively small force acting for a long time. Students should not get the idea that an impulse has to be a sharp blow.

5) Classroom work should emphasize both the magnitude and the vector nature of momentum. For example, a 2-ton car moving 30 mph north does not have the same momentum as a 1-ton car moving 60 mph south.

You may wish to introduce changes in momentum even at this stage. Thus, a 1 1/2-ton car going north at 40 mph suddenly turns and goes west at 40 mph. What is its change in momentum? If the tires squeal a bit, that's fine. It emphasizes that forces must act to change the momentum of an object.

The above example may be made quantitative if given in metric units. A 1500-kg car going 20 m/sec for instance. What impulse is necessary to change its direction from north to west? The necessary impulse is $3\sqrt{2} \times 10^4$ kg-m/sec or nt-sec directed southwest.



COMMENTS Some students will accept the vector nature of impulse without going through the detailed development given in the text. The best way to present this to your class depends upon how much time you devoted to vectors in Chapter 6. For example, if your students know that the product of a scalar and a vector is another vector, the fact that $\vec{F}\Delta t$ is a vector is obvious. Force always has direction. Including the time the force acts does not change its directional nature. (Incidentally, the equation $\vec{F}\Delta t = m\Delta\vec{v}$ would be meaningless if $\vec{F}\Delta t$ were not a vector and $m\Delta\vec{v}$ were a vector. If we did not specify the direction of the force, we would not know the direction of the resulting change in velocity.

If you emphasized the graphical interpretation of distance and speed when you covered Chapter 5, your students may get additional insight into the additive effect of many impulses if you review the graphical material briefly. The graphical presentation of impulse as the area under an F vs t graph need not be stressed merely to enable the students to do calculations involving impulse. In fact, the importance of the concept of impulse is that, although we may know neither $\vec{F}$ nor Δt, we do have a real experimental handle on the product, namely the change in momentum of an object. (Note that we measure $m\Delta\vec{v}$ and infer $\vec{F}\Delta t$ only if we want it.) Hence, the graphical presentation of impulse is not of prime importance. Furthermore, most students will not understand how to picture graphically the total impulse produced by a force which changes direction. (It is not worth the time to introduce the idea of graphing separately the three components, x, y, and z, of the forces.)

* * *

Beyond the recognition that momentum is $m\vec{v}$, students should see that any two objects which have the same momentum, no matter how it was acquired, have a common property. Their common property is that both could be stopped by the same impulse, $\vec{F}\Delta t$. Students should see that it is the product $m \times \vec{v}$ that is important. Any given momentum can be produced by an infinite number of combinations of masses and velocities. All the combinations resulting in the same momentum have a common property. They can all be stopped by the same impulse, $\vec{F}\Delta t$.

Although two objects might have equal velocities, there is no simple relation between their velocities and the impulses needed to stop them; the impulse would depend on their masses.

(Although it should not be taken up now, later chapters will show that kinetic energy is another dynamic property; two objects with equal kinetic energies can be stopped by the same amount of work $F\Delta x$.)

* * *

The unit used for impulse serves equally well for momentum. That the unit is not given a special name, and that it can be made of two obvious combinations of more familiar units, may bother some students. When one uses $\vec{F}\Delta t$ to calculate impulse or momentum, it is most convenient to use the unit, newton-sec. On the other hand, if $m\Delta\vec{v}$ is used, the simplest way to express the unit is kg-m/sec. These two ways of writing the unit are identical; 1 newton-sec = 1 kg-m/sec. This is clearly true because 1 newton = 1 kg-m/sec^2.

CAUTION Students must have a firm idea of the definition of momentum, that it is a vector, and that momenta are changed by impulses – forces acting for a time. However, at this stage of the chapter they are only learning definitions without the motivation of new physics. Do not delay too long over these new expressions of old ideas. Get them only firmly enough in hand to go on to the next two sections which are the meat of the chapter.

QUIZ QUESTIONS 1. A force of ten newtons acts on a body for a total time of 5 seconds. What is the magnitude of the impulse? What is the momentum change of the body? If the body was initially at rest, and has a mass of 2 kg, what is its final speed? (50 newton-sec; 50 kg-m/sec; 25 m/sec.)

2. A golf ball of mass 100 grams is struck by a club. After the impact the ball moves off with a speed of 50 m/sec. If the ball and club were in contact for 5×10^{-3} sec, what was the average force acting on the ball? (1000 newtons.)

3. Two forces act on a 10 kg-mass which is initially at rest. First the body is subjected to a 20 newton force for 3 seconds which acts in a northward direction. One second after the first force is applied the body is subjected to a force of 30 newtons which acts in an eastward direction for two seconds.

(a) What is the northward component of the impulse? (60 newton-sec.)

(b) What is the eastward component of the impulse? (60 newton-sec.)

(c) What is the final momentum of the body? (60 $\sqrt{2}$ kg-m/sec, northeast.)

(d) If the eastward force had been applied initially at the same instant as the northward force, instead of a second later, what would be the final momentum of the body? (same as (c).)

4. A 2-kg ball moving with an initial speed of 3 m/sec to the right is acted upon by a constant force F to the left for a period of 3 seconds. After the interaction, the ball is moving to the left with a speed of 6 m/sec. What is the magnitude of F? (6 newtons.)

5. A body is subjected to a varying force which increases linearly with time from zero to 10 newtons in 0.1 sec, remains constant at 10 newtons for 0.5 sec, and then decreases linearly to zero in 0.2 sec. What is the total impulse? If the body has a mass of 3 kg and was initially at rest, what is its final speed? (6.5 newton-sec; 2.17 m/sec.)

Section 3 – Changes in Momentum when Two Bodies Interact
Section 4 – The Law of Conservation of Momentum

PURPOSE To present the law of conservation of momentum and to develop some idea of its usefulness.

CONTENT a. When two objects "explode" apart from rest, their momenta are equal and opposite in direction.

b. In a head-on collision of two equal masses the momentum of one is transferred completely to the other.

c. In a glancing collision the change in momentum of one object is equal and opposite in direction to the change in momentum of the other.

d. A new way to state all the above results is that the total momentum of the two bodies is the same before and after the collision.

e. This law is tested for cases of greater generality.

EMPHASIS These sections contain the central ideas of this chapter. If they are understood thoroughly your job is almost done. The following sections are merely special cases or simple generalizations of the law of conservation of momentum. Allow at least three class periods, and more if necessary, for the presentation of the material of these two sections

LABORATORY Experiment III-9, The Cart and the Brick, can be performed at any time from here on in the course of this chapter.

COMMENT It is important to understand that, as presented in this course, momentum conservation is an empirically established law and cannot be proven or derived by any set of logical arguments. While a proof can be based on Newton's third law, the text establishes the conservation of momentum on purely empirical grounds, and then deduces Newton's third law* (in Section 8). This approach is especially appropriate in leading toward the observation and use of momentum conservation in analyzing collisions between atomic particles where little or nothing is known about the forces of interaction.

DEVELOPMENT The development of the content of these sections can be handled in three steps:

1) Discussion of observed changes in momenta of two exploding or colliding bodies and development of the idea of conservation of momentum.

2) Practice in applying and understanding the law of conservation of momentum.

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* In an interaction between two bodies, the forces are equal and opposite.

3) Introduction of the idea of an isolated system. This last point is not discussed explicitly in the text but should be developed in class. Suggestions along this line are given near the end of this section of the guide.

1) In order to stress the fact that momentum conservation is a law which is based directly upon experimental observations, it is useful to focus student's attention on the direct evidence given in the text, Figure 23-7. Even though this figure is analyzed in the text, it deserves the additional emphasis of a class discussion to make sure they properly interpret the experiment.

Students can analyze the motion of each disc during ten time intervals (or 2 seconds). The large disc traveled 20.0 cm; the small disc traveled 41.2 cm. Ask the students for these distances.

There are two qualitative features of Figure 23-7 which may not be understood by a few students:

a. The pattern formed by the small disc looks odd becuase it happened to travel almos exactly one half its diameter in each interval between flashes.

b. The motion during the "first" time interval is less than in the others because the explosion occurred about midway between two flashes.

Here are two questions which you might ask students about Figure 23-7 to make sure they are "seeing" the figure properly:

a. Estimate when the explosion occurred relative to the flashes.

Students who estimate this by looking at the positions of the small disc will easily see that the explosion came about halfway between flashes. However, some students who look at the positions of the large disc may think the explosion occurred just a bit before a flash. This common error is based on the fact that the dark space between the first two positions of the large disc appears so small. Ask students to explain this; you can point out that the large disc moved not only through this dark space, but also through a distance equal to the width of the white rim.

b. Using only the velocity given in the text, find the size of the small disc.

This can be done easily and quickly if the students use the fact that the small disc happened to move a diameter in two flashes. Since $v = 0.206$ m/sec and since two flashes correspond to 2/5 sec, the distance traveled in two flashes or the diameter is 2/5 sec × 20.6 cm/sec = 8.15 cm.

After a brief preliminary discussion such as the above, the parallelism between Figures 23-7 and 23-5 can be pointed out and the actual momentum changes discussed. An alternate class discussion might be based on student data from Experiment III-8, Momentum Changes in an Explosion.

It will be wise to also go over the analysis of momentum changes in Figures 23-9 and 23-11, or, alternatively, discuss HDL Problem 16.

After discussing momentum changes in these situations, and developing the law of conservation of momentum, you can return to each example and analyze it from the conservation point of view. Of course, you will only be repeating your previous work but you will be looking at each interaction as a whole from the standpoint of momentum rather than at the individual momenta.

2) The next step is practice. Although it is rather easy for students to learn how to use momentum conservation, you should give them a few simple examples similar to those described in the text or problems such as HDL 9 through 19. Initially, use simple numerical values and ask for numerical answers. Such use of momentum conservation will not be a challenge to some students, but there is no point to trying to discuss the more subtle questions with the class, until they have mastered the straightforward application of momentum conservation.

3) The final step in understanding momentum conservation is to understand when two bodies may be considered to be a complete system, isolated from external forces.

There are two fundamental bases of student understanding of momentum conservation. The most obvious and most important is the idea that when only two objects interact, the total momentum of the two does not change. This is extremely general and does not depend on the interaction; the interaction can be simple or complex, it can be caused by chemical explosion, elastic forces, muscular pushes, etc. As long as only two objects are involved, momentum is conserved. This aspect has been treated up to this point. The second aspect is more subtle. It involves the question of when tow objects are indeed isolated. You will have to spend some time discussing this or your students may think that momentum conservation applies only in the physics course and not in nature.

Before going into a detailed discussion of isolated systems, let us indicate the source of student confusion. When students are given an artificial problem involving, say, the collision of two balls, they may apply momentum conservation to get the answer they are after. But, they know that a single ball will not maintain its velocity; it will slow down. Hence, instead of being a universal, powerful law, momentum conservation may seem to be an expedient to be used to get the correct answer for a particular class of problems. Considerable class discussion will be needed to relate momentum conservation to nature as the students see it day after day.

COMMENTS ON ISOLATION In dealing with momentum problems, the isolation of a system (what to include in the system, and what to exclude) is not always simple. The only case in which near-perfect isolation occurs is in an interaction somewhere in outer space . Such cases are somewhat artificial, but easy to understand.

A second kind of isolation is illustrated by smooth objects on a frozen lake (assuming the ice is frictionless), or billiard balls on a table (again neglecting friction). Here external forces are not small. Gravity pulls down, while the ice or the table pushes up. However, because the surface is rigid, the forces automatically adjust themselves to be equal and opposite, and the net force up or down on the object is zero. In such cases, then, there is a kind of pseudo-isolation with large external forces acting, but adding up to zero.

A third and very useful case (and one which students should understand in order to appreciate the broad utility of momentum conservation) involves isolating an interaction in time – applying momentum conservation during the time interval that just covers the interaction. Consider the question of changes of momentum between two billiard balls colliding on a billiard table. No one doubts that a billiard ball set rolling on the table will eventually come to rest because of friction. Hence, two billiard balls on a table do not form a really isolated system. But consider just the instant of collision. The table, an object external to the system, exerts frictional forces on the balls during the collision. These forces may not be small. The important thing is that the impulse imparted by these forces during the brief instant of the collision is negligibly small compared to the impulses of the forces of interaction between the balls during the collision. Therefore the momentum of the two-ball system may be said to be conserved during the collision even though it is not conserved if longer times are considered. If we need to consider frictional effects we can apply momentum conservation to the interaction, then consider the effect of friction on these results.

Another illustration of this case is an exploding rocket. Here, gravity acts on the rocket as an external force. However the explosion takes place in such a short time that the impulse imparted by gravity to the rocket during the explosion is negligible and momentum conservation within the rocket system may be used to describe the situation immediately before and after the explosion. We can then, if necessary, consider the effect of gravity on a given mass with a given velocity. Also, the gravitational case has a special property. In an interaction in a constant gravitational field, the accelerations of all bodies are equal so the gravitational force can be neglected. See HDL Problem 22.

DEVELOPMENT Use of the Principle of Conservation in Practical Situations. Return in class discussion to each of the examples given in the text in Section 3. Ask individual students to tell exactly what the two objects are in each case, and why these two objects form an "isolated" system. Also have them restate the law of conservation of momentum specifically as it applies to each system.

For example, the boy and the man are the interacting objects in the first example of Section 3 (Figure 23-5). They can be considered as an isolated system because they are standing on smooth ice; the term smooth indicates that we can neglect friction. The effect of the downward pull of gravity is exactly cancelled by upward push of the ice. Since both the man and boy were stationary before the push, the initial momentum was zero. Hence, the total momentum (vector sum) must always be zero. If the man acquires momentum in one direction, the boy must acquire an equal and opposite momentum. Ask a student what would happen if the boy pushed downward on the man so that the boy lifted himself into the air. Would the man move down? Are the man and boy "isolated" insofar as vertical motion is concerned?

Ask students what effect escaping gases would have on momentum conservation applied to a gun and bullet. When the bullet leaves the gun barrel and the gases do escape, they usually carry a negligible amount of momentum. However, whether or not the momentum of the gases is negligible is not so important as is the realization by students that they are considering this momentum negligible. If someone devised a gun in which the gases carried significant momentum, these gases would have to be included together with the gun and the bullet in calculating momenta.

COMMENT Figure 23-9, 23-11 and 23-26 show billiard balls colliding. Most students think of these balls as rolling on a table. Actually they were suspended on long threads. That is why spots and stripes appear to stay fixed or rotate very slowly.

Some students may be interested in the fact that when an off-center collision occurs between a moving (non-spinning) billiard ball and a stationary ball of equal mass, the two balls, after the collision, move off along paths which are at right angles to each other. Students may note this when they perform Experiment III-10, A Collision in Two Dimensions. This fact depends on the conservation of kinetic energy (toward which students are led by the final paragraph of Experiment III-10, see the comments in the yellow pages on that experiment) and can easily be proved after they have studied energy. This proof is given in Appendix 6 at the end of this volume of the guide.

* * *

QUIZ QUESTIONS

1. A 5-kg ball of putty moving at a speed of 6 m/sec strikes a 1-kg block of wood which was initially at rest. The putty sticks to the wood. What is the final speed of the wood? (5 m/sec.)

2. A 10-gram bullet is fired with a speed of 1000 m/sec from a freely hanging 2-kg gun. What is the recoil velocity of the gun? (5 m/sec.)

3. A 5-kg body at rest is torn apart by an explosion. It breaks into two pieces. A 4-kg piece is seen to move off to the right at a speed of 50 m/sec. Describe the motion of the remaining 1-kg piece. (Moves to left at a speed of 200 m/sec.)

4. A 10-kg mass moving to the right at a speed of 20 m/sec interacts with a second mass which was initially moving to the left at a speed of 20 m/sec. After the interaction both masses are found to be moving to the right at a speed of 5 m/sec. How large is the

second mass? (6 kg.)

5. Can momentum conservation alone determine motion after a collision? If you have time at this stage you can use this question to motivate the search for other dynamical quantities in later chapters. Ask students whether any other final velocities of the two balls in Figure 23-8 could have kept the total momentum unchanged. Students will be able to answer this more easily if you use specific numerical values. Say that the balls each had a mass of 0.5 kg and that the initial speed was 4.4 m/sec. (Note that this speed corresponds to the ball going a little less than 27 cm in 3/48 seconds. Note too that the apparent elongation of the ball by about 2 cm combined with this speed implies that the camera shutter was open for about 1/220 second during each frame.) The students should realize that equal momentum could have been realized by many other combinations. Ask them how fast ball 1 would have been moving after the collision if ball 2 had attained a speed of 3.3 m/sec rather than 4.4 m/sec. They should recognize quickly that ball 1 would have continued with a speed of 1.1 m/sec instead of stopping. What if ball 2 had attained a speed of 4.1 m/sec? Ball 1 would have retained a velocity of 0.3 m/sec forward.

You can leave students with the question of how ball 2 manages to make a choice with all of these possibilities. Physically it is clear that the detailed interaction at collision must determine the result. In Chapter 24, we shall include energy in our analysis of collisions and thereby eliminate some of the ambiguity.

Section 5 — Rockets

PURPOSE To illustrate the use of momentum conservation by considering one application of great current interest, the operation of rockets.

CONTENT A rocket ship gains forward momentum equal to the backward momentum of the exhaust gases.

EMPHASIS This material constitutes one single example of the use of momentum conservation. It can be covered as a reading assignment. If you have extra time, your students will be interested in a discussion of some examples using the graph of Figure 23-15b.

DEVELOPMENT A Sample Rocket Problem. If a fuel were available with an exhaust speed of 3.3 km/sec, what mass rocket would be needed to give a 1000-kg load a final speed of 10 km/sec. (Neglect air resistance and gravity.) Since $v/v_e = 3$, Figure 23-15b shows that $m/m_o = 0.05$. Therefore a 20,000 kg total would be needed. (Note that if air friction and gravity were included a fuel load of about 50,000 kg would be required to bring the 1,000 kg pay load to a speed of 10 km/sec; 11 km/sec is the velocity needed by an object so that it can escape from the earth.)

You can get other sample problems from the supplementary information below.

SUPPLEMENTARY MATERIAL ON ROCKETS The estimates given in the text for the ratio of total rocket plus fuel mass to final rocket mass (or "pay load") are conservative. Recent American and Soviet rockets have used fuels with exhaust speeds considerably above the 2 km/sec used for illustration in the text. The rocket fuels which were conventional in 1957 had exhaust velocities of about 2.6 km/sec. Scientists are working on chemical fuels which are expected to raise this value to 4 km/sec within, perhaps, 5 years. Preheating gaseous fuels may raise the exhaust speed to 6 km/sec. (It is unlikely that achievable fuel efficiencies will be made public while rockets play a key role in military preparations.)

When exhaust speeds of from 4 km/sec to 6 km/sec are achieved, manned earth satellites will be practical. However, much higher exhaust speeds, say about 100 km/sec, will be needed to make space travel feasible. Such exhaust speeds would require something more than chemical fuels. One possible alternative is to exhaust a stream of ions which has first been accelerated electrically. Presumably some advanced, compact power source such as a nuclear reactor would have to be carried to supply the required electrical power. Another proposal for the future involves the use of atomic bombs to hurl fragments opposite to the direction of the rocket motion.

* * *

If you read about rocket fuels in newspapers or magazines, you will probably come across the term "specific impulse" which is used to describe fuel efficiency. For example, a fuel in a particular rocket might have a specific impulse of 250 pound seconds per pound. (This unit is pounds force times seconds per pound of mass.) In MKS units this specific impulse would be $9.8 \times 250 = 2.45 \times 10^3$ newton-seconds/kg which is exactly what the text would refer to as an exhaust speed of 2.45 km/sec.

$$\frac{1\,\text{nt sec}}{\text{kg}} = \frac{1\,\text{nt sec}}{\text{kg}} \times \frac{1\,\text{kg m/sec}^2}{1\,\text{nt}} = 1\,\text{m/sec}.$$

The unit for specific impulse is expressed in newton seconds per kilogram to show explicitly the impulse (in newton seconds) derived from burning and exhausting 1 kg of fuel. In thinking about rocket design, it is sometimes convenient to rewrite 2.45×10^3 nt sec/kg as 2.45×10^3 nt per kg/sec. That is, 2.45×10^3 nt of force would be generated if the fuel were burned at the constant rate of 1 kg/sec. Part of the engineering problem of making rockets is to burn and exhaust the fuel fast enough.

Consider for example, a 100 ton rocket (about 10^5 kg). The force of gravity on this rocket would be 9.8×10^5 newtons. If a fuel of specific impulse 3×10^3 nt sec/kg (or an exhaust speed of 3 km/sec) were used, burning 1 kg/sec would produce only 3×10^3 nt. To overcome gravity, a minimum rate of burning would be 3.3×10^2 kg/sec. Since this would merely balance gravity, a burning rate of 400 kg/sec would be more reasonable. Note that after only 100 seconds the mass of ejected fuel would be 4×10^4 kg or 40% of the original mass.

Actually one of the most surprising results derived in this section is that the final speed of the rocket depends only on the exhaust velocity and the ratio of pay load to initial mass of the rocket, and does not depend on the rate at which the rocket fuel is burned. There is thus no real need in an interplanetary rocket for a great flaming, roaring exhaust unless the rocket must accelerate rapidly. Why, then, are rockets built as they are? The answer is given in the previous example. Fuel must be burned at a rapid rate in order to overcome the pull of gravity.

Another practical necessity in rocket design is to avoid excessive acceleration which could damage some of the intricate equipment in the rocket itself (e.g., the steering mechanism or the fuel system) or in the pay load. As the fuel is burned, the rocket mass decreases and the acceleration increases. In multistage rockets, a new engine which burns fuel more slowly (and therefore probably more efficiently) takes over once the mass has decreased enough. Of course, when this switch occurs, the mass can be further reduced by ejecting the larger, faster-burning rocket engine together with the storage tanks that had held the ejected fuel.

Figure 23-15b is a graph of the ratio of the remaining mass, m, to the original total mass of fuel and rocket, m_o, as a function of the ratio of the final speed attained, v, to to the exhaust speed, v_e. The equation relating these is:

$$\frac{m}{m_o} = e^{-v/v_e}$$

$$\text{or } 2.30 \log_{10} \frac{m}{m_o} = -\frac{v}{v_e}$$

$$\text{or } \log_{10} \frac{m_o}{m} = 0.434\frac{v}{v_e}.$$

(The derivation of this equation (for teacher reference) is in Appendix 5 of this volume of the guide.)

This formula assumes that all the mass which left the rocket $(m_o - m)$ moved with a velocity, v_e, relative to the rocket in the direction opposite to the rocket motion.

A lower limit on the exhaust speed of the fuel in the 80-ton Atlas rocket which put a 4-ton load in orbit (on Dec. 18, 1958) can be obtained by setting $v = 8$ km/sec (the speed of a satellite in equilibrium orbit near the earth's surface) and using either the equation or Figure 23-15b. From the equation,

$$\log_{10}(20) = 0.434\,(8/v_e)$$

$$\text{or } v_e = \frac{0.434\,(8)}{1.3} = 2.67\,\text{km/sec}.$$

Using Figure 23-15b, we find the value of v/v_e when $m/m_o = 0.05$. This value is about 3. Therefore, $v_e = \frac{8}{3} = 2.67$ km/sec. This must be a lower limit of the exhaust speed because (a) there was some air resistance, (b) the force of gravity was present, and (c) some of the original mass was fuel container and the first stage rocket engine.

Section 6 – The Center of Mass

PURPOSE To introduce the idea of center of mass through consideration of a system composed of two objects.

CONTENT a. The center of mass of two masses is the point that divides the distance between them in inverse proportion to the masses. Thus, $m_1x_1 = m_2x_2$.

b. If a system of two objects is isolated, the velocity of the center of mass, v_{cm}, does not change. The total momentum of the system is $(m_1 + m_2)\,v_{cm}$.

EMPHASIS This section may be treated very briefly or omitted without seriously affecting the continuity of the course. This does not mean that the idea of center of mass is of secondary importance. On the contrary, it is basic to the understanding of all mechanics beyond the most elementary. If you have plenty of time, you should surely include this section. It may be abbreviated only because most students do not think seriously about the motion of extended bodies but in some way assume that the "average" motion is given by Newton's law, an intuitive feeling which happens to be correct.

COMMENTS The actual forces acting on an extended object may be quite complex. If such an object is pushed "off center" (i.e., so the net force does not go through the center of mass) it will rotate as well as translate. In addition one part of the object may push on another part and the action of these forces should be taken into account. It can be proved that the center of mass of such an object moves as if the entire mass of the object were concentrated at the center of mass, and as if the only forces acting were the external applied forces. This can be demonstrated by taking a very irregularly shaped piece of wood and painting its center of mass with a white dot. When thrown into the air the object twists and wobbles, but the white dot describes a smooth parabolic path.

* * *

Not only is the motion of the center of mass particularly simple, but motion relative to the center of mass often may be very simply described. See the exercise on Figure 23-9 suggested below.

DEVELOPMENT a. One way to treat this subject is to discuss Problem 22 with the class, drawing the diagram suggested in the solution for that problem.

b. You can discuss Figure 23-21 so that students appreciate how the motion of the center of mass is simple even though the wrench appears to be moving in a complicated way. The following questions can be used to lead students to consider the motion as the resultant of two simple components.

1) If the wrench is 20 cm long, what is the average speed of the center of mass? The students should be able to find that the wrench moves a distance equal to its length (or very slightly over that) in two flashes. Hence, $v = 0.20/0.067 = 3\,\mathrm{m/sec}$. (Note that in doing this the students will confirm the fact that $\vec{v}_{cm}$ is essentially constant.)

2) Which way is the wrench moving? The only clue is that the wrench is probably slowing down due to friction. It will take rather careful measurements to show that the wrench is moving from left to right.

3) Is the picture distorted by the camera? Yes, the wrench size seems different at the two ends of the picture.

4) What is the speed of rotation of the wrench about its center of mass? It rotates about once in 14 flashes or has a rotational speed of just more than 2 revolutions per second.

5) What is the instantaneous velocity of the hole at some particular time? The students can get an estimate of this graphically by dividing the displacement by the time. But to get this accurately the students must first find the tangential speed and the corresponding direction due to the rotation. They can get this by knowing the rotational speed and the distance from the hole to the center of mass (which is the axis of rotation). To this velocity (relative to the center of mass) must be added (vectorially) the velocity of the center of mass.

c. A graphical exercise to be done at home (time consuming but informative).

The following exercise gives students an interesting insight into collisions and into how phenomena can appear different when viewed from different reference frames.

Analyze Figure 23-9, a collision between two equal masses, in the following way.

Make a tracing of the positions of the center of the two balls at various times. This might look like Figure (a). Join the positions of the two balls after the collision as shown at (b) and mark the positions of the midpoint of the line, i. e., the center of mass. Do this for the two balls before the collision as at (c). This may be a little messy; but can be done. Point out that the center of mass moves with uniform velocity.

Figure (a)

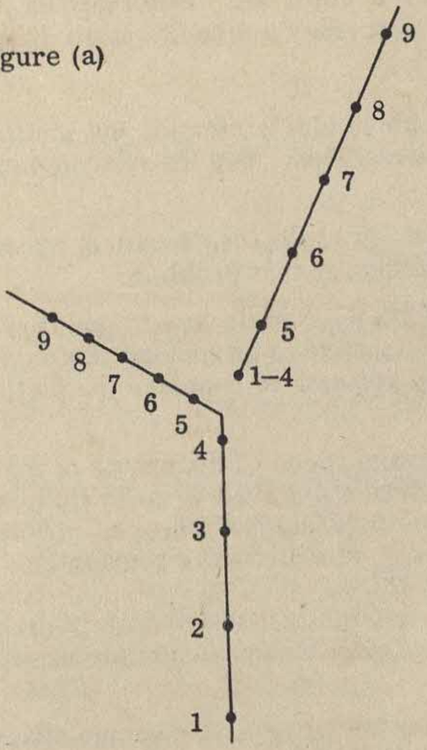


Figure (b)

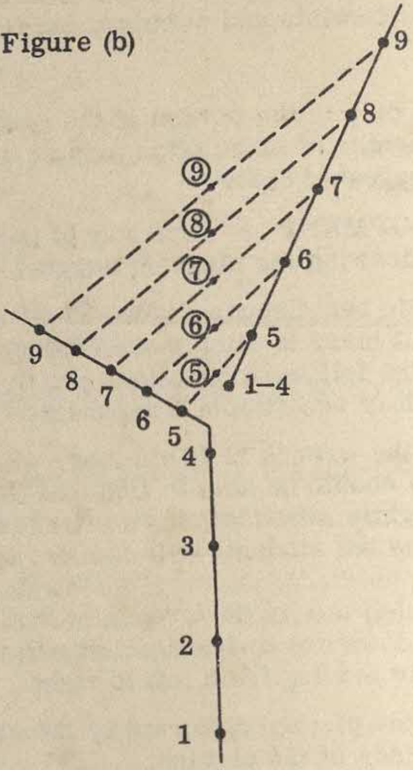


Figure (c)

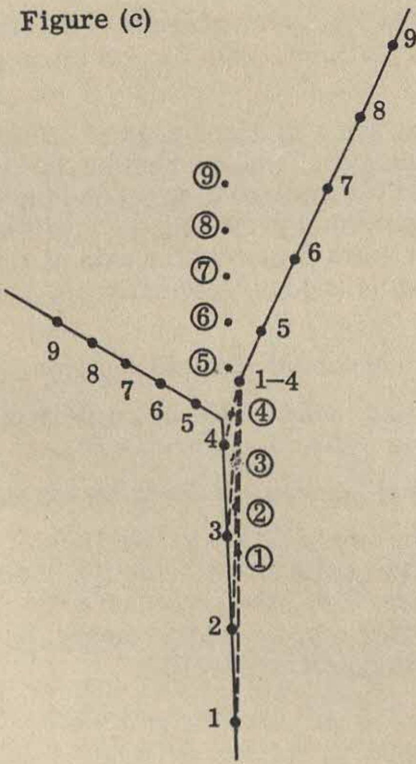
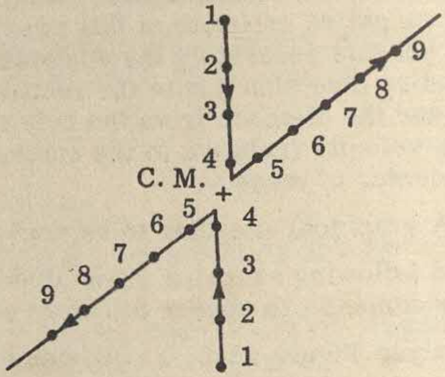


Figure (d)



An extension of this exercise might be to suppose we were riding on the center of mass, and see what the collision would look like. Take a second sheet of tracing paper and mark a spot, C.M., for the center of mass. Place it over the center of mass in the previous tracing and mark the position of the balls at this time. Do the same for each successive position of the center of mass. A final picture will look like (d). The things to notice are that the balls seem to come in from opposite directions with equal velocities (actually equal momenta, but here we are dealing with equal masses) and to go out in a different direction with the opposite velocities. Actually the velocities as they leave are less than they were when they came in since the kinetic energy was not conserved.

d. Additional exercise similar to c.

Some teachers have found it useful to do the exercise described in c above in class, and to give the students the problem of analyzing Figure 23-11 by the same technique at home. One thing that emerges immediately upon trying to begin such an analysis of Figure 23-11 is that the center of mass location is not so obvious because the mass of each ball is not known. However, if you tell the students to assume that the center of mass moves with uniform velocity, it is quite easy for them to determine that the larger ball has more than twice the mass of the smaller ball, but less than three times the mass. This can be judged most easily by using a trial ratio at the two extremes and seeing whether this ratio also holds when the balls almost touch. The best value is about 2.7, but a value of 2.5 will give good results and is easier to use. Once the relative masses are known as 2.5 to 1 or 5 to 2, the students should mark the center of mass near the large ball about 2/7 of the total separation distance. They can then proceed to find the motion relative to the center of mass by using a second tracing as in Exercise c.

Section 7 — The Conservation of Momentum in General

PURPOSE To extend momentum conservation to systems composed of many objects.

CONTENT Momentum is conserved in any isolated system independent of the number of objects in the system.

EMPHASIS It is important for students to know that momentum conservation is general. However, most of them will probably be convinced from the earlier material. Detailed understanding of the argument is not essential. This section can be handled as a reading assignment with a brief discussion of the conclusions if you choose.

COMMENT Since momentum conservation for a two-body system is introduced as a fact based on experiment, there is only modest additional gain in extending momentum conservation to include any isolated system. Inasmuch as momentum conservation depends on experiment it cannot really be proved. However, once it has been established for a two-body system, we need only a principle of additivity to extend it to an n-body system. Since the parameters involved, force and momentum, are vector quantities which have been established to be additive, we can conclude that the generalization will be valid. Even if this plausibility argument does not satisfy every student, you should be sure that they recognize that momentum conservation has been established experimentally for all types of systems.

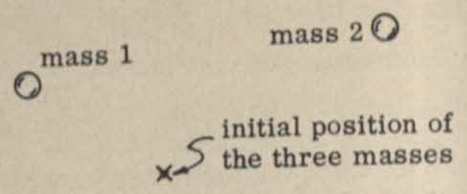
DEVELOPMENT One example, quickly worked out in class, should suffice. Here is one of many possibilities:

> A billiard ball, moving directly east at 2 m/sec, hits two other identical balls. After the collision it is found that the original ball is moving north at 1/2 m/sec, and one of the struck balls is moving east at 1m/sec. How fast and in what direction is the third ball moving?

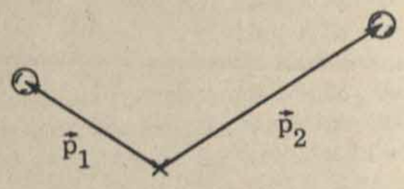
It will be quite sufficient, even desirable, to do this problem graphically. The final momentum is composed of three momenta and must equal the initial momentum. (Note that this collision is not elastic, because the final kinetic energy is less than the initial kinetic energy.)

QUIZ QUESTION A slightly simpler problem may be given as a quiz:

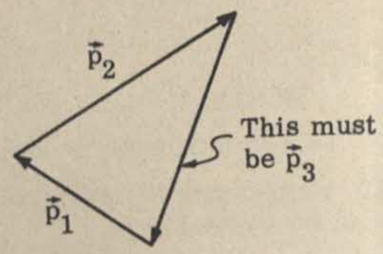
1. Three identical masses are close together and at rest on a frictionless table when an explosion drives them apart. A photograph a moment later shows two masses. Where is the third?



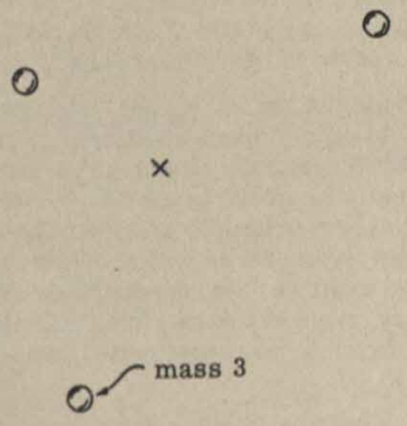
Since the distance traveled is proportional to the velocity and the masses are the same, the momentum vectors are just:



The initial momentum was zero. Therefore, the final momentum must be zero.



The final positions are then:



Section 8 – Forces of Interaction

PURPOSE To introduce the concept of action and reaction forces.

CONTENT If object A exerts a force on object B, object B exerts an oppositely directed force of equal magnitude on A.

EMPHASIS If Experiment III-10, A Collision in Two Dimensions, has not been done earlier it is a good laboratory experience to lead students into the material of Chapter 24.

DEVELOPMENT Parts (c) and (d) of HDL Problem 13 will provide a good introduction to the subject matter of this section.

COMMENT As was mentioned earlier in the Guide on momentum conservation, the traditional approach is to take "equal and opposite" forces as an experimental law and to prove momentum conservation from it. This course takes the approach of finding momentum conservation as the experimental law and proving Newton's third law from it. Hence, the material of this section plays a subsidiary role in this course. Nevertheless the section should be discussed since the ideas will be used again, for instance, in the next chapter on energy. Extensive practice should not be necessary, and there are no HDL problems specifically on this subject.

* * *

If your students have come to consider momentum conservation only in cases where the force acts for a very short time, there may be some misunderstanding about the equal and opposite forces of this section. They should now understand that when two bodies interact, $\Delta p_1 = -\Delta p_2$. Then if $\Delta p_1 = F_{2-1}\Delta t$, and $\Delta p_2 = F_{1-2}\Delta t$, it follows that $F_{2-1} = -F_{1-2}$. Students who think only of the complete change can only see that the average value of F_{2-1} equals minus the average value of F_{1-2}. In order to appreciate this section, they must realize that $\Delta p_1 = -\Delta p_2$, for any interval of time they may wish to consider. Then if the particular Δt is chosen small enough so that the forces do not change appreciably during the time interval, the two forces are equal and opposite at that arbitrary time. They are therefore equal and opposite at all times. The Δt used in this proof is not ''the time taken by a collision or explosion''. It is any small interval of time we choose.

* * *

The words ''equal and opposite forces'' may seem contradictory – in fact, they should seem contradictory if students have become accustomed to proper vector notation. Two vectors are equal only if they have the same direction. The words just occur so often in physics that ''equal in magnitude and opposite in direction'' has been contracted to ''equal and opposite''.

* * *

It might be helpful, when discussing the isolation of a system, to introduce the following well-known paradox if you have not used it before:

> A horse is harnessed to a wagon and tries to pull it down the street. However, as hard as he pulls on the wagon, the wagon pulls back just as hard on him. Then how can he ever get going?

The paradox vanishes as soon as you remember that you must consider only the system in which you are interested. If you wish to understand the motion of the wagon, consider the wagon and the external forces which act on it. The horse pulls it forward and the friction between wheels and axles and between wheels and ground holds it back. When the horse's pull exceeds the force of friction, the wagon accelerates forward.

If you are interested in the motion of the horse, consider only the forces acting on the horse. The external forces which act are the backward pull of the wagon, and the forward push of the ground on the horse's feet (in reaction to their backward push on the ground). When the latter exceeds the former, the horse accelerates forward.

The horse and wagon problem is just a special case of a more general statement about equal and opposite forces: The two equal and opposite forces mentioned in this section always act on different bodies. If you consider both bodies together as a single system, the forces do cancel. It is only this exact cancellation which allows us to ignore the tremendously complex interatomic forces in, say, a stone falling in a gravitational field. In the case of horse and wagon considered together as an object, the pull of the horse on the wagon and the wagon on the horse cancel, leaving only the push of the ground on the horse's feet (forward) and the frictional forces in the wheels (backward). When the one exceeds the other, the horse and wagon (one body) accelerate forward.

* * *

Do not bother to try to give a rigorous definition that will distinguish Newtonian from non-Newtonian forces. As physicists analyze matter and interactions in more detail, a time lag is found to exist in the action of any type of force Most physicists do not believe that any signal can travel faster than light; hence many of the familiar force fields propagate very much like the light emitted by a super nova as discussed in the text.

* * *

It is not necessary to analyze the super nova example in any detail. If you attempt to be precise, you may find some students asking questions which are almost semantic:

> Does the light emitted by a super nova, comprise a material part of the super nova? If not, how can the center of mass of the super nova (at rest before the light was emitted) be moving after the light is emitted? Is something wrong with momentum conservation?
>
> If the light is part of the super nova, why is it so strange that we give the super nova a counter-shove?
>
> What is the nature of the force with which the super nova shoves the light? Since light must always travel with the speed of light, how can you speak of the light as being shoved?

You can probably never satisfactorily answer any of these questions since the students simply will not know enough electromagnetic theory (nor can you promise them they will learn enough in Part IV). The whole situation straightens itself out, as mentioned in the text, when the momentum of the light is taken into account. A light beam of energy E, carries a momentum E/c, where c is the speed of light. Some of the students may ask whether light, if it has momentum, has mass. The answer is yes, it has a mass given by $E = mc^2$. More "light" will be shed on this in Part IV.

Chapter 23 — Momentum and the Conservation of Momentum

For Home, Desk and Lab — Answers to Problems

Even if you do not emphasize the content of Sections 5 through 8 in your classwork, the problems for these sections may be assigned. The techniques developed in Sections 1 through 4 plus the vocabulary of later sections are sufficient.

Problems 11 and 12 constitute a pair which illustrates that, in some cases, either conservation of momentum or F = ma may be equally well applied.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Section	Easy	Medium	Hard	Class Discussion
1, 2	1, 2, 3, 5	4*, 6*, 7	8*	6*
3, 4	10, 11*, 12, 13, 14, 18*	9*, 15* 17, 19	16	10, 13, 16
5		20*		20
6	21, 23*		22*	22
7		24*	25, 26*	24*, 25, 26*

SHORT ANSWERS

1. 18.0 newton-sec.

2. 32.0 kg-m/sec.

3. (a) 0.333 m/sec.
   (b) 0.667 m/sec.

4. 8.0 newton-sec.

5. (a) 750 newton-sec.
   (b) 500 newtons.

6. (a) -120 kg-m/sec.
   (b) 30 newtons, in a direction opposite to the initial velocity.
   (c) 100 kg-m/sec before;
   -20 kg-m/sec after.

7. (a) 1.5 kg-m/sec.
   (b) Zero.
   (c) 1.5 newton-sec, down;
   0.31 sec.
   (d) a: double, b: zero, c: impulse double, time same.

8. (a) 0.40 kg-m/sec.
   (b) 9.6 kg-m/sec.
   (c) 19.2 m/sec.

9. See discussion on page 23-23.

10. Throw something; fan or blow air; etc.

11. (a) No change.
    (b) 6.0 m/sec.
    (c) -6.0 m/sec or 18 m/sec.

12. (a) 100 newtons.
    (b) 3.0 m/sec, forward.
    (c) 1.7 m/sec, in the direction of cart.

13. (a) $v_a/v_b = 0.52$.
    (b) $m_a/m_b = 1/0.52$.
    (c) Equal but opposite.
    (d) $\vec{a}_a/\vec{a}_b = 0.52$.

14. -1 m/sec.

15. (a) 6.68×10^{-27} kg.
    (b) Force cannot be computed.
    (c) See discussion on page 23-28.

16. (a) See diagrams on page 23-29.
    (b) Nearly opposite.
    (c) No.
    (d) $\dfrac{\Delta v_{big}}{\Delta v_{small}} = \dfrac{m_{small}}{m_{big}}$.
    (e) 85 ± 5 gm.

17. See discussion on page 23-31.

18. 0.60 m/sec.

19. 2.0×10^6 m/sec. in the direction of the proton. For the effect of the X ray, see discussion on page 23-32.

20. (a) $v_{truck} = 2$ m/sec,
    $mv_{truck} = 9$ ton-m/sec.
    (b) 5/8 ton.
    (c) 5/9 ton.

21. Remains constant (fixed or moves uniformly).

22. (a) 5,700 m/sec, 47° from vertical.
    (b) 3000 m/sec.
    (c) See discussion on page 23-35.

23. (a) 0.20 m/sec.
    (b) 0.20 m/sec.

24. (a) 6000 kg-m/sec.
    (b) 6000 kg-m/sec in opposite direction.
    (c) 600 newtons.
    (d) See graphs on page 23-36.
    (e) The areas are equal.

25. (a) See diagram on page 23-36.
    (b) 1/2 kg.

26. (a) 3.0 m/sec.
    (b) Remains fixed at center of frame.
    (c) Zero.
    (d) See detailed solution on page 23-37.
    (e) 0.15 L, where L is the length of the frame.

COMMENTS AND SOLUTIONS

PROBLEM 1

How great is the impulse exerted by a 3.00-newton force for 6.00 sec?

Impulse = force × time = 3.00 newtons × 6.00 sec = $\underline{\text{18.0 newton-sec}}$.

PROBLEM 2

How great is the impulse that gives an 8.00-kg mass a change in velocity of 4.00 m/sec?

Impulse = change in momentum = mass × change in velocity = 8.00 kg × 4.00 m/sec = $\underline{\text{32.0 kg - m/sec}}$.

PROBLEM 3

What happens to the velocity of an object when an impulse of 2.00 newton-sec is applied to it? Suppose this impulse is applied
(a) to a 6.00-kg object
(b) to a 3.00-kg object.

When an impulse of 2.00 newton-sec is applied to an object, the momentum of that object changes by 2.00 newton-sec or 2.00 kg-meter/sec.

a) Applying an impulse of 2.00 newton-sec to a 6.00-kg object changes the momentum by 2.00 newton-sec. Since the mass is constant, the velocity changes:

$$m\Delta\vec{v} = \vec{F}\Delta t, \text{ and } \Delta\vec{v} = \frac{\vec{F}\Delta t}{m} = \frac{2.00 \text{ kg-m/sec}}{6.00 \text{ kg}} = \underline{0.333 \text{ m/sec}}.$$

Students should recognize that the unit kilogram-meter per second is the same as a newton-second.

If the body were initially at rest, the 2.00 newton-second impulse would give it a velocity of 0.333 m/sec. If the body were already in motion its velocity (a vector) would be altered by 0.333 m/sec, but the exact nature of the change in speed (magnitude of velocity vector) would depend on the relation between the direction of the original motion and the direction of the impulse.

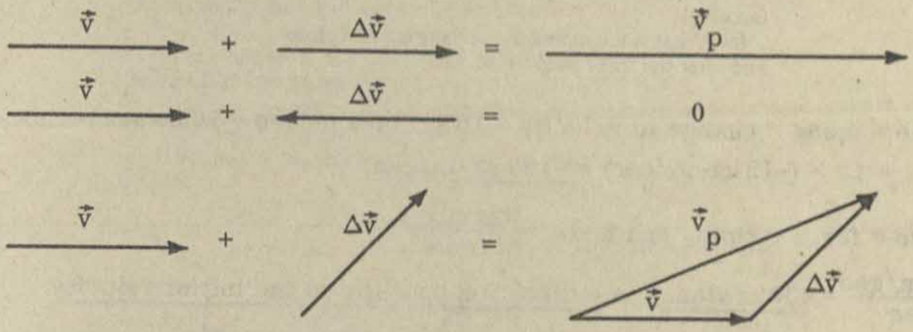
b) If the mass were 3.00 kg, the mass would 1/2 of the mass in part(a). Consequently the same 2.00 newton-sec impulse, imparting the same change in momentum, would result in a change of velocity twice as great as in part (a).

$$\Delta\vec{v} = \frac{\vec{F}\Delta t}{m} = \frac{2.00 \text{ kg-m/sec}}{3.00 \text{ kg}} = \underline{0.667 \text{ m/sec}}.$$

* * *

Note: While this is a simple problem, because of its simplicity it may be worth exploiting to show the generality of the relation between $\vec{F}\Delta t$ and $m\Delta\vec{v}$ when the vector nature of the relation is considered.

If the object has an initial velocity $\vec{v}$, $\Delta\vec{v}$ must be added vectorially:



PROBLEM 4

A constant force applied to a 2.0-kg object at rest moves it 4.0 m in 2.0 sec. What impulse was applied to the object?

There are two approaches to this problem. The first involves finding the force and multiplying it by the time to get the impulse; the second involves finding the change in momentum directly and equating this to the impulse.

Since the force is constant, the acceleration is constant and can be found from $d = \frac{1}{2}at^2$.

$$a = \frac{2d}{t^2} = \frac{2 \times 4.0 \text{ m}}{2.0^2 \text{ sec}^2} = 2.0 \text{ m/sec}^2.$$

Then since $F = ma$, $F = 2.0 \text{ kg} \times 2.0 \text{ m/sec}^2 = 4.0$ newtons. The impulse is then

$$F\Delta t = 4.0 \text{ newtons} \times 2.0 \text{ seconds} = \underline{8.0 \text{ newton-sec}}.$$

In the second approach we note that the object's average velocity during the period is 4.0 meters ÷ 2.0 seconds or 2.0 meters per sec. Its final velocity must be twice this, or 4.0 meters per sec. [Alternatively we could write $v = at$, $d = \frac{1}{2}at^2$, and eliminate a, obtaining $v = \frac{2d}{t}$ as above.] If the object has a velocity of 4.0 m/sec and a mass of 2.0 kg, its momentum change was 8.0 kg-m/sec. The impulse therefore must have been 8.0 kg-m/sec or newton-seconds as before.

PROBLEM 5

A skier with a mass of 75 kg is moving on level ground at a constant speed of 10 m/sec. Through some miscalculation, he finds himself brought to a stop in a snowbank during a Δt of 1.5 sec.

(a) What impulse did the snow apply to the skier?

(b) What was the average force exerted by the snowbank to produce this change of speed?

a) Since the skier had a momentum of 75 kg × 10 m/sec = 750 kg-m/sec, and was completely stopped by the snow, his change of momentum was 750 kg-m/sec. Therefore the impulse applied by the snow was 750 newton-seconds.

b) $F\Delta t$ = impulse. Then 1.5 sec × F = 750 newton-sec, and the average force was 500 newtons. The force may have been much larger at times and smaller at other times, but the average was 500 newtons.

PROBLEM 6

An object with a mass of 10 kg moves at a constant speed of 10 m/sec. A constant force then acts on the object for 4.0 sec, giving it a speed of 2.0 m/sec in the opposite direction.

(a) Calculate the impulse acting on the object.

(b) What is the magnitude and direction of the force?

(c) What is the momentum of the object before and after the force acts?

a) Impulse = mass × change in velocity = 10 kg × (-2 m/sec - 10 m/sec)

= 10 × (-12 kg-m/sec) = -120 kg-m/sec.

b) Impulse = force × time, and force = $\frac{\text{impulse}}{\text{time}}$.

$F = \frac{-120 \text{ kg-m/sec}}{4.0 \text{ sec}} = \underline{-30 \text{ newtons}}$, in a direction opposite to the initial velocity.

c) Momentum = mv,

Initial momentum: mv_1 = 10 kg × 10 m/sec = 100 kg-m/sec.

Final momentum: mv_2 = 10 kg × (-2) m/sec = -20 kg-m/sec.

Thus, the change in momentum is equal to the impulse.

PROBLEM 7

A 0.50-kg ball is thrown straight up at 3.0 m/sec.

(a) What is the initial momentum of the ball?

(b) What is the momentum at the top of the rise?

(c) What impulse stopped the ball? For how long did the impulse act?

(d) If the ball had a mass of 1.0 kg, what would be different in (a), (b), and (c)?

This problem demonstrates that the idea of impulse is useful in cases involving forces acting for an appreciable time as well as in its common application to cases where forces act for a very short time.

a) momentum = mv = 0.50 kg × 3.0 m/sec = 1.5 kg-m/sec.

b) Since the ball stops momentarily at the top, its velocity at that point is zero. Consequently its momentum at that point is zero.

c) The initial momentum was 1.5 kg-m/sec upward, therefore the impulse which stopped the ball was 1.5 newton-sec directed down. The force which supplied this impulse was the gravitational force which is 0.50 kg × 9.8 newtons/kg.

$$\Delta t = \frac{m\Delta\vec{v}}{\vec{F}} = \frac{1.5 \text{ kg-m/sec}}{0.50 \text{ kg} \times 9.8 \text{ m/sec}^2} = \underline{0.31 \text{ sec}}.$$

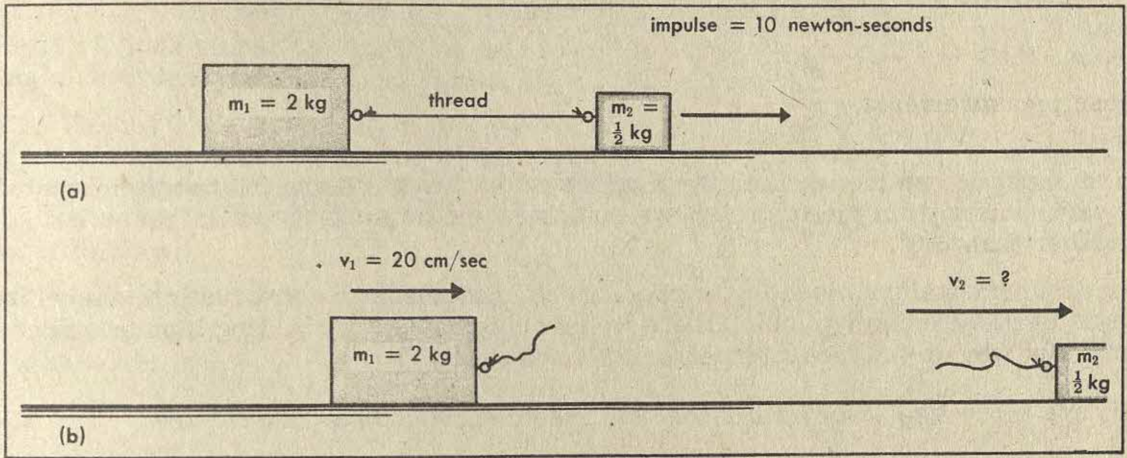
d) Momentum is proportional to both mass and velocity. Hence, in part (a), doubling the mass (but keeping the same velocity) would double the momentum. In part (b), since the velocity is zero, the momentum will be zero no matter what the mass.

In part (c) the impulse required to stop a body is equal to the body's momentum, therefore doubling the mass doubles both the momentum and the impulse required to stop it. However, since F_g = mg, doubling the mass would double the force of gravity, and with doubled force, the same time gives the needed doubled impulse.

PROBLEM 8

A body is made of two masses m_1 and m_2, 2.00 kg and 0.500 kg respectively, tied together by a light thread. [Fig. 23-23 (a).] It moves on a frictionless table starting from rest. We set it in motion by applying an impulse of 10 newton-sec to the body, but unfortunately the thread connecting m_1 and m_2 breaks during the course of the impulse. [Fig. 23-23 (b).] As a result, m_2 goes off with great speed while m_1 at the end is only moving at a speed of 0.20 meters per second.

(a) How big an impulse did m_1 receive?
(b) How big an impulse did m_2 receive?
(c) How fast is m_2 moving at the end of the impulse?



The final momentum of m_1 is 2.00 kg × 0.20 m/sec = 0.40 kg-m/sec. The total momentum of the two bodies is 10 kg-m/sec since that was the total impulse applied. The final momentum of m_2 is then 10 - 0.40 = 9.6 kg-m/sec.

Therefore,

a) m_1 received an impulse of $\underline{0.40 \text{ kg-m/sec}}$.

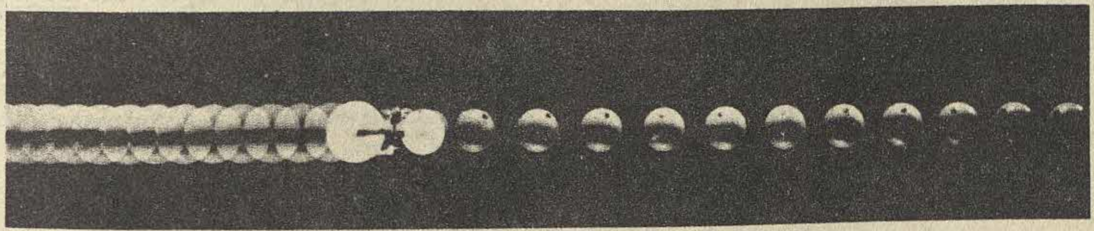
b) m_2 received an impulse of $\underline{9.6 \text{ kg-m/sec}}$.

c) $m_2 v_2 = 9.6$ kg-m/sec.

$$v_2 = \frac{9.6 \text{ kg-m/sec}}{0.500 \text{ kg}} = \underline{19.2 \text{ m/sec}}.$$

PROBLEM 9

Fig. 23-24 shows a multiple-flash photograph of an "explosion" of two billiard balls. The mass of the larger ball is 201 gm and that of the smaller ball 85 gm. The light flashes were $\frac{1}{30}$ sec apart. Your measurements on the photograph should give you a simple check on the conservation of momentum. If you find that the momenta of the two balls are not equal in magnitude, how would you explain this?



Since the balls started from rest (zero momentum) the total momentum after the explosion should be zero. This can be true only if the two individual momenta are equal in magnitude and opposite in direction. The ratio of the two momenta is

$$\frac{m_1 v_1}{m_2 v_2} = \frac{201\ v_1}{85\ v_2} = \frac{201\ d_1 t_2}{85\ d_2 t_1},$$

where d_1 is the distance traveled by the larger ball in a time t_1, and d_2 is the distance traveled by the smaller ball in a time t_2. Measured directly from the figure,

$$d_1 = .79'',\quad t_1 = 5 \text{ flashes},$$

$$d_2 = 1.69'',\quad t_2 = 5 \text{ flashes}.$$

Using these measurements, $\frac{m_1 v_1}{m_2 v_2} = 1.1.$

We also must determine whether the motions of the two balls are in opposite directions. Direct measurement from the figure shows that the angle between the balls' paths is 179.5°, rather than 180°.

The measurements are close to the predictions of momentum conservation, therefore, on the basis of this experiment only, there would be no basis for doubting momentum conservation. But why do the measurements not check exactly?

There are three likely sources of the observed discrepancies. These are:

a) Some external force acts on one or both of the balls after the explosion. Certainly this is so. It is easy to see that the balls are slowing down. This could be the result of air drag. If the balls were hanging from strings, the backward pull of the strings would constitute such a force. If the balls were rolling, frictional forces could be responsible for part of the discrepancy. If the table were slightly tilted the force of gravity could give rise to the other observed effects.

b) A third object might have shared in the "explosion" interaction. A small explosion mechanism is seen initially between the two balls. We know neither its mass nor how much momentum it acquired. This may have been appreciable and could account for the observed discrepancy.

c) There could be distortion in the photograph. Ordinarily this would not be large enough to produce the discrepancy observed here.

PROBLEM 10 A man is at rest in the middle of a pond on perfectly frictionless ice. How can he get himself to shore?

This standard problem can serve nicely as the basis for reviewing isolated systems as well as conservation of momentum.

If we suppose for the moment that there is no air, then there is no way for the whole man to get to shore. He can move in one direction only if he throws something in the other. Fortunately it can be very small, since he will move at a constant speed once he gets going. As long as we are supposing, if there is air friction he can use his hands as "propellers". With a still wilder supposition of no air friction but still air to breathe, he can make a jet with his breath – and turn his head to breathe in.

PROBLEM 11

A 20-kg cart is moving with a speed of 2.0 m/sec. A boy whose mass is 60 kg jumps off the cart. When he hits the ground, he is

(a) moving at the same velocity as the cart
(b) not moving relative to the ground
(c) moving with twice the initial velocity of the cart.

In each case, what is the change in velocity of the cart?

Initially cart (20 kg) and boy (60 kg) are moving with a velocity of 2.0 meters/sec. Hence the total momentum of the system is (20 + 60 kg) × 2.0 m/sec = 160 kg-m/sec.

a) In this case the boy has a final momentum of 60 kg × 2.0 m/sec = 120 kg-m/sec which leaves 160 - 120 = 40 kg-m/sec for the cart. Since it has a mass of 20 kg it has a velocity of 40 ÷ 20 = 2.0 m/sec. There is no change in the velocity of the cart. This is almost trivially obvious since, to "land" at the same speed as the cart, the boy would have to drop off the side of the cart, imparting no impulse to the cart.

b) If the boy ends up with no velocity and hence no momentum, the cart must have all the original 160 kg-m/sec. It then is going with a velocity = 160/20 = 8.0 meters/sec. Therefore the change is 6.0 m/sec. Physically this comes about because the boy must jump backward off the cart so as to arrive on the ground with no momentum. In doing this he pushes the cart forward.

c) If the boy moves with twice the initial velocity of the cart he is moving with a velocity of 4.0 meters/sec. His momentum is then 60 × 4.0 = 240 kg-m/sec. If the final momentum of the cart is p_f

$$p_f + 240 = 160; \quad p_f = -80 \text{ kg-m/sec.}$$

The cart has a velocity of -80/20 = -4.0 or 4.0 meters/sec in the direction opposite to that in which it was originally moving. The change is therefore -6.0 m/sec.

The above holds if we assume that "twice the initial velocity of the cart" means twice the magnitude in the same direction. This is logical. However, some students may argue that it could have been twice the magnitude in the opposite direction. In that case

$$p_f - 240 = 160; \quad p_f = 400, \text{ and } v_c = \frac{400}{20} = 20 \text{ m/sec.}$$

The change in the cart's velocity would then be 20 - 2 = 18 m/sec.

PROBLEM 12

The same boy gets on the same cart as in Problem 11 and moves along at the same initial speed. Then he jumps off the cart again, applying a steady force to the cart for 0.20 sec and giving the cart a forward impulse of 20 newton-sec.

(a) What force did he apply?
(b) What was the final velocity of the cart?
(c) With what horizontal velocity did he hit the ground?

a) Since the impulse is the product of the force and the time,

$$F = \frac{\text{Impulse}}{t} = \frac{20 \text{ newton-sec}}{.20 \text{ sec}} = \underline{100 \text{ newtons}}.$$

b) The final velocity of the cart is the sum of the initial velocity and the change in velocity imparted by the jump.

$$v_f = v_i + \Delta v.$$

A force of 100 newtons acting for 0.20 sec would result in a change of velocity found from $\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}$. Solving for Δv, and then the final velocity, we find

$$\Delta v = \frac{F\Delta t}{m} = \frac{100 \text{ newtons} \times 0.20 \text{ sec}}{20 \text{ kg}} = 1.0 \text{ m/sec}.$$

$$v_f = v_i + \Delta v = 2.0 \text{ m/sec} + 1.0 \text{ m/sec} = \underline{3.0 \text{ m/sec, forward}}.$$

Alternatively, and more directly, the final velocity can be found from momentum conservation:

$$\vec{p}_i = m\vec{v}_i = 20 \text{ kg} \times 2.0 \text{ m/sec} = 40 \text{ kg-m/sec}.$$

An impulse of 20 newton-sec was applied, giving a final momentum of 40 + 20 = 60 kg-m/sec.

$$\vec{p} = m\vec{v}, \text{ and } \vec{v} = \frac{\vec{p}}{m} = \frac{60 \text{ kg-m/sec}}{20 \text{ kg}} = \underline{3.0 \text{ m/sec}}.$$

c) The momentum of the boy was initially 60 kg × 2.0 m/sec = 120 kg-m/sec. The change in the boy's momentum was -20 newton-sec (negative because the impulse was directed oppositely to his original motion). The boy's final momentum is 120 - 20 = 100 kg-m/sec. Since $\vec{p} = m\vec{v}$,

$$\vec{v} = \frac{\vec{p}}{m} = \frac{100 \text{ kg-m/sec}}{60 \text{ kg}} = \underline{1.7 \text{ m/sec, in the direction of the cart}}.$$

PROBLEM 13

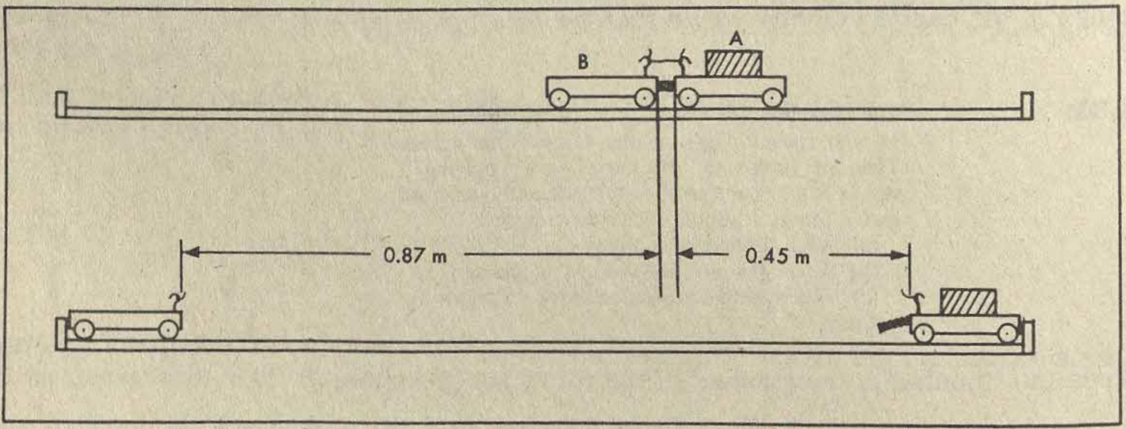
Two heavy frictionless carts are at rest. They are held together by a loop of string. A light spring is compressed between them (Fig. 23-25). When the string is burned, the spring expands from 2.0 cm to 3.0 cm, and the carts move apart. Both hit the bumpers fixed to the table at the same instant, but cart *A* moved 0.45 meters while cart *B* moved 0.87 meters. What is the ratio of:

(a) the speed of *A* to that of *B* after the interaction?

(b) their masses?

(c) the impulses applied to the carts?

(d) the accelerations of the carts while the spring pushes them apart?



This problem deals with momentum conservation, but also can be used to introduce the ideas of Newton's third law, which appears in Section 8 of the text.

a) Cart A moved 0.45 meters during a time t, and Cart B moved 0.87 meters in the same time. Their velocities then were

$$v_a = \frac{0.45}{t}, \text{ and } v_b = \frac{0.87}{t}.$$

Dividing the first equation by the second, the ratio of the velocities is

$$\frac{v_a}{v_b} = \frac{0.45}{0.87} = \underline{0.52}.$$

b) Since the two momenta must be equal in magnitude:

$$m_a v_a = m_b v_b, \text{ or}$$

$$\frac{m_a}{m_b} = \frac{v_b}{v_a} = \underline{\frac{1}{0.52}}.$$

c) Since the momenta were equal and opposite, the impulses were also equal but opposite.

d) Since average $a = \frac{\Delta v}{\Delta t}$, $\frac{a_a}{a_b} = \frac{\Delta v_a/t}{\Delta v_b/t} = \frac{\Delta v_a}{\Delta v_b} = \underline{0.52}$, but opposite in direction.

PROBLEM 14

A freight car of mass 10^4 kg is coasting along a track at 2 m/sec. A second freight car of twice the mass comes toward it in the opposite direction. If both cars come to rest upon collision, how fast was the second car moving?

This is the exact opposite of an "explosion" problem. Here the final momentum is zero. The initial momentum is therefore also zero.

From momentum conservation, the answer to this problem can be seen directly. Since the final momentum is zero, the initial momenta of the cars must have been equal but opposite. Since the ratio of their masses is 1/2, the ratio of their velocities must be 2/1.

$$m_1 v_1 + m_2 v_2 = 0$$

$$m_2 v_2 = -m_1 v_1$$

$$v_2 = \frac{-m_1 v_1}{m_2}$$

$$= \frac{-(10^4 \text{ kg} \times 2 \text{ m/sec})}{2 \times 10^4 \text{ kg}}$$

$$= \underline{-1 \text{ m/sec}}.$$

The minus sign specifies that the direction of travel is opposite to that of the lighter car.

PROBLEM 15

A proton (mass 1.67×10^{-27} kg) with a speed of 1×10^7 m/sec collides with a motionless helium nucleus and the proton bounces back with a speed of 6×10^6 m/sec. The helium nucleus moves forward with a speed of 4×10^6 m/sec after the bombardment.

(a) Can you compute the mass of the helium nucleus? If so, what is it?

(b) Can you compute the force that acted during the collision? If so, what is it?

(c) If you answered "no" to either (a) or (b), be prepared to discuss in class why you gave this answer.

This is a conservation of momentum problem with a nuclear physics flavor.

a) Let m_p be the mass of the proton and v_p its velocity before the collision. Since the helium nucleus is at rest, the total momentum before the collision is $m_p v_p$. After the collision it is $m_{he} v_{he} - m_p v'_p$ where v'_p is the final velocity of the proton, the minus sign occuring because velocity v'_p is opposite in direction to all the other velocities

$$m_p v_p = m_{He} v_{He} - m_p v'_p, \text{ and } m_{He} = \frac{m_p v_p + m_p v'_p}{v_{He}} = \frac{m_p(v_p + v'_p)}{v_{He}}$$

$$m_{He} = \frac{1.67 \times 10^{-27}\ \text{kg}\ (10^7 + 6 \times 10^6\ \text{m/sec})}{4 \times 10^6\ \text{m/sec}}$$

$$= 4 \times 1.67 \times 10^{-27}\ \text{kg} = \underline{7 \times 10^{-27}\ \text{kg}}.$$

b) No.

c) The force during the collision cannot be computed unless the length of time of the collision interaction is known.

Note 1. We can estimate the force as follows: From other types of experiments we know that the forces are exerted over distances of about 2.0×10^{-15} m. If we divide 2.0×10^{-15} m by 0.5×10^7 m/sec we get 4.0×10^{-22} sec for the time duration of the collision. Using this value,

$$F = \frac{mv}{t} = \frac{1.67 \times 10^{-27}\ \text{kg} \times 1 \times 10^7\ \text{m/sec}}{4 \times 10^{-22}\ \text{sec}} \approx 40 \text{ newtons.}$$

This is an order of magnitude calculation. The average velocity might be smaller; the distance might be smaller since the proton and helium nucleus might barely touch. Even if v is 10 times smaller, the force is of the order of 10 newtons which is fantastically large considering that this is the force between two such small particles. However, for this problem the correct answer is that the force cannot be computed.

Note 2. There is some significance to the answer "no" to part (b). It shows the real power of momentum conservation techniques. Here is a case in which no one knows the actual force in question, but momentum conservation is applicable and can give useful information — such as the mass of a struck nucleus.

PROBLEM 16

In Fig. 23-26 the large ball came in at the top of the picture and the little ball at the bottom. As you see, a collision took place in the middle.

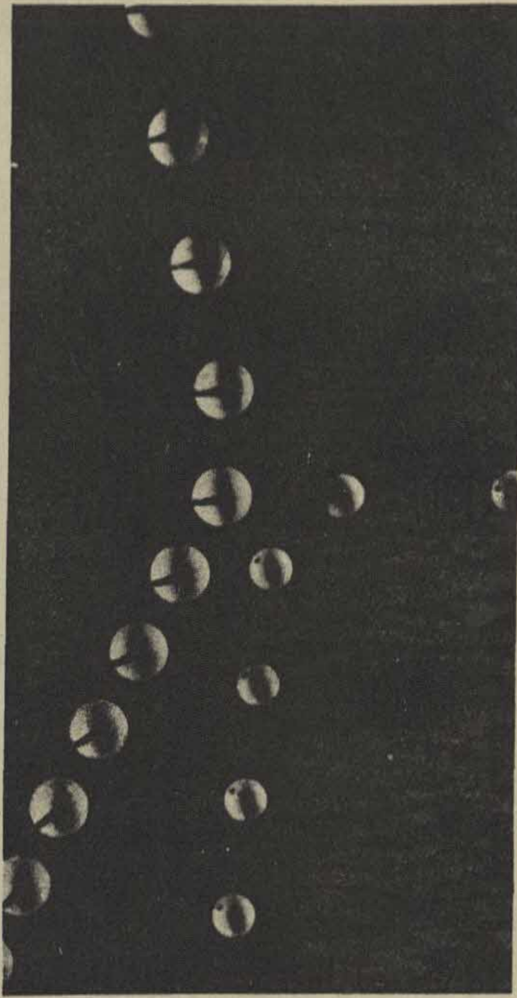
(a) Draw the vectors which represent the change in the velocity of the large ball and the change in velocity of the small ball. Plot these vectors to the same scale and make sure that each one is in the right direction.

(b) Are these changes of velocity opposite in direction?

(c) Are they equal in magnitude?

(d) If their magnitudes differ, what should be their ratio?

(e) The mass of the large ball is 201 gm. What is the mass of the small ball?

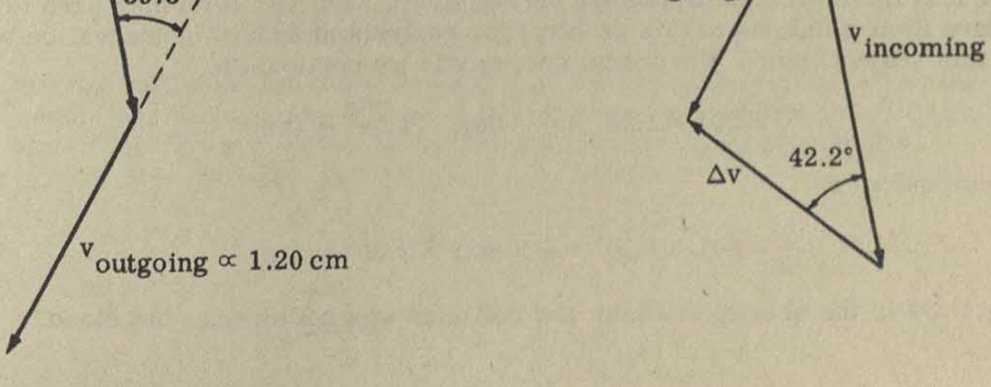


Students should be encouraged to work to 0.1 millimeter and 0.1° accuracy although the distortions in the picture are probably greater than this. Depending on how much practice students have had with this sort of graphical problem it may be well to point out in assigning it that they can plot velocity vectors proportionally even though they do not know either the real distance scale or the time between flashes. The velocity of an object may be plotted as proportional to the distance between successive flashes. Directions of travel can be determined by drawing along one side of the line of images of the ball, using tracing paper if it is not permissible to mark the text. Distances can be determined by using the line of centers, measuring between the more brightly lighted edges.

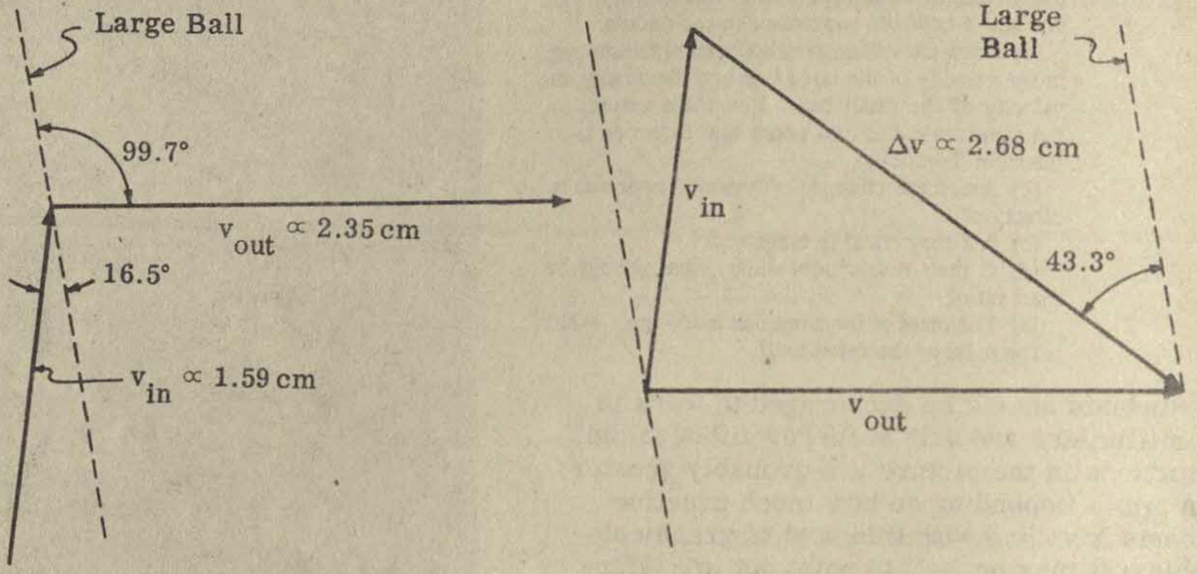
a) The vector diagrams are given below.

The vector equation is $\vec{v}_{in} + \Delta\vec{v} = \vec{v}_{out}$. Therefore $\Delta\vec{v}$ is the vector from the head of $\vec{v}_{in}$ to the head of $\vec{v}_{out}$.

Big Ball


$v_{incoming} \propto 1.76$ cm
39.5°
$v_{outgoing} \propto 1.20$ cm
$\vec{v}_{outgoing}$
$v_{incoming}$
42.2°
Δv

Small Ball



b) The changes of velocity are within 0.5° of opposite directions. Any measurement between 0 and 1.5° might be accepted.

c) No. From the vector diagrams, their ratio is

$$\frac{\Delta v_{big}}{\Delta v_{small}} = \frac{1.13}{2.68} = .422.$$

d) From momentum conservation, the ratio of the changes in velocity should be inversely proportional to the ratio of the masses:

$$m_{big}\Delta v_{big} = m_{small}\Delta v_{small}$$

$$\frac{\Delta v_{big}}{\Delta v_{small}} = \frac{m_{small}}{m_{big}} .$$

e) Since

$$\frac{m_{small}}{m_{big}} = \frac{\Delta v_{big}}{\Delta v_{small}} = .422,$$

$m_{small} = .422 \times 201 = \underline{84.8 \text{ gm}}$, (a value between 80 and 90 is acceptable). Clearly these balls are the 201 and 85 gm pair that has been used in other figures.

* * *

Note: You may be interested in whether energy was conserved. You may wish to ask students to save their solutions to this problem for analysis of energy conservation when you deal with this subject later. The initial energy was proportional to

$$\frac{1}{2} \times 201 \times 1.76^2 + \frac{1}{2} \times 84.8 \times 1.59^2 = 418$$

and the final energy to

$$\frac{1}{2} \times 201 \times 1.20^2 + \frac{1}{2} \times 84.8 \times 2.35^2 = 378.$$

Thus about 9.5% of the energy was lost; the collision was not elastic, but close.

PROBLEM 17

A double star consists of two large masses that attract each other gravitationally. By observing the motion of both masses, we can see that they rotate around each other.

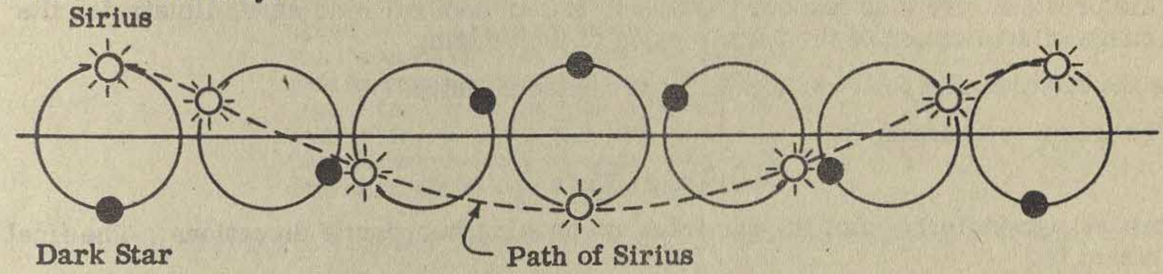
(a) What do you think happens to the momentum of each of the masses in a double star as time goes on? Explain your answer.

(b) When observed carefully, the bright star Sirius seems to wobble about, instead of having a uniform motion of the center of mass. From this and other evidence astronomers believe that Sirius has a dark companion. It is really a double star. How does this explain the peculiar observed motion?

This problem asks the student to discuss conservation of momentum in an astronomical problem.

a) The momentum of one member of a double star does not have to remain constant because there is an external force – the gravitational attraction of the other member – acting on it. (Momentum changes due to emitted light are negligible.) Therefore the momentum of each member changes, but the sum of the two momenta remains constant since we presume the double star system is far enough away from all other stars that it may be considered isolated from them. This means that when one momentum changes, the other must change by an equal amount in the opposite direction. The changes are periodic, i.e., repeat themselves as the two stars revolve about each other.

b) Assume that Sirius and its dark companion have equal masses and are revolving around each other as together they move through space. The point half-way between the two stars would move uniformly:



How this motion would appear to an observer depends upon the magnitude and frequency of revolution of the binary system, and upon the angle from which the motion is viewed. If the observer is in the plane of revolution, one star periodically eclipses the other star. This is called an eclipsing binary. From other angles, the visible star will appear to wobble in its path. The magnitude and frequency of wobble will depend on the stars' relative masses and the shapes of their orbits.

PROBLEM 18

A stationary refrigerator car with mass 2.0×10^4 kg is rammed by a loaded gondola car with mass 3.0×10^4 kg. Before impact, the gondola car was going 1.0 m/sec. If they lock together, what is the new velocity?

Before the collision, the total momentum of the two-car system is just the momentum of the gondola car because only the gondola car is moving. After the collision the total momentum of the two-car system is the same as before. Since the collision couples the two cars, both move with the same velocity after the collision.

$$p_f = p_i$$

$$(m_g + m_r)\, v_f = m_g v_g, \text{ and}$$

$$v_f = \frac{m_g v_g}{m_g + m_r} = \frac{3.0 \times 10^4 \text{ kg} \times 1.0 \text{ m/sec}}{3.0 \times 10^4 \text{ kg} + 2.0 \times 10^4 \text{ kg}} = \underline{0.60 \text{ m/sec}}.$$

Most students have seen the jerky banging together of two freight cars being coupled. It may be well to call attention to the fact that although complicated forces are at work, the over all result is simple to comprehend. If the question is raised in discussion, you might tell your students that it is kinetic energy that is lost in the banging around. However, try to steer clear of energy at this point. It will be the subject of the next three chapters.

PROBLEM 19

When a proton (mass 1.67×10^{-27} kg) collides with a neutron (mass 1.67×10^{-27} kg) the two can stick together forming a deuteron (whose mass is 3.34×10^{-27} kg). (Actually the neutron is slightly more massive than a proton and the deuteron is slightly less massive than the sum of the two. However, these differences are less than 1 part in 400.) With what velocity will the deuteron move if it is formed from a proton moving with a velocity of 7.0×10^6 m/sec to the right and a neutron moving with a velocity of 3.0×10^6 m/sec to the left?

Note: Actually, in this collision an X ray is given off. The momentum carried in the X ray is about 20 per cent of the final momentum of the deuteron. An answer that neglects this is therefore somewhat in error.

This problem involving nuclear particles, rather than railroad cars, illustrates the wide range of application of the conservation of momentum.

In the solution that follows, the X ray is neglected initially.

The initial momentum is

$$p_i = m_p v_p - m_n v_n.$$

The minus sign indicates that the particles are moving in opposite directions. The final momentum is

$$p_f = m_d v_d.$$

Since momentum must be conserved, $p_f = p_i$, and because all the motions are along a line, we can write

$$m_d v_d = m_p v_p - m_n v_n$$

$$v_d = \frac{m_p v_p - m_n v_n}{m_d} = \frac{1.67 \times 10^{-27} \text{ kg} \times (7.0 \times 10^6 \text{ m/sec} - 3.0 \times 10^6 \text{ m/sec})}{3.34 \times 10^{-27} \text{ kg}}$$

$$= \underline{2.0 \times 10^6 \text{ m/sec, in the direction in which the proton was traveling.}}$$

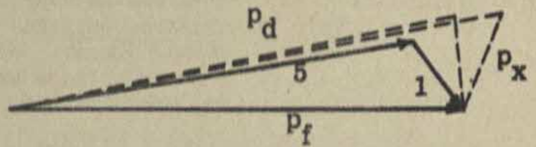
* * *

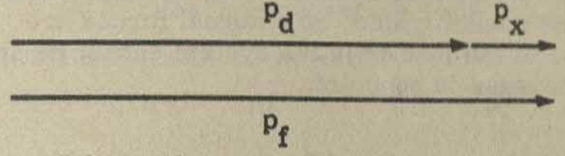
In order to obtain a solution which takes into account the emission of an X ray, it must be remembered that the total momentum of the system is still conserved, i.e., the vector sum of the momenta of the deuteron and the X ray must still be

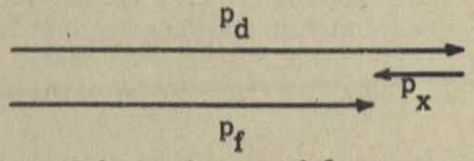
$$m_p v_p - m_n v_n = 1.67 \times 10^{-27} \text{kg} \times (7.0 \times 10^6 \text{m/sec} - 3.0 \times 10^6 \text{m/sec}) = 6.7 \times 10^{21} \text{kg-m/sec}.$$

It is given that $p_x = 0.2\ p_d$, but since the direction in which the X ray will be emitted cannot be predicted, the direction of motion of the deuteron cannot be predicted. Several

possible combinations of p_d and p_x that have amplitude ratios of 5:1 are shown at the right. Two extreme solutions are shown below for which the values of p_d are calculated as follows:







Solution 1: $p_d + 0.2\,p_d = p_f$

$$p_d = 0.83\,p_f$$

Solution 2: $p_d - 0.2\,p_d = p_f$

$$p_d = 1.25\,p_f$$

PROBLEM 20

A fire hose with a nozzle that we can turn on and off squirts bursts of water into a truck from behind. The truck can roll along a level surface with very little friction. The body of the truck is not leaky, and the water stays in it.

(a) Suppose the truck has a mass of 4.5 tons (4.5×10^3 kg) and that it is at rest when the first burst of water goes into it. The water comes squirting in at 20 meters per second horizontal speed, and the first burst is half a ton. What is the momentum of the truck after receiving the first burst? With what speed is it moving?

(b) We now squirt a second burst into the truck. The water moves at the same speed, 20 meters per second, with respect to the ground. How large does the second burst have to be so that the truck gains the same amount of speed the second time?

(c) If you increase the speed with which the water is shot out so that each burst moves with the same relative velocity with respect to the truck, what would this do to your answer to part (b)?

a) For the first burst of water the momentum is (1/2 ton)(20 m/sec) = 10 ton m/sec.

After the water is in the truck, the total mass is 1/2 ton (water) + 4 1/2 tons (truck) = 5.0 tons.

Momentum is conserved, therefore the final velocity of truck and water, v, is

$$10 \text{ ton m/sec} = 5 \text{ tons} \times v,$$

$$v = \underline{2 \text{ m/sec}}.$$

The momentum of the truck, itself, now is (4.5 tons)(2 m/sec) = 9 ton m/sec.

b) For the second burst, the velocity of the water is still 20 m/sec. Let M be the mass of water in the second burst. Then the momentum of the second burst is 20 M m/sec. The final velocity of this burst must be the same as the final velocity of the system, 4 m/sec. Therefore the momentum change suffered by the second burst is simply 4 M - 20 M = -16 M m/sec.

The truck plus the first burst of water had an initial momentum of 10 ton m/sec, and since their velocity is doubled upon receipt of the second burst, their final momentum must be 20 ton meters/sec. Thus their change in momentum is 10 ton meters/sec.

Conservation of momentum tells us that the change in momentum of the second burst plus the change in momentum of the truck plus first burst, must sum to zero.

$$10 \text{ ton m/sec} - 16\,M \text{ m/sec} = 0$$

$$\therefore\ M = \underline{5/8 \text{ ton}}.$$

PROBLEM 21

A rocket is out in free space shooting out a stream of exhaust gases and picking up speed in the opposite direction. What happens to the center of mass of all the matter, that which is ejected and that which is left in the rocket?

A rocket moving in "free space" represents an isolated system. The system consists of the ejected matter and that which is left in the rocket. Since no external forces are acting on the rocket system, the center of mass of the matter in the system cannot be accelerating. Therefore the velocity of the center of mass is constant.

The first burst of water (0.5 ton) gave the truck a velocity of 2 m/sec. As would be expected, more than 0.5 ton of water would be required in a second burst to increase by another 2 m/sec the velocity of the truck plus the original water.

c) If the second burst were to move at 20 meters per second with respect to the truck, it would have to move at 22 m/sec with respect to the ground. In this case, the velocity of the water in the solution for part (b) would be 22 m/sec, and the required mass of water would be 5/9 ton, a greater mass of water than required in part (a) but less than required in part (b).

It may be interesting to consider how fast the truck could ever be pushed using the method of parts (a) and (b). The answer is clearly that it can never move faster than 20 m/sec. Using the method of part (c), however, it could be pushed arbitrarily fast – neglecting the limitations of relativity and, more particularly, available fire hoses!

PROBLEM 22

In an unsuccessful attempt to put a satellite in orbit, the carrier rocket moving vertically upward at 3000 meters/sec explodes and breaks into two pieces. One of the pieces continues upward at an angle of 45° with the vertical, and with a speed of 3500 meters/sec immediately after the explosion.

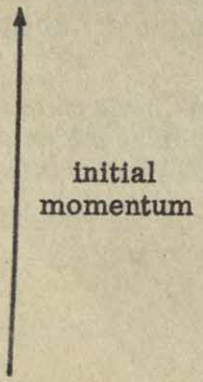
(a) What is the velocity of the second piece if its mass is 0.60 that of the first piece?

(b) What is the velocity of the center of mass of the rocket immediately after the explosion?

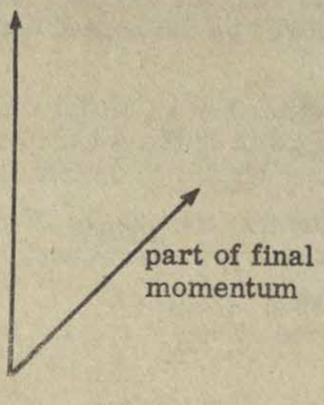
(c) What is the change in center-of-mass velocity in the next second?

a) Let the mass of the first piece of rocket be M, the second piece 0.60M. The original mass was then 1.6M. For a graphical solution of the problem we construct a vector proportional to the original momentum of the complete rocket, i. e., proportional to 3000 m/sec × 1.6M = 4,800M m/sec. We may draw an arrow 4.8 cm vertically up as at (a). After the explosion we know that one piece had a momentum 3,500 m/sec × 1.0M = 3,500M m/sec, 45° up to the right, as at (b). The second piece must have a momentum p, as at (c). Measurement shows the momentum of the second piece to be 3,400M m/sec, at an angle of 47° to the vertical.

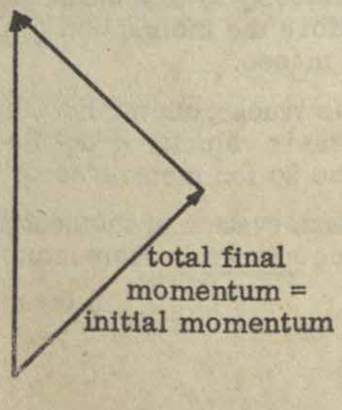
(a)



(b).



(c)



Then, using v for the speed of the second piece, we get 3,400M m/sec = 0.60M × v; and the final velocity of the second piece is 5,700 meters/sec, upward at 47° from vertical, and on the opposite side of the vertical from the first piece.

We can also solve this problem by resolving the momenta into vertical and horizontal components:

initial vertical component = 4,800M m/sec,

initial horizontal component = 0,

final vertical component of first piece = $3500 \left(\frac{1}{\sqrt{2}}\right) \times 1.0M = 2{,}475M$ m/sec,

final horizontal component of first piece = $3500 \left(\frac{1}{\sqrt{2}}\right) \times 1.0M = 2{,}475M$ m/sec.

Consequently from the conservation of momentum we get for the second piece,

$$P_h = -2{,}475M \text{ m/sec},$$

$$P_v = 4800M \text{ m/sec} - 2475M \text{ m/sec} = 2325M \text{ m/sec}.$$

Then $$P = \sqrt{(2475M)^2 + (2325M)^2} = 3{,}295M \text{ m/sec},$$

and $$\tan\theta = \frac{2475}{2325}, \ \theta = 46.8°.$$

The velocity is then $$\frac{3395M \text{ m/sec}}{0.60M} = 5660 \text{ m/sec, } 46.8° \text{ from vertical}.$$

b) The velocity of the center of mass of the rocket was 3000 m/sec just before the explosion and since no external forces acted on the rocket, the velocity of the center of mass after the explosion is still 3000 m/sec.

Since we are concerned only with the velocity of the center of mass immediately after the explosion, the impulse due to gravity acting for the short time of the explosion will be neglibible and the velocity will still be 3000 m/sec.

c) To understand how the motion of the center of mass would change after the explosion, we need only think of how the whole rocket would behave if it did not break into pieces and if its fuel were shut off. If no external force acts, the rocket would continue to move with the same speed and direction. This was seen in the solution to part (b).

PROBLEM 23

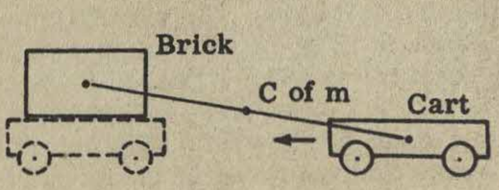
A 2.0-kg brick with no horizontal motion is dropped on a 2.0-kg cart moving across a frictionless table at 0.40 meter/sec.

(a) What is the change in velocity of the cart?

(b) What is the velocity of the center of mass of the system composed of cart and brick before the brick is dropped?

a) The initial speed of the cart is 0.40 m/sec. To conserve momentum, when its mass is doubled, its speed is halved. Therefore its final speed is 0.20 m/sec.

b) The center of mass of the 2-kg brick and 2-kg cart lies on the line joining them and halfway between them. Consequently, the center of mass of cart and brick moves half as fast as the cart does before the "collision". Therefore the center of mass moves at 0.20 m/sec before the collision (and after the collision).



PROBLEM 24

A machine gun fires ten 100-gram bullets per second at a speed of 600 meters/sec.

(a) What impulse does the gun apply to the bullets in ten seconds?

(b) What impulse do the bullets apply to the gun in ten seconds?

(c) What is the average force applied to the gun?

(d) Sketch rough graphs of:
- (i) *Average* force on gun *vs.* time
- (ii) *Actual* force on gun *vs.* time

(e) How are the graphs (i) and (ii) related?

a) Each bullet has a momentum:

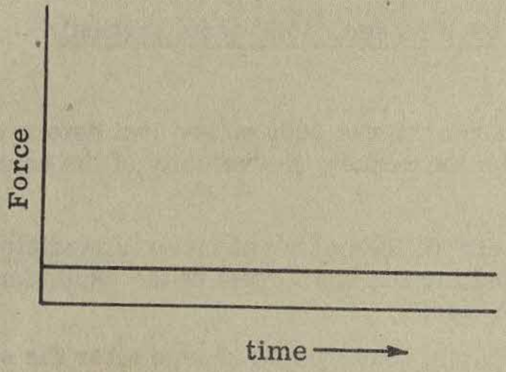
$$0.1\text{ kg} \times 600\text{ m/sec} = 60\text{ kg-m/sec}.$$

In ten seconds 100 bullets are fired so that the total impulse applied by the gun to the bullets is 60 kg-m/sec × 100 = 6,000 kg-m/sec or newton-sec.

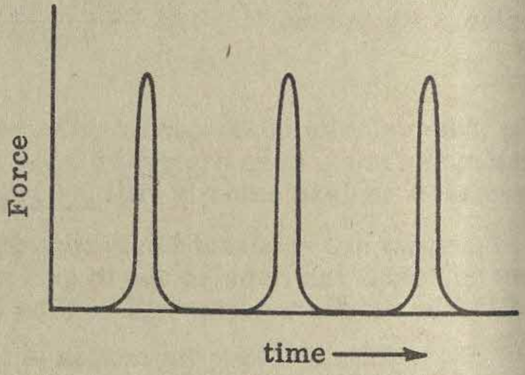
b) Since there is no net impulse on the system of the gun and the bullets, momentum is conserved. Hence the impulse applied by the bullets to the gun is 6,000 newton-sec, in a direction opposite to that of part (a).

c) Neglecting the reaction of the gases (which may be done with a well designed gun), the total impulse applied to the gun in 10 seconds is 6,000 newton-seconds. This is 600 newton-seconds per second, which gives 600 newtons as the average force.

d) (i)



(ii)



A rough graph of the actual force vs. time is shown at (ii). The time duration of the force intervals, as compared to the time between force intervals, should be even shorter than shown; and we know little about the actual shape of the spikes.

e) The areas under the two graphs are equal.

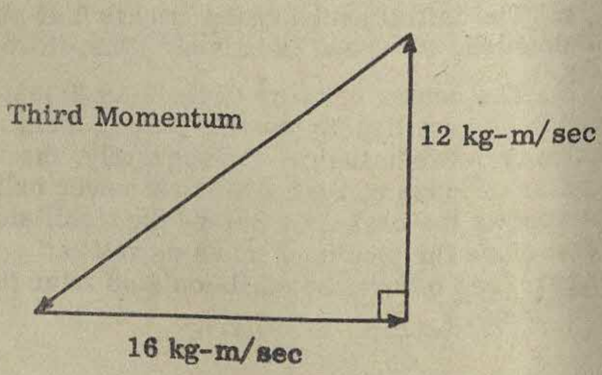
PROBLEM 25

An explosion blows a rock into three parts. Two pieces go off at right angles to each other, a 1.0-kg piece at 12 m/sec and a 2.0-kg piece at 8.0 m/sec. The third piece flies off at 40 m/sec.

(a) Draw a diagram to show the direction in which it goes.

(b) What is its mass?

a) We must presume the rock to be initially at rest. It then had zero momentum before the collision and hence must have zero momentum after the collision. The momentum of the 1.0-kg piece is 12 kg-m/sec; the momentum of the 2.0-kg piece is 16 kg-m/sec at right angles to the first. The third piece must have a momentum $\vec{p}$ given by the vector diagram at the right which shows that the three momenta add to zero.



b) The diagram shows the direction in which the third piece flew off, and from the 3, 4, 5 right triangle it is seen to have a momentum of 20 kg-m/sec. Since its velocity was 40 m/sec, its mass was 1/2 kg.

PROBLEM 26

The system shown in Fig. 23-27 consists of a 5.0-kg frame with a 1.0-kg and a 4.0-kg mass in the middle. At either end of the frame are stops made of putty. The two masses and the frame are mounted on Dry Ice discs. (The mass of the Dry Ice is included in the values given above.) The center of mass of the whole system is at the center of the frame. An explosion pushes the two masses apart. The 1.0-kg mass goes off with a speed of 12 m/sec. Eventually each mass is trapped by the putty.

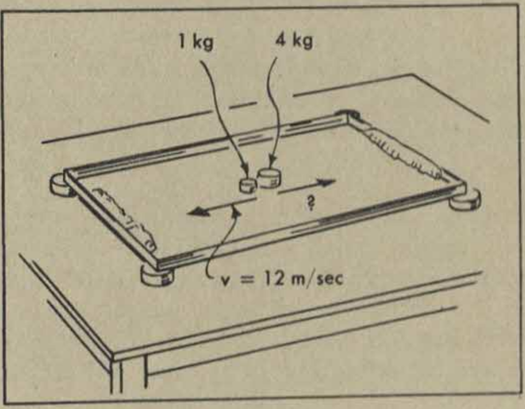
(a) What is the speed of the 4.0-kg mass immediately after the explosion?

(b) What is the center of mass of the whole system after 1 sec? After 2 sec?

(c) What is the velocity of the frame and the attached masses after 100 sec?

(d) Describe the motion qualitatively from the time of explosion to 100 sec.

(e) How far does the frame move?



a) Momentum must be conserved in the explosion between the 1.0 and 4.0-kilogram masses. The 1.0-kg mass has a momentum of 12 kg-m/sec after the explosion. The 4.0-kg mass must have an equal and opposite momentum and therefore a velocity of 3.0 meters/sec.

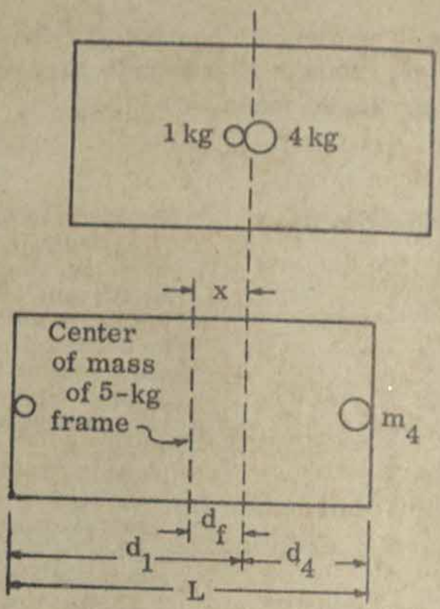
b) Since there are no external forces acting on the system, the center of mass of the whole system remains fixed at all times.

c) Since there are no external forces acting, and since before the explosion the momentum of the whole system was zero, the momentum remains zero. When the frame and two masses are joined they all have the same velocity – zero. (With reasonable dimensions the masses have become attached to the frame long before 100 sec has passed.)

d) With the explosion, the 1.0 and 4.0-kilogram masses fly apart with speeds of 12 and 3 meters/sec respectively. The 1.0-kg mass hits first and sticks, moving the frame in its direction. The momentum of frame and 1.0-kg mass is 12 kg-m/sec, the momentum of the mass before the collision. Next the 4.0-kg mass hits and transfers 12 kg-m/sec in the opposite direction, thus stopping the frame and masses, which then remain at rest.

e) The distance the frame moved can be calculated by doing part (d) quantitatively. However, the simplest method is to remember that the center of mass does not move.

Suppose the frame moved a distance x. Then it acts like a 5.0-kg mass a distance x from the old (and the new) center of mass. Where L is the length of the frame, the 1.0-kg mass is (L/2 + x) from the center of mass, and the 4.0-kg mass is (L/2 - x) from the center of mass.



$$d_1m_1 + d_fm_f = d_4m_4$$

$$\left(\frac{L}{2} + x\right) m_1 + xm_f = \left(\frac{L}{2} - x\right) m_4$$

$$\frac{Lm_1}{2} + xm_1 + xm_f = \frac{Lm_4}{2} - xm_4$$

$$x(m_1 + m_4 + m_f) = \frac{L(m_4 - m_1)}{2}$$

$$x(1.0 + 4.0 + 5.0) = \frac{L(4.0 - 1.0)}{2}$$

$$x = \underline{0.15\,L}.$$

Many students who are unable to solve this part of the problem using L will be able to do so if they are given a specific value for L.

Chapter 24 – Work and Kinetic Energy

Chapter 24 introduces energy, one of the most useful concepts in physics. The idea of energy will be extended in the next two chapters, and is used extensively throughout the rest of the course. Application of the law of conservation of energy is a powerful tool for solving a wide range of problems, particularly those in which the forces of interaction are not known in detail (e.g., in collisions).

For an over all commentary on the three energy chapters you may want to refer to the summary of Part III at the front of this volume of the Guide.

CHAPTER SUMMARY

Work is introduced as a measurable form of energy transfer, starting from ideas about fuel consumption. Work which goes into the acceleration of a body produces a corresponding change in the quantity $\frac{1}{2}mv^2$, the kinetic energy. The elastic collision is analyzed in detail.

Sections 1 through 3 From common intuitive feelings about "energy", "job", "work", and "fuel", a definition of the scientific concept of work is developed. Work is equal to the product of a force and the displacement produced in the direction of the force.

Section 4 The work done in accelerating a body produces a change in the quantity $1/2\ mv^2$, the kinetic energy. The same amount of work always produces the same change in kinetic energy. An object in motion can do an amount of work exactly equal to its initial $1/2\ mv^2$.

Sections 5 through 7 The changes in kinetic energy in two-body collisions are analyzed and stress is given to elastic collisions (i.e., collisions in which kinetic energy is conserved). Finally, a collision is proved to be "elastic" whenever the force between two bodies is a function only of their distance of separation.

Sections 8 through 11. The general consequences of the conservation of momentum and kinetic energy are applied to elastic interactions to find solutions to a certain class of problems. Some of the characteristics of inelastic collisions are also discussed.

SCHEDULING CHAPTER 24

Chapter 24 contains a great deal of new and important material which will require considerable class time for a full development. Time should be spent on the solution of problems involving work and kinetic energy in order to drive home the quantitative use of these concepts in physical situations. The Home, Desk and Lab section contains a large number of problems which are suitable for stimulating class discussion. A few supplementary problems useful for quizzes or class discussion are contained in the following pages.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part III. Sections which are enclosed by brackets [ ] can be deemphasized in class discussion without loss of continuity.

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2, 3	3	0	-	2	0	-
Sec. 4	1	0	-	1	0	-
Secs. 5, [6], [7]	3	1	III-11	1	1	III-11
Secs. 8, [9], 10, 11	2	0	-	2	0	-

RELATED MATERIALS FOR CHAPTER 24

Laboratory. Experiment III-11. Slow Collisions. This experiment employs two massive carts with soft spring "bumpers" in a head-on collision. The "collision" occurs sufficiently slowly that students are able to follow it in detail, making quantitative measurements. This experiment should be integrated directly with the class discussion of Sections 5 through 7.

Films. "Energy and Work", by Professor Dorothy Montgomery of Hollins College demonstrates that the work that goes into raising a heavy ball transfers to kinetic energy when it falls. The relationship between work done and change in kinetic energy is further underscored by demonstration with a device in which a varying (but replicable) force acts through a given distance. This is done by finding the work from the area under a force-distance curve and the kinetic energy from measurement of mass and velocity. An amusing "Rube Goldberg" device is used to remind students that mgh expresses gravitational potential energy. In closing, the film leads toward heat energy by showing that, as the falling ball is brought to a stop, the loss of kinetic energy is accompanied by an increase in heat. This film may be used in connection with Chapter 24 or 25. Most teachers prefer to use it about the middle or toward the end of Chapter 25. Running time: 28 minutes.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Section	Easy	Medium	Hard	Class Discussion
1, 2, 3	5*	1, 2*, 3, 4*		1, 2*, 4*, 5*
4	7, 9, 10	6*, 8*, 11*, 12 13, 14, 15*		6*, 8*, 11*
5, 6, 7	20	17*, 18*, 21*, 22	16*, 19	16*, 17, 21*
8, 9, 10, 11	24, 26, 27*, 29	25, 28, 30, 31*, 32	23, 33, 34	23, 24, 25, 28, 31*

Section 1 – Energy Transfer
Section 2 – Work: A Measure of Energy Transfer
Section 3 – More About the Definition of Work

PURPOSE To introduce work as a quantitative measure of energy transfer.

CONTENT a. The quantity of energy transferred can be measured in terms of work performed or fuel used – two identical jobs require twice as much fuel or work as one job.

b. Fuel consumption apparently varies with the magnitude of the force exerted and the distance through which the force acts. Work is defined as the product of force × distance. The joule is defined as the unit of work when the force is expressed in newtons and the distance through which the force acts is expressed in meters.

c. The quantity of work is independent of the time a force acts.

d. A force which does not move an object does no work. Work is done only when a force act parallel to the displacement.

EMPHASIS This is very important material, and will require about two, perhaps three, class periods for development. It will be wise not to try to define energy in terms of work. Let the concept grow during this and the next two chapters. However, this is the time for an exact definition of work, since work is a purely mechanical concept.

DEVELOPMENT Students will generally accept the idea of fuel consumption being a quantitative measure of something, at least the cost of doing a particular job. This is a good idea to stress, since, after all, the law of conservation of energy, which we are developing, is little more than a total cost accounting of the full price (including "hidden" terms) of doing a wide variety of jobs. Cost is an intuitively quantitative concept, and the extrapolation from fuel consumption to the mechanical definition of work can be kept quantitative.

The logic of the initial development of work and energy has been worked out carefully in the text, and it will be wise to follow this logic rather closely. Otherwise it is very easy to become trapped in circular dedinitions of "work" and "energy", apparently using the two terms to describe the same thing. The term "work" should be confined to external forces which change the energy of a system. The concept becomes terribly muddled if work is done by something inside the system, since then it is very difficult to isolate the agent or force doing the work (e.g., when a pendulum swings, energy is continually being converted from potential to kinetic and back – it makes no sense to speak of the pendulum doing work on itself, or on the gravitational field of the earth; the pendulum and earth constitute a system, and the total energy of the system does not change when the pendulum swings).

COMMENTS Some students may be bothered by the idea that no work is done when a force is applied to an object which does not move. A steel girder holds up a building. It exerts a huge force, but moves nowhere. It requires no fuel. It is not doing work. Yet, all of us have at some time or other held a heavy load overhead, waiting for someone to lift it from us. It is hard to hold an appreciable load up there for more than a short time. It makes us tired. It is natural to want to say that we have worked. However, the only work that we have done is work against internal frictions (pumping blood), and generating heat by combustion. The result is that we heat our bodies, but we do no useful work on an outside system.

We frequently find that our technical scientific vocabulary overlaps our less-precise everyday use of language. Watch out for "work". We have now given it a precise definition. Students simply need to realize that "work" in physics does not cover many of the ideas for which it is used colloquially. Solving a problem is "hard work", but the energy consumed is very small.

Work is only one method of energy transfer. Others will be developed in the following chapters.

* * *

Work is done only by that component of a force which lies along the direction of the displacement. The text develops this idea by pointing out that no fuel is consumed when an object slides at constant speed across a frictionless table. The force exerted by the table in supporting the object is perpendicular to the motion, and does no work. (See Appendix 6 at the back of this volume of the Guide for a discussion of work as the dot product of two vectors.)

The same problem can also be approached from the point of view of the final state of the system. Has the energy content been changed by the action of a force? Consider a dry-ice puck held by a string so that it moves in a circle of constant radius. Again, this is a case of a constant force acting perpendicular to the motion. Suppose a flame burns through the string after 5 revolutions. The puck flies off tangentially with the speed it had in its circular travel. Now repeat the experiment, but this time burn through the string after 10 revolutions. The centripetal force has acted twice as long in this case. It has been accelerating the puck continuously. Yet the puck still flies off in exactly the same way as the first time. The force acted perpendicularly to the direction of motion. No work was done.

* * *

The text points out that the work done depends only on the force and the distance moved in the direction of the force, but not on the time that the force acts. Some students may not feel comfortable with this assertion because they will think of examples where time seems to be a factor (e.g., it takes more gasoline to drive a given distance at 60 mph than at 30 mph). The difference in fuel consumption is not associated with the difference in kinetic energy of the car in the two cases. Initial and final kinetic energies are zero in both cases. Frictional forces encountered in moving one solid across another (e.g., tires on a road) are essentially independent of the speed of the object, so tire friction does not affect the difference in fuel consumption. The answer lies in the effects of fluid friction, the drag of the air on the car, and the drag between various parts of the car and the lubricating oil. When a body is moved through a fluid, the frictional force increases with the speed of the body, roughly in proportion to the speed. (It is this same drag effect that is responsible for the phenomenon of terminal velocity.) Thus a high-speed car uses more fuel than a slow car in covering the same distance, because a larger force must be applied to move the fast car at constant speed against its frictional load than is needed for the slow car. Thus more work is done by the fast car, and more fuel is consumed.

QUIZ PROBLEMS

1. Calculate the work done by a boy who pulls a sled a distance of 20 m while exerting a 100-newton force on a rope which makes an angle of 45° with the horizontal. ($1000\sqrt{2}$ joules)

2. A man pulls a 20-kg mass up a frictionless inclined plane which makes an angle of 30° with the ground. He moves the mass to a point 5.0 meters in height above the floor.

   (a) How much work has he done? (980 joules)

   (b) How large a force was required to raise the mass? (98 newtons)

   (c) How far was the mass moved along the plane? (10 m)

   (d) How much work would be required to do the same job if the plane were not frictionless, but instead exerted a constant retarding force of 50 newtons along the plane? (1480 joules)

3. How much work is done by a man who takes a 7.5-kg bowling ball and bag from a shelf 2.0 m high and lowers it to the floor? (150 joules is done on the man by the ball)

4. How much work is required to carry a 15-kg suitcase a horizontal distance of 50 m? (Zero)

5. A 60-kg boy pushes a 50-kg girl around a dance floor. He exerts an average force of 80 newtons on his partner. During the dance they move at a rate of 0.50 m/sec for 5.0 minutes. How much work does the poor boy do? (12,000 joules)

Section 4 – Kinetic Energy

PURPOSE To introduce kinetic energy.

CONTENT Work is a form of energy transfer. When work is done on an object in such a way that it starts the object in motion and produces no other change, then all of the energy transferred has become energy of motion, or kinetic energy. Then the work done is identically equal to the change in magnitude of the quantity $\frac{1}{2}mv^2$. This quantity is defined as the energy inherent in motion, kinetic energy. The same work will always produce the same change in kinetic energy, whether a small force acts through a great distance or a large force acts through a short distance. An object in motion can do work. The amount of work extracted in bringing an object to rest is equal to that which was expended in producing the motion. The kinetic energy of an object, $\frac{1}{2}mv^2$, is an important parameter describing a dynamical characteristic of the object. It does not depend in any way on the history of how the object reached that condition.

EMPHASIS These ideas are basic to an understanding of kinetic energy. At least one full class period should be spent in discussing this section.

DEVELOPMENT This section introduces the first quantitative definition of any of the forms of energy. To give students a good feeling for kinetic energy and its relationship to work, it will be worthwhile to follow through a number of examples in class. The text suggests a pattern. As a beginning, you might work through in detail the examples of a 1-kg mass accelerated by a 5-newton force acting through 10 meters, and by a 10-newton force acting through 5 meters. Show that the same work is done and the same kinetic energy is developed in the two cases.

* * *

Students sometimes have difficulty in understanding that kinetic energy is a completely different physical quantity from momentum. This difference should be clarified. The following two examples may be helpful. The first example develops kinetic energy as a property of motion. The example shows how the same work performed in two different cases produces the same change in kinetic energy, but quite different changes in momentum. The second example emphasizes the different properties of two objects moving with the same momenta but widely different energies.

Example 1. a) Let a force of 1500 newtons act through a distance of 200 meters on a 1500-kg car which starts from rest. The work done is

$$W_1 = F_1 x_1 = 1500 \times 200 = 3 \times 10^5 \text{ joules.}$$

To find the final speed of the car after it has traveled these 200 meters, we note that the acceleration was

$$a_1 = \frac{F_1}{m} = \frac{1500}{1500} = 1 \text{ m/sec}^2, \text{ so that}$$

$$v_1^2 = 2a_1x_1 = 400 \text{ m}^2/\text{sec}^2, \text{ and}$$

$$v_1 = 20 \text{ meters/sec.}$$

The momentum of the car will be

$$p_1 = mv_1 = 1500 \times 20 = 3 \times 10^4 \text{ kg-m/sec.}$$

The kinetic energy will be

$$(E_K)_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}1500 \times 400 = 3 \times 10^5 \text{ joules,}$$

which is equal to W_1, the work which was done (see above).

b) Now let a 1500-newton force act again, in the same direction, through 200 more meters.

$$W_2 = F_2 x_2 = 1500 \times 200 = 3 \times 10^5 \text{ joules.}$$

To find the final speed we note that the acceleration again is

$$a_2 = \frac{F_2}{m} = \frac{1500}{1500} = 1 \text{ meter/sec}^2.$$

The final speed v_2, is given by

$$v_2^2 = v_1^2 + 2a_2x_2 = 400 + 2 \times 1 \times 200 = 800 \text{ m}^2/\text{sec}^2.$$

$$\therefore v_2 = 28.3 \text{ m/sec.}$$

The new momentum of the car will be

$$p_2 = mv_2 = 1500 \times 28.3 = 4.24 \times 10^4 \text{ kg-m/sec.}$$

Thus the change in momentum during the second acceleration interval was

$$(\Delta p)_2 = p_2 - p_1 = 1.24 \times 10^4 \text{ kg-m/sec.}$$

Whereas as a result of the initial acceleration

$$(\Delta p)_1 = p_1 - p_i = 3 \times 10^4 \text{ kg-m/sec.}$$

The final kinetic energy will be

$$(E_K)_2 = \frac{1}{2} mv_2^2 = \frac{1}{2} 1500 \times 800 = 6 \times 10^5 \text{ joules.}$$

Thus the change in E_K was the same during both acceleration intervals while the change in momentum was quite different. In each case,

$$\Delta(E_K) = W = 3 \times 10^5 \text{ joules.}$$

Why should this be? In the first period, the car accelerated from zero speed to 20 m/sec with constant acceleration. Therefore the elapsed time must have been

$$v_1 = 20 = v_i + a_1 t_1 = 0 + (1 \times t_1),$$

$$\therefore t_1 = 20 \text{ seconds.}$$

The elapsed time during the second acceleration must have been

$$v_2 = 28.3 = v_1 + a_2 t_2 = 20 + (1 \times t_2),$$

$$\therefore t_2 = 8.3 \text{ seconds.}$$

Thus although identical forces acted through identical distances, the time these forces were applied was quite different in the two cases. This is because, for the second acceleration, there was a large initial velocity which resulted in the given distance being covered in a much shorter time. As was shown in Chapter 23, change of momentum stems from an impulse. In the two intervals in question, the work done was the same, but the impulse, force × time, was quite different.

Example 2. In order to emphasize the difference between the two measures of motion, momentum and energy, it is often helpful to cite an example such as a 100-gram bullet and a 1500-kilogram car, each striking a block of wood. The bullet might have an initial speed of 300 meters per second; the car, .02 meters per second. Thus both have an initial momentum of 30 kg-meters/sec, but the bullet's kinetic energy would be $(E_K)_b = 4.5 \times 10^3$ joules whereas for the car, $(E_K)_c = 0.3$ joules. This difference in kinetic energies results in the car giving the block of wood a slight nudge while the bullet, with the same momentum but much more energy, rips the wood apart.

It is interesting to recall an argument which arose in the 18th century. Descartes, the French philosopher and mathematician, proposed that the quantity mv was an important characteristic of a moving body, and was a constant for an isolated body. The German philosopher and mathematician, Leibnitz, attacked him bitterly, claiming that Descartes was wrong – the important quantity was mv^2, not mv. As we can see now, both were right, but were talking about completely different properties.

Another interesting and sometimes confusing point about changes in kinetic energy concerns their measurement in different frames of reference which move relative to each other. The kinetic energy of an object is a relative thing. With respect to a viewer in a moving train, a baseball has a very different velocity (and therefore also momentum and kinetic energy) than it has relative to a ground observer. Since momentum varies with the first power of the velocity, changes in momentum are the same in any observer's system. However, kinetic energy varies with the square of the velocity, therefore changes in kinetic energy are different when viewed from different frames of reference. The following example illustrates this situation.

Example 3. Picture a man riding on a flatcar of a freight train. Suppose it to be moving at 10 m/sec. He throws a baseball in the direction of the train's motion, exerting a 50-newton force throughout a 1-meter motion of his band. The ball attains a speed of 10 m/sec relative to the flatcar. The kinetic energy of the ball relative to the flatcar is $\frac{1}{2} \times 1\text{ kg} \times (10\text{ m/sec})^2 = 50$ joules. But to an observer on the ground, what appears to have happened? The ball initially was moving with a speed of 10 m/sec as a result of the motion of the flatcar. Its original kinetic energy was therefore 50 joules. After the man threw it, it had a speed of 20 meters per second. Its new kinetic energy was therefore $\frac{1}{2} \times 1\text{ kg} \times (20\text{ m/sec})^2 = 200$ joules, and its change of kinetic energy, in the throwing process, was $200 - 50 = 150$ joules. What is the source of the work that produced this energy change? The force which acted on the ball was still 50 newtons. However, the distance through which the force acted is quite different as seen from the ground. It can be calculated as follows.

On the train, the man's hand moved through 1 meter. If his force was constant, the acceleration was constant. His hand's final speed was 10 m/sec. Thus the average speed was 5 m/sec and the time of the acceleration was $t = \frac{s}{v} = \frac{1}{5}$ seconds. Now in 1/5 seconds the train itself moved $1/5 \times 10 = 2$ meters relative to a ground observer. Therefore in pushing the ball, the man's arm moved 1 meter as a result of his own motion and 2 meters as a result of the train's motion, a total of 3 meters relative to the ground observer. Thus the work done from the ground observer's point of view was 50 newtons × 3 meters = 150 joules, just equal to his observation of the change in kinetic energy. This is not a fictitious amount of work. The man did only 50 joules in pushing with a 50-newton force through 1 meter. But the man's feet pushed back on the train, and the train pushed forward on his feet, with this same 50-newton force. While exerting this force on the man's feet the train traveled 2 meters. Thus the train itself supplied 100 joules of the work done on the ball. This work came from the energy liberated from the burning of the train's fuel.

QUIZ PROBLEMS

1. A 10-kg mass is moving with a constant speed of 10 m/sec.

   a) How much work must be done to double the speed to 20 m/sec? (1500 joules is transferred to the moving mass)

   b) How much work must be done to halve its speed to 5.0 m/sec? (-375 joules is transferred from the moving mass)

2. A constant 20-newton force acts for 10 seconds on a 5.0-kg object which is initially at rest.

   (a) What is the object's final momentum? (200 kg-m/sec)

   (b) What is the object's final kinetic energy? (4000 joules)

   (c) How much work was done in accelerating the body? (4000 joules)

3. A force which increases from 0 to 100 newtons at a rate of 10 newtons/sec acts on a body for 10 sec. 12,500 joules of work are done by this force in accelerating the body from rest to a final speed of 50 m/sec.

(a) What is the mass of the body? (10 kg)

(b) How large a constant force would have been required to give the body the same final speed if the force acted during one meter of travel? (12,500 nt)

4. A 2.0-kg mass moving at a speed of 20 m/sec has the same momentum as a 10-kg mass.

(a) What is the speed of the 10-kg mass? (4 m/sec)

(b) What is the kinetic energy of each of the masses? (400 joules, 80 joules)

5. A 10-kg body has a kinetic energy of 500 joules. What is its momentum? (100 kg-m/sec)

6. A 10-kg body at rest on a frictionless table receives an impulse of 30 newton-sec. What is its final kinetic energy? (45 joules)

7. With what kinetic energy does a 2-kg rock strike the ground if it falls from a height of (a) 5m? (b) 10m? (c) 20m? (d) Compare the momenta of the rock just before striking the ground for each of the cases (a), (b), and (c).
a 100 joules; b 200 joules; c 400 joules; d mv(20 m) = $\sqrt{2}$ × mv(10m) = 2 × mv(5m).

Section 5 – The Transfer of Kinetic Energy from One Mass to Another
Section 6 – Another Look at a Simple Collision
Section 7 – Conservation of Kinetic Energy in Elastic Interactions

PURPOSE To show that kinetic energy is conserved when the force of interaction depends only on the distance of separation of the interacting masses.

CONTENT In collisions between two bodies the total kinetic energy of the system is the same before and after the collision. However, the kinetic energy may change during the interaction. When the force is repulsive, the amount of kinetic energy which "disappears" is given by FΔs, where F is the constant repulsive force of the interaction, and Δs is the change in the separation distance between the two bodies. For a given force of repulsion, when the separation distance decreases, the total kinetic energy decreases. The total kinetic energy increases when the separation distance increases. The criterion for an "elastic" collision is that the force between two objects be a function only of their distance of separation. Then, after interaction is complete, the total kinetic energy is the same as it was before the interaction started.

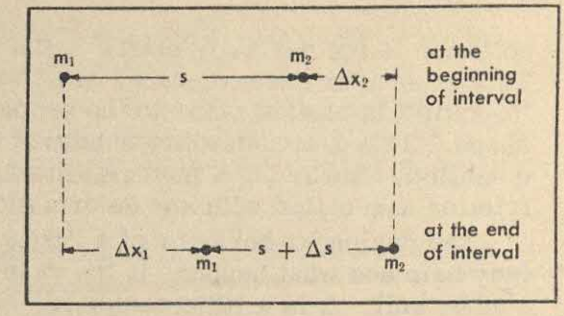
EMPHASIS The point of these sections is important and somewhat difficult. Plan to spend at least one full day in class discussion, even if you completely omit the general solution of Section 7. For a full treatment of the entire unit, allow at least 3 days class discussion.

LABORATORY Experiment III-11, Slow Collisions, should be integrated with class discussion of Sections 5 through 7.

DEVELOPMENT The example illustrated in Figures 24-8 and 24-9 can be reviewed in class after students have studied Sections 6 and 7

Some students may feel that the conditions specified for an elastic collision occur more commonly than is actually the case. To clarify this point you may wish to cite an example of an inelastic interaction. A collision of two automobiles should serve the purpose. As the bumpers hit, springs are compressed. If this is all that happened there would be no difficulty. As the springs re-extend themselves, the force would repeat the pattern followed during compression. However, if something is deformed in the collision so that it does not return to its original configuration, then the force as the cars bounce apart differs from the force that acted as they came together. This is the more common case, an inelastic collision.

The following type of discussion briefly recapitulates and may facilitate understanding the problem discussed in the text and illustrated in Figures 24-8 and 24-10. Consider Figure 24-10 from the text. We will begin with the analysis that is made in the caption for that figure.



Δx_1 = distance moved by m_1 during interval between t_o and t_1.

Δx_2 = distance moved by m_2 during interval between t_o and t_1.

s = distance of separation of m_1 and m_2 at time t_o.

Δs = the change in s during the time interval between t_o and t_1

From the figure, it can be seen that:

$$s + \Delta x_2 = \Delta x_1 + (s + \Delta s), \text{ or}$$

$$\Delta s = \Delta x_2 - \Delta x_1.$$

As pointed out in the text, when m_1 moves through Δx_1, it does so against the retarding force F. Therefore it loses kinetic energy. The change is equal to the work done:

$$(\Delta E_K)_1 = -F\Delta x_1.$$

For m_2, the force, F, is an accelerating force, and the change in its kinetic energy is given by

$$(\Delta E_K)_2 = +F\Delta x_2.$$

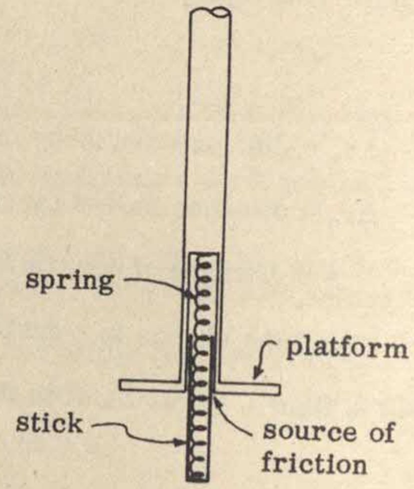
The total change in kinetic energy of the two masses is simply the sum of the individual changes:

$$(\Delta E_K)_{\text{total}} = F\Delta x_2 - F\Delta x_1 = F(\Delta x_2 - \Delta x_1) = F\Delta s.$$

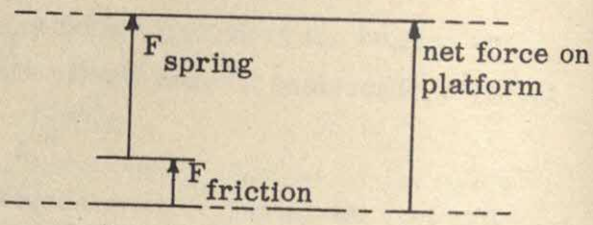
During any short interval of the interaction of two objects, the change in kinetic energy is given by F (t) Δs where F (t) is the force which is acting and Δs is the change in the separation distance. F is written here as F (t) to show that in general it will be different at various times during the interaction. In this case which involves a repulsive force, the kinetic energy decreases when Δs is decreasing, and kinetic energy increases when Δs is increasing. If the interaction force is determined only by the separation of the two objects, then we can write F (t) as F (s). Then, as two mutually-repelling objects part from a separation distance, s, to s + Δs, the change in the total kinetic energy will be exactly equal and opposite to the change in their total kinetic energy which occurred as they approached each other from a separation, s + Δs, to a separation, s. This is true because the forces were identical at these two instants—because F was a unique function of s. Whenever this is true, kinetic energy will be conserved over the full course of a collision (but not at every instant during the collision). Collisions between objects whose interaction forces are of this type are called elastic collisions.

COMMENT You may be asked to explain why most collisions are inelastic. The example of the automobiles is a simple case. A more subtle example is that of a rubber ball. We know that when a rubber ball is dropped, it does not bounce to its initial height. The collision is not perfectly elastic. But the ball does return perfectly to its original shape. This is another case where we must watch our vocabulary. Colloquially, the word "elastic" is used to refer to the property of a rubber ball that returns it to its original shape. This is a necessary condition for an elastic collision. But it is not a sufficient condition. Rubber is a material in which there are appreciable internal forces of friction associated with any deformation.

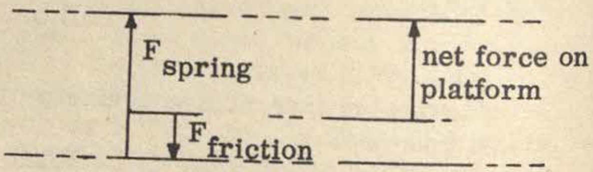
The analogous behavior of a "pogo stick" may help see what happens in the case of a rubber ball. It is a little easier to "separate" the forces acting on the various parts of a pogo stick than those on a rubber ball of uniform composition. A pogo stick can be thought of as a stick coupled to a platform through an ideal (lossless) spring, but with the motion of the stick relative to the platform involving appreciable friction.



Consider the forces acting on the part of the stick to which the platform is attached. When the pogo stick hits the ground, compression of the spring begins. At some given compression point of the spring on the downward motion, the forces acting are shown at the right.



The spring eventually reaches maximum compression, and the platform then starts moving up. At the same point of absolute compression as that considered on the downward motion above, F_{spring} is the same as above. But now the force of friction is reversed, and the net force as Δs increases is less than the net force acting as Δs decreases. This relation between the forces is shown at the right.



In the same way, the net force acting on the ball at any separation distance, s, as it approaches the floor is greater than that which acts on it at the same distance, s, as it leaves the floor. The force does not depend only on the distance between the ball and floor, but also on the direction in which the ball is moving relative to the floor. Kinetic energy disappears, dissipated in internal heat in the ball. You might ask your students if any of them have ever felt a squash ball or a handball after it has been batted around for a while. The temperature rise is striking.

QUIZ PROBLEMS

1. A 200-gm ball, moving at 6.0 m/sec, approaches a second ball head on. The second ball has a mass of 100 grams. While the balls are within 5.0 cm of each other, the repel each other with a constant force of 40 newtons. The second ball leaves the collision with a speed of 8.0 m/sec.

(a) How much kinetic energy does the second ball gain in the collision?
(3.2 $\text{kg-m}^2/\text{sec}^2$ or 3.2 joules)

(b) What is the kinetic energy of the first ball after the collision?
(0.4 $\text{kg-m}^2/\text{sec}^2$ or 0.4 joules)

(c) How far did each ball move during the collision, i.e., while they remained within 5.0 cm of each other? (8.0 cm)

2. A block having a mass of 5.0 kg slides on a frictionless surface with a speed of 1.0 m/sec toward a second block which is at rest. The second block has a mass of 1.0 kg. When the separation of two blocks is 10 cm and less, they repel one another with a force of 50 newtons.

(a) What is the magnitude and direction of the acceleration of each block while they are colliding, i.e., closer together than 10 cm?

($a_1 = 10\ \text{m/sec}^2$, backward; $a_2 = 50\ \text{m/sec}^2$, forward)

(b) The beginning of the collision is at the instant when the two blocks first get within 10 cm of each other. From their positions at that instant, how far will each block move in 1/60 sec? In 1/30 sec?

1/60 sec		1/30 sec
$\Delta x_1 = \frac{11}{12} \times \frac{1}{60}\ \text{m} = 0.015\ \text{m}$,	and	$\frac{5}{6} \times \frac{1}{30}\ \text{m} = 0.028\ \text{m}$.
$\Delta x_2 = \frac{5}{12} \times \frac{1}{60}\ \text{m} = 0.0070\ \text{m}$,	and	$\frac{5}{6} \times \frac{1}{30}\ \text{m} = 0.028\ \text{m}$.

(c) Calculate the change in the total kinetic energy 1/60 sec after the beginning of the collision, and 1/30 sec after the beginning of the collision.

$$1/60\ \text{sec}: (\Delta E_K)_{\text{total}} = 50\left(\frac{5}{12} \times \frac{1}{60} - \frac{11}{12} \times \frac{1}{60}\right)$$

$$= -50 \times \frac{1}{2} \times \frac{1}{60} = -\frac{5}{12}\ \text{newton-meters} = 0.42\ \text{joules}.$$

$$1/30\ \text{sec}: (\Delta E_K)_{\text{total}} = 0\ \text{joules}.$$

3. A 10-kg mass moving with a speed of 5.0 m/sec collides elastically with a 5.0 kg mass which in initially at rest. What is the final kinetic energy of the system? (125 joules)

Section 8 – Kinetic Energy and Momentum

PURPOSE AND CONTENT To show that the application of conservation of energy and conservation of momentum to elastic head-on collisions enables us to solve for the final velocities without knowing the details of the force of interaction. An application of this method is illustrated in Chadwick's discovery of the neutron.

EMPHASIS The ideas of this and the following sections of this chapter are important, but they are more in the nature of an application of previous ideas than of basic building blocks for future developments. The complete solution of the elastic collision is interesting and useful, but somewhat difficult. About 2 class periods will be required for a minimum treatment of the elastic collision and summarizing the ideas of the entire chapter.

COMMENT Some students may not see the importance of studying "collisions". A wide variety of problems in modern physics basically concern what is called the two-body problem. Problems dealing with the orbit of a planet around the sun, the dynamics of a rocket, and the collision between two protons are examples of the two-body problem. Fortunately this relatively simple physical situation, the interaction of two objects, has rather broad applicability. The general problem which involves the interaction of more than two bodies can still not be solved exactly.

Two-body problems dealing with the motions of the planets can be solved by applying Newton's law and thus obtaining solutions for the motion along successive very small intervals of an orbit. This kind of solution is possible because the interaction force is well defined at all points of space by the law of universal gravitation. Even so, the procedure is a tedious one. In the case of the propulsion of a rocket, or of the collision of two protons, a solution by application of $\vec{F} = m\vec{a}$ is impossible because the interaction force is not known. We can make considerable progress with these problems through the use of the conservation laws. When the collision is elastic, we can use the conservation of momentum and of kinetic energy.

DEVELOPMENT Class discussion could begin with an analysis of a head-on collision of m_1 and m_2, m_2 being initially at rest and m_1 approaching with the velocity, v_1. The total momentum before the collision must then be $m_1 v_1$.

After the collision, m_1 and m_2 will have velocities v_1' and v_2', respectively (both in the same direction as v_1). The principle of momentum conservation indicates that (1) $m_1 v_1 = m_1 v_1' + m_2 v_2'$. However this does not give a solution to the problem. That is, knowing the quantities m_1, m_2, and v_1 still does not enable us to predict final values of v_1' and v_2'. There is an infinite number of combinations which will satisfy the momentum equation. For any arbitrary v_1', there is a corresponding value of v_2' that will satisfy momentum conservation. To see this clearly, consider the case where $m_1 = m_2 = m$. Then the equation becomes:

$$(2) \quad mv_1 = m(v_1' + v_2'), \text{ or}$$

$$(3) \quad v_2' = v_1 - v_1'.$$

A few sample solutions would be:

$$v_1' = 0, \text{ and } v_2' = v_1;$$

$$v_1' = v_1, \text{ and } v_2' = 0;$$

$$v_1' = \frac{v_1}{2}, \text{ and } v_2' = \frac{v_2}{2}, \text{ etc.}$$

Thus there is no unique solution to the problem. In mathematical language, the difficulty is simply that there are two unknowns, v_1' and v_2', but only one relationship between them, corresponding to momentum conservation. If a second equation is added, the indefiniteness of the solution can be removed. The equation that is available for an elastic collision is the one which expresses the conservation of kinetic energy:

$$(4) \quad \frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2, \text{ or}$$

$$(5) \quad v_1^2 = v_1'^2 + v_2'^2.$$

But we had, from momentum conservation, the relation:

$$(3) \quad v_2' = v_1 - v_1'.$$

Substituting in (5):

$$v_1^2 = v_1'^2 + (v_1 - v_1')^2 = v_1'^2 + v_1^2 - 2v_1 v_1' + v_1'^2,$$

$$\text{or } v_1'^2 - v_1 v_1' = 0.$$

Therefore $v_1' = v_1$, or $v_1' = 0$.

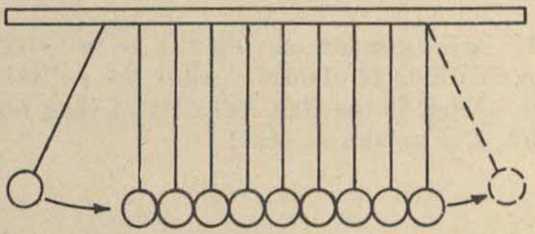
In the first case, from (3), $v_2' = 0$. Then we simply have not had a collision; m_1 has somehow passed m_2, and we can ignore this case. The other solution is $v_1' = 0$, and from (3), $v_2' = v_1$; the first ball stops and the second ball takes on the initial velocity of the first.

Students should notice that the solution has been found without any detailed knowledge of the force law between m_1 and m_2. It is only necessary that the interaction be elastic.

(Appendix 7 to the Guide for this volume treats the non-head-on collision of two equal masses.)

* * *

DEMONSTRATION There is a good standard demonstration that illustrates the uniqueness of the solution to this type of problem. A row of billiard balls or golf balls is suspended with strings tied to a bar so that they hang in contact with each other as in the figure at the right.



Draw back one ball and let the class guess the solution to the problem. If the one ball has a velocity, v, when it hits, what will happen at the other end? With momentum conservation only, we could have one ball bounce off with velocity, v; two balls with v/2, three with v/3, etc. Applying conservation of kinetic energy we find that only the one-ball solution works. If the balls are fairly elastic, this will turn out to be the case experimentally. Likewise, if two balls are drawn back initially, the only possible solution is for two to bounce off, etc.

* * *

Your students may wonder if these two conservation laws can be used to analyze collisions that are not head-on. You can indicate the method of attack as follows.

We can choose the x-axis as the direction of the incoming particle. Then the x-components of the two particles' momentum after the collision must add up to the initial momentum:

$$m_1v_1 = m_1v_{1x}' + m_2v_{2x}'. \qquad (1)$$

After the collision, both particles must move in the same plane since whatever momenta they possess that is not parallel to the x-axis must add up to zero (two vectors not in the same plane cannot add up to an x-component plus zero). Let us call the plane of motion after the collision the x-y plane. Then

$$m_1v_{1y} = 0 = m_1v_{1y}' + m_2v_{2y}',$$

and the velocities perpendicular to the x-y plane are all zero:

$$v_{1\perp} = v_{1\perp}' = v_{2\perp}' = 0. \qquad (2)$$

Equations (1) and (2) express the conservation of momentum.

The conservation of energy can be expressed as

$$\frac{1}{2}mv_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = \frac{1}{2}m_1(v_{1x}'^2 + v_{1y}'^2) + \frac{1}{2}m_2(v_{2x}'^2 + v_{2y}'^2) \qquad (3)$$

These three equations enable us to solve for three unknowns – the other three quantities in the equations must be measured. For example, if a particle known to be a neutron (m_1 known) were to strike a proton in a cloud chamber, the visible tracks of the proton would enable us to measure v_{2x}' and v_{2y}'. We could then calculate the initial and final velocities of the invisible neutron, v_1, v_{1x}' and v_{1y}'.

QUIZ PROBLEMS

1. A 10-kg mass moving with a speed of 5 m/sec collides head-on with a stationary 10-kg mass. The collision is elastic. What is the final velocity of each of the masses? (0, 5 m/sec)

2. A 2-kg mass moving at a speed of 20 m/sec collides elastically, head-on, with a 5-kg mass initially at rest. Find the final speeds and directions of both masses. (2 kg: $-\frac{60}{7}$ m/sec; 5 kg: $+\frac{80}{7}$ m/sec)

3. A 10-kg mass moving toward the right with a speed of 5 m/sec collides elastically, head-on with a second 10-kg mass moving initially at 10 m/sec to the left. What is the final velocity of each of the masses? (10 m/sec left, 5 m/sec right)

4. A 5-kg mass moving at 4 m/sec strikes a glancing blow on a stationary 5-kg mass. The collision is elastic. After the collision one mass leaves at 45° to the incident direction. What is the final velocity of each mass (magnitude and direction)? ($2\sqrt{2}$ m/sec at +45°, $2\sqrt{2}$ m/sec at -45°)

Section 9 – Work and Kinetic Energy When More Than One Force Acts

PURPOSE To show that the kinetic energy transferred to or from a body when several forces act on it is the same as the energy that would be transferred if the resultant of those forces acted on the body for the same part of the motion.

CONTENT The conclusion stated above is based on a consideration of the action of the components of the forces along one direction of motion at a time.

EMPHASIS This section is simply a comment which generalizes the conclusions from the previous consideration of single interactions.

Section 10 – Loss of Kinetic Energy in a Frictional Interaction

PURPOSE To show that if a frictional force acts on a body, some of the energy transferred may go into heat rather than into kinetic energy.

CONTENT Two examples are carefully chosen for this discussion. This topic is sticky in more ways than one, so additional examples should be chosen with care to avoid getting involved in time-consuming and premature explanations. (This section leads toward the next chapter.) The first example is of a block sliding on a table. The table doesn't move but the block exerts a force on it, and the block is slowing down. Where does the "lost" kinetic energy of the block go? Into kinetic energy of the molecules of the table, that is, heat. Notice that the "collision" of the block with the table is different from an elastic collision; the force depends on the velocity of the block, not its position relative to the table

The second example is the bouncing of a ball of putty. Here the ball deforms during the collision so the force on rebound behaves differently from the force on impact. The net kinetic energy of the ball changes as a result of the collision, and the ball becomes warm – kinetic energy has been transferred to heat.

Collisions of this type are called inelastic; they are characterized by an over all loss of kinetic energy of the bodies as a whole, and a gain of kinetic energy of internal motion of the molecules.

DEVELOPMENT Problems 25, 26, 28, 29, 20, 31 and 34 in HDL relate to inelastic collisions. The following might help as a quiz problem.

A 20-kg mass moving at a speed of 3 m/sec collides inelastically with a stationary 10-kg mass. The two stick together and move off at the same speed.

(a) What is the final speed of the two masses?
(b) How much kinetic energy is "lost" in the collision?

Solution: (a)

$$m_1 = 20\text{ kg}$$

$$m_2 = 10\text{ kg}$$

$$v_1 = 3\text{ m/sec}$$

$$m_1 v_1 = (m_1 + m_2) v_2$$

$$v_2 = \frac{m_1}{m_1 + m_2} v_1 = \frac{20}{30} \times 3 = 2\text{ m/sec.}$$

(b)

$$E_{K_1} = \frac{1}{2} \times 20 \times 3^2 = 90\text{ joules}$$

$$E'_{K_{1+2}} = \frac{1}{2} \times 30 \times 2^2 = 60\text{ joules.}$$

Hence, 30 joules of energy is "lost" in the collision.

COMMENT It will be wise to make sure that students do not confuse the loss of kinetic energy into heat with the temporary "disappearance" of kinetic energy during an elastic collision. They will learn in the next chapter that, in an elastic collision, "disappearing" kinetic energy is going into potential energy from whence it can return. Kinetic energy transferred into heat can be recovered only partially.

Chapter 24 – Work and Kinetic Energy

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 24-18 to 24-37.

Section	Easy	Medium	Hard	Class Discussion
1, 2, 3	5*	1, 2*, 3, 4*		1, 2*, 4*, 5*
4	7, 9, 10	6*, 8*, 11*, 12 13, 14, 15*		6*, 8*, 11*
5, 6, 7	20	17*, 18*, 21*, 22	16*, 19	16*, 17, 21*
8, 9, 10, 11	24, 26, 27* 29	25, 28, 30 31*, 32	23, 33, 34	23, 24 25, 28, 31*

SHORT ANSWERS

1. See discussion on page 24-12.

2. (a) 6×10^6 joules.
   (b) 6×10^6 joules.

3. (a) 6 gals.
   (b) Probably more, certainly not half.

4. (a) None.
   (b) None.
   (c) 120 joules.
   (d) -1500 joules.

5. (a) 0.2 joules.
   (b) 0.2 joules.

6. (a) 30.0 joules.
   (b) 30.0 joules.
   (c) 5.48 m/sec.

7. Both 1200 joules.

8. (a) $(E_k)_1 = 4\,(E_k)_2$.
   (b) Equal.
   (c) Equal.
   (d) Equal.
   (e) $(E_k)_A = 2\,(E_k)_B$.

9. (a) 1200 joules.
   (b) 11 m/sec.

10. (a) 1.4 joules.
    (b) 1.2 m/sec.

11. (a) -32 joules.
    (b) 8.0 meters.
    (c) 32 joules.

12. (a) 3.86×10^5 joules.
    (b) 3.86×10^5 joules.
    (c) No way of knowing.

13. (a) 39 joules.
    (b) 160 newtons.
    (c) None.

14. (a) 40 newtons.
    (b) Perpendicular to the motion.

15. (a) 90.0 joules.
    (b) 160 joules.
    (c) See discussion on page 24-24.

16. (a) $\Delta x_1 = (0.16t - t^2)$meters.
    $\Delta x_2 = 3.0t^2$meters.
    (b) 0.04 sec.
    (c) $v_1' = v_1 - 0.08$ m/sec,
    $v_2' = v_1 + 0.08$ m/sec.

17. (a) 4 joules.
    (b) 14.5 joules.
    (c) 38 joules.
    (d) 42 joules.

18. 2.67 joules.

19. 0.020 joules/second.

20. (a) 0.10 m/sec for each body.
    (b) 1.5 kg body: Forward
    0.5 kg body: Backward.

21. 0.2%

22. (a) 0.02 joules.
    (b) 1.0 joules.
    (c) 0.02 joules.
    (d) 1:1.
    (e) When $\frac{m_2}{m_1}$ is either very large or very small.

23. (a) $(E_k)_1 = 0.20$ joules,
    $(E_k)_2 = 0$.
    (b) $(E_k')_2 = 0.18$ joules,
    $(E_k')_1 = 0.02$ joules.
    (c) 10-kg mass: 0.089 joules.
    5-kg mass: 0.044 joules.
    (d) 0.083 m.

24. (a) Equal to initial.
    (b) Greater than initial.

25. (a) 0.22.
    (b) 0.70.
    (c) 1.09.

26. Yes.

27. (a) $v_1 = 3.. \times 10^7$ m/sec,
    $v_1' = 0$.
    (b) 4.4×10^6 m/sec.
    (c) After proton collision, velocity is 0. After nitrogen collision, velocity is -2.9×10^7 m/sec.

28. (a) 5.0×10^4 kg m/sec.
    (b) 0.67 m/sec.
    (c) 5.0×10^4 joules,
    1.7×10^4 joules, No.

29. Yes, it heats your hands.

30. See discussion on page 24-33.

31. 78 joules.

32. (a) 5.0 newtons.
    (b) 12.5 joules.
    (c) 4.5 joules., 8.0 joules, Equal.

33. (a) Same as initial velocity.
    (b) 0.

34. (a) 0.
    (b) $4.5\,mv^2$.
    (c) $\frac{1}{2}mv^2$.
    (d) $4\,mv^2$.
    (e) $3\,mv^2$.
    (f) Difference due to energy transferred from tail to head of rocket.

COMMENTS AND SOLUTIONS

PROBLEM 1

Coal is burned in a city electric power plant to make steam to run a steam turbine that drives an electric generator. The city water department uses this electric supply to run an electric motor to pump water from a well into a standpipe on a hilltop. List, in order, all the energy changes that take place.

a) Chemical energy of the coal is changed to thermal energy of the flame.

b) The energy of the flame is transferred to the water, which takes the form of steam. From the microscopic view, the thermal energy of the steam resides in the kinetic energy of the steam molecules. They in turn impart kinetic energy to the turbine rotor by colliding with its vanes. The rotation of the turbine shaft turns the generator rotor, thus transferring some of the chemical energy of the fuel into electrical energy.

c) Electrical energy is converted to kinetic energy by the electric motor which turns the pump. This kinetic energy is transferred to the water causing it to flow up into the standpipe.

d) As the water rises into the standpipe, it moves upward against the force of gravity, acquiring potential energy. Potential energy of the water in the standpipe can be converted into kinetic energy as it flows down out of the standpipe, through pipes, and out of a faucet.

PROBLEM 2

A cable pulls a car up a mountain with a force of 4×10^3 newtons at a velocity of 5 m/sec. It takes the car 5 minutes to reach the top.

(a) How much work is done in getting the car to the top of the mountain?

(b) How much work would be done to get the car up the mountain if it traveled 2.5 m/sec?

a) In going to the top of the mountain the car travels 5 min × 60 sec/min × 5 m/sec = 1500 meters. The work done is equal to the force multiplied by the distance through which it acts.

$$4 \times 10^3 \text{ newtons} \times 1500 \text{ m} = 6 \times 10^6 \text{ newton meters or joules}.$$

b) If the same force is required to pull the car up the mountain at the slower speed, both force and distance are the same for both speeds. Hence the work done in getting the car up the mountain at the slower speed would be the same as at the higher speed: 6×10^6 joules. (Sources of friction like bearings, pulleys, etc., usually exert less drag at slow speeds than at high speeds. Hence, while the amount of work required to move the car up the mountain is the same regardless of speed, the amount of energy transferred into heat in bearings etc. will depend upon speed.)

PROBLEM 3

A rotary snow plow uses four gallons of gasoline to plow 10 miles of road in 20 minutes.

(a) About how many gallons would be used in plowing 15 miles at the same speed?

(b) Do you think it would take half as much fuel to plow 15 miles in twice the time? Why? Be prepared to discuss your reason in class.

a) Students should be able to solve this part of the problem even if they know nothing of the nature of a snow plow. In plowing 15 miles instead of 10 miles with no change of speed, all conditions are the same except distance. Hence plowing 15 miles involves the same driving force acting through 3/2 the distance, and the engine (or engines) will have to do 1 1/2 times the amount of work done in plowing 10 miles. Since the fuel requirement is proportional to the work done, 6 gallons of gasoline will be required.

b) The energy required to do the same job is the same at both speeds. The energy required for other things (like heating the engine, bearings, and the surrounding air) varies with the speed of the plow and depends upon the design of the plow. There is no particular reason to expect the total fuel consumption to be cut in half. It may be greater or less than before depending upon the design of the plow.

To see that the fuel requirement may be greater, consider what would happen when the time the plow runs is greatly lengthened. The engine would clearly burn more fuel just heating the engine and, eventually, far more of the surrounding air.

PROBLEM 4

How much energy is transferred to a 15-kg suitcase while

(a) you hold it 5.0 minutes, waiting for a bus?

(b) you run with it a horizontal distance of 10 meters in two seconds at constant velocity in catching the bus?

(c) you lift it 0.80 meter getting into the bus?

(d) you carry it 20 meters down a ramp that makes an angle of 30 degrees with the horizontal?

This problem points up the fact that transfer of energy is measured by work, the product of the force times the distance moved in the direction in which the force acts.

a) Despite the tiring effect of holding a suitcase, you do no work and hence transfer no energy to the suitcase since work is defined as force × distance moved. If you had placed the suitcase on a stand while you waited for the bus, you would not think of the stand as having transferred energy to the suitcase.

b) Similarly, although not so obviously, you do no work and transfer no energy to the suitcase while running with it along a horizontal path. (A negligible amount of work will be required to "push" the suitcase through the air). Work is only done when the force acts in the direction of the motion. As before, you can reason that if the suitcase was on frictionless wheels, you would not think of the sidewalk as transferring energy to the suitcase while it rolls along on its wheels.

c) When you lift the suitcase 0.80 meters, you exert a force of 15 kg × 9.8 nt/kg = 150 nt. Since the force is in the direction of motion, you do 150 nt × 0.80 m or 120 joules of work, thereby transferring 120 joules of energy to the suitcase.

d) When you carry the suitcase 20 meters down a ramp, its weight has a component of force along the direction of your motion down the ramp; it therefore does work on you. The component of the weight parallel to the ramp is 1/2 × 150 newtons; the distance down the ramp is 20 meters; the work done by the suitcase on you is 75 newtons × 20 m = 1500 joules. A literal answer to part (d) would say that the energy transferred to the 15 kg suitcase is -1500 joules. (This energy is transferred to you and to the ramp in the form of heat.)

PROBLEM 5

A meter stick, mass 0.20 kg, is lying on a table near two blocks 10 cm high (Fig. 24–17).

(a) If you lift the stick, holding it horizontal, and put it on the blocks, how much work have you done?

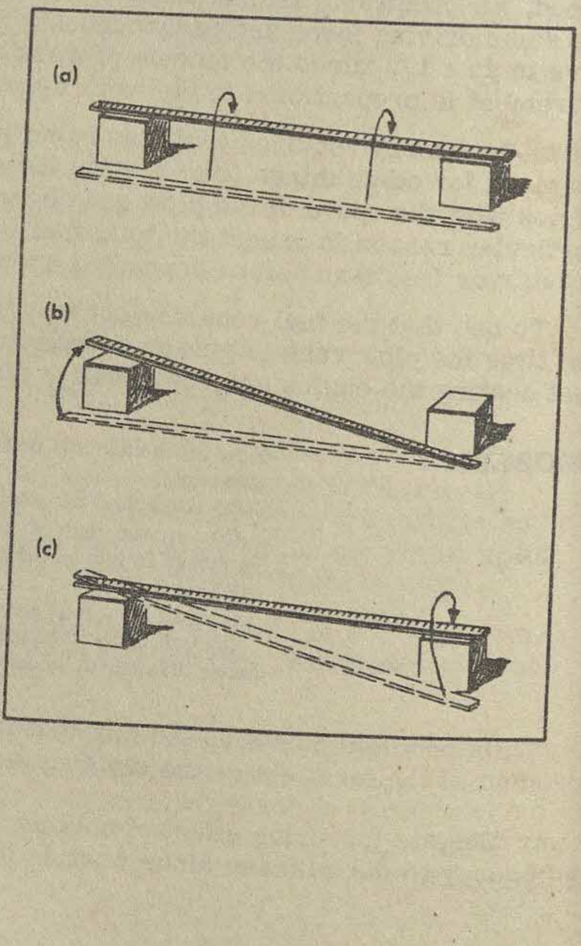
(b) If you lift one end and set it on one block, and then lift the other end, setting it on the other block, how much work have you done in moving the stick?

a) If you lift the stick, holding it horizontal, you are lifting the whole weight of the stick and must be exerting a force of 0.20 kg × 9.8 nt/kg or 2.0 nt. This force, moving the stick 10 cm in the direction of the force, does 2.0 nt × 0.1 m = 0.2 joules of work.

b) If you lift the stick one end at a time and put it up on the blocks, you have done the same amount of work, but in two steps. In the first step you lift half of the weight of the stick, the table supporting the other half of the weight; you thus do

$\frac{1}{2}$ × 2.0 nt × 0.1 m = 0.1 joules of work.

Then when you lift the other end you again lift half of the weight, the other half being held up by the block on which it is resting. In this step you also do 0.1 joules of work, and the total work is 0.2 joules as before. (You can make the point that, although the energy transfer is accomplished in two steps, the energy transfer is the same because the same job is done.



PROBLEM 6

A force of 10.0 newtons acts on a 2.00-kg roller skate initially at rest on a frictionless table. The skate travels 3.00 meters while the force acts.

(a) How much work is done?

(b) How much energy is transferred to the skate?

(c) What is the final speed of the skate?

a) The work done is 10.0 nts × 3.00 m = $\underline{30.0 \text{ joules}}$.

b) The energy transferred to the skate is measured by the work done, $\underline{30.0 \text{ joules}}$.

c) The final speed of the skate is obtained by knowing that the kinetic energy of the moving skate, $\frac{1}{2}mv^2$, equals the energy that has been transferred to the skate:

$$\frac{1}{2}mv^2 = 30.0 \text{ joules, or } v = \sqrt{\frac{2 \times 30.0}{2.00}} = \underline{5.48 \text{ m/sec}}.$$

PROBLEM 7

Compare the work done in lifting a 6.0-kg mass 20 meters and the kinetic energy of the same mass when it has fallen a distance of 20 meters from rest.

The work done in lifting a 6.0 kg mass 20 meters is:

$$6.0 \text{ kg} \times 9.8 \text{ nt/kg} \times 20 \text{ m} = \underline{1200 \text{ joules}}.$$

When a 6 kg mass falls 20 meters under the force of its weight, its velocity is given by:

$$v^2 = 2as = 2gh = 2 \times 9.8 \text{ nt/kg} \times 20 \text{ m} = 400 \text{ (m/sec)}^2.$$

The kinetic energy of the falling 6.0-kg mass is:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 6 \text{ kg} \times 400 \text{ (m/sec)}^2 = \underline{1200 \text{ joules}}.$$

The kinetic energy gained by a body in falling from a certain height is thus shown to be equal to the work done in raising it to that height.

Note. While the mass is being lifted, the net force on it (lifting force plus gravity) is zero. Therefore no energy is transferred to kinetic energy during that process. However, energy evidently is transferred from the lifter to some stored form which can later (upon dropping) be transferred to kinetic energy.

PROBLEM 8

Compare the kinetic energies of two objects, *A* and *B*, identical in every respect except one. Assume that the single difference is:

(a) *A* has twice the velocity of *B*.

(b) *A* moves north, *B* south.

(c) *A* moves in a circle, *B* in a straight line.

(d) *A* is a projectile falling vertically downward; *B* is a projectile moving vertically upward at the same speed.

(e) *A* consists of two separate pieces attached by a light string, each equal in mass to the mass of *B*.

a) Since the kinetic energy is $\frac{1}{2}mv^2$, A has 2^2 or $\underline{\text{4 times}}$ the kinetic energy of B.

b) Kinetic energy is a scalar, independent of direction; hence the kinetic energy of A is $\underline{\text{equal}}$ to that of B.

c) Same as (b), the kinetic energies are $\underline{\text{equal}}$, since the magnitudes of the velocities are equal.

d) Again, same as (b). the kinetic energies are equal, since the magnitudes of the velocities are equal. (Of course, in this case, the kinetic energies will not be equal a moment later, since that of A is increasing, while that of B is decreasing.)

e) Since the mass of A is twice that of B, the kinetic energy of A is twice that of B.

PROBLEM 9

A force of 30 newtons accelerates a 2.0-kg object from rest for a distance of 3.0 meters along a level, frictionless surface; the force then changes to 15 newtons and acts for an additional 2.0 meters.

(a) What is the final kinetic energy of the object?

(b) How fast is it moving?

a) The kinetic energy of the object equals the energy transferred to it by the net force that acts on it. The energy transferred to the object equals the work done on it by the force:

$$30 \text{ nt} \times 3.0 \text{ m} + 15 \text{ nt} \times 2.0 \text{ m} = \underline{120 \text{ joules}}.$$

b) The object's speed can be determined by making use of the fact that its kinetic energy is $\frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 120}{2.0}} = \underline{11 \text{ m/sec}}.$$

PROBLEM 10

A force of 4.0 newtons applied at an angle of 45 degrees with the horizontal accelerates a 2.0-kg Dry Ice disc, initially at rest, for a distance of 0.50 meter on a horizontal table.

(a) Find the work done on the disc.

(b) What is the velocity of the disc?

a) The horizontal component of the 4.0 newton force has a magnitude of 4.0 × 0.707 = 2.8 newtons. Since the disc moves in a horizontal plane, it is only this component of the 4.0 newton force that does work transferring kinetic energy to the disc. This work is $2.8 \text{ nt} \times 0.50 \text{ m} = \underline{1.4 \text{ joules}}$.

b) The velocity of the disc is obtained from $E_k = \frac{1}{2} mv^2$.

$$v = \sqrt{\frac{2 E_k}{m}} = \sqrt{\frac{2 \times 1.4}{2}} = \underline{1.2 \text{ m/sec}}.$$

(Note that the vertical component of the 4 newton force does no work on the disc; it merely relieves some of the weight of the disc from the table, or presses it down on the table, depending on whether the force is directed upward or downward at 45°.

PROBLEM 11

A 3.0 kg mass moving 5.0 m/sec is acted upon by a force of 4.0 newtons opposite to the direction of motion. If the object is slowed to a speed of 2.0 m/sec, what is

(a) the change in kinetic energy?

(b) the distance moved while the force acts?

(c) the energy gained by the system supplying the force?

a) The kinetic energy of the mass before the force acts is 1/2 × 3.0 × 25.0 = 38 joules. The kinetic energy of the mass after the force acts is 1/2 × 3.0 × 4.0 = 6.0 joules. The change in kinetic energy is therefore $\underline{-32 \text{ joules}}$.

b) Since the loss in kinetic energy is equal to the work done by the 3.0-kg mass,

$$Fd = 32 \text{ joules}, \quad d = \frac{32 \text{ joules}}{4.0 \text{ newtons}} = \underline{8.0 \text{ meters}}.$$

c) According to the law of conservation of energy, the energy gained by the system applying the force is equal to the loss in energy of the 3.0-kg mass, or 32 joules.

PROBLEM 12

An automobile of mass 1000 kg is moving at 100 km/hour. We read this speed on the speedometer.
(a) What is its kinetic energy?
(b) How much work was done to provide this kinetic energy?
(c) Can you determine what force acted on the car to provide this kinetic energy? Can you determine through what distance this force acted?
Be prepared to defend your answer in class.

a) 100 km/hr = 10^5 m/3600 sec = 27.8 m/sec. The kinetic energy of a mass of 1000 kg moving at this speed is $\frac{1}{2} \times 1000 \times (27.8)^2 = \underline{3.86 \times 10^5 \text{ joules}}$.

b) Just this much work must have been done to provide this kinetic energy, with the car starting from rest. (Much more work was done over all, since automobile engines transfer energy into the form of heat in the exhaust gases, heat in the bearings, etc.)

c) There is no way of knowing what force acted on the car, or through what distance it moved; we know only that the product of the force and the distance is 3.86×10^5 joules. Any combination of force and distance whose product is 3.86×10^5 joules would produce a speed of 100 km/hr.

PROBLEM 13

A 2.0-kg stone whirls around on the end of a 0.50-m string with a velocity of 2.0 revolutions per second.
(a) What is its kinetic energy?
(b) What is the centripetal force on it?
(c) How much work is done by the centripetal force in one revolution?

a) To find the kinetic energy of the stone we have to know its speed as well as its mass. The stone traverses the circumference of a circle of radius 0.50 m in 1/2 sec. Hence its speed is

$$\frac{2\pi R}{T} = \frac{2\pi \times 0.50 \text{ m}}{1/2 \text{ sec}} = 2.0\pi \text{ m/sec}.$$

Its kinetic energy is then:

$$E_k = \frac{1}{2} \times 2.0 \text{ kg} \times (2.0\pi \text{ m/sec})^2 = 4.0\pi^2 = \underline{39 \text{ joules}}.$$

b) The centripetal force is $F = \frac{mv^2}{R} = \frac{2.0 \times 39}{0.50} \approx \underline{160 \text{ newtons}}$.

c) No work is done by the centripetal force since it always acts at right angles to the direction of motion and thus has no component in the direction of motion. This is checked by the fact that the stone does not speed up; its kinetic energy does not change.

PROBLEM 14

A body moves along a curve with a constant kinetic energy of 10 joules. Part of the curve is the arc of a circle of radius 0.50 meter.

(a) What is the net force acting on the body while in this part of the curve?

(b) What is the direction of this force?

a) Since the centripetal force is given by $F = \frac{mv^2}{R}$, and the kinetic energy, E_k, is $\frac{1}{2}mv^2$, then $F = \frac{2E_k}{R} = \frac{2 \times 10}{0.50} = \underline{40 \text{ newtons}}$.

b) Since the kinetic energy is constant, the speed must be constant, and the force required to produce the deflection must be perpendicular to the motion, toward the center of the circle.

PROBLEM 15

A man pulls with a string on a 20.0-kg mass initially at rest on the floor. He exerts a force of 20.0 newtons horizontally, and the mass moves through 8.0 m. The mass then has a velocity of 3.00 m/sec.

(a) What is its final kinetic energy?

(b) How much energy has been transferred from the man?

(c) How do you explain the difference in your answers to (a) and (b)?

a) The final kinetic energy of the mass is $\frac{1}{2} \times 20.0 \text{ kg} \times (3.00 \text{ m/sec})^2 = \underline{90.0 \text{ joules}}$.

b) On the other hand, the man has done 20.0 newtons × 8.0 meters = 160 joules of work.

c) It is the net force × distance that gives the energy transfer into the kinetic energy of the body's motion. From the difference in energies we conclude that the net force (on the average) is less than the force applied by the man; there must have been another force acting, possibly frictional.

PROBLEM 16

Fig. 24–8 illustrates the interaction of two bodies, m_1 and m_2, where $m_1 = 3m_2$. The repulsive force F is constant as long as the separation is less than the distance d, and the force is zero when the separation is greater than d.

Suppose F is 6.0 newtons; d is 20 cm; m_1 is 3.0 kg; and m_2 is 1.0 kg. Also, the initial speed v_1 of m_1 is 16 cm/sec.

(a) Find expressions for Δx_1 and Δx_2 (the distances moved by m_1 and m_2 during the interaction) as a function of time t. Use Newton's law and the relation $\Delta x = v_i t + \frac{1}{2}at^2$. The initial speed, v_i, is the speed v_1 for m_1 and zero for m_2.

(b) At the end of the interaction the masses are again d apart and $\Delta x_1 = \Delta x_2$. How long a time does the interaction last?

(c) Find expressions for v_1' and v_2', the speeds at the end of the interaction, in terms of v_1. Remember that for constant acceleration $v = v_i + at$.

Curiosity may impel someone to ask whether m_1 hits m_2. If m_2 were fixed so that it could not move, then to hit m_2, m_1 would have to approach m_2 against a repulsive force of 6 newtons for a distance of 0.2 m, thus losing 1.2 joules of energy. Since m_1 has a kinetic energy of only $\frac{1}{2} \times 3.0 \times (0.16)^2 = .038$ joules, it would stop before hitting m_2, even if m_2 did not move.

a) The acceleration of m_1 is $\frac{-F}{m_1} = -2.0\ \text{m/sec}^2$, the minus sign indicating m_1 is decelerating; Δx_1 is then $\underline{\Delta x_1} = 0.16\,t - \frac{1}{2} \times 2.0 \times t^2 = \underline{\left(0.16t - t^2\right)\text{meters.}}$

The acceleration of m_2 is $\frac{F}{m_2} = 6.0\ \text{m/sec}^2$; Δx_2 is then

$$\underline{\Delta x_2} = 0 + \frac{1}{2} \times 6.0 \times t^2 = \underline{3.0\ t^2\ \text{meters.}}$$

b) To find the time at which $\Delta x_1 = \Delta x_2$, equate the two expressions found in part (a):

$$\Delta x_1 = 0.16t - t^2 = \Delta x_2 = 3.0t^2$$

$$4.0t^2 = .16t$$

$$t = \underline{.04\ \text{seconds.}}$$

c) In terms of v_1, at $t = 0.04$ sec:

$$v_1' = v_1 + at = v_1 - (2.0\ \text{m/sec}^2 \times 0.04\ \text{sec}) = \underline{v_1 - 0.08\ \text{m/sec}}$$

$$v_2' = v_2 + at = 0 + (6.0\ \text{m/sec}^2 \times 0.04\ \text{sec}) = 0.24\ \text{m/sec} = \underline{v_1 + 0.08\ \text{m/sec.}}$$

There is nothing mysterious about the fact that v_1' is less than v_1 by the same amount that v_2' is greater than v_1. This comes about because the ratio of the two masses is 3/1. Hence $\Delta v_2 = 3\Delta v_1$.

Note. This problem deals quite directly and thoroughly with the material in Section 24-6. It can be extended for class discussion to examine the loss in kinetic energy of m_1 and the gain of energy of m_2 to show that they are equal.

PROBLEM 17

A 3.0-kg mass accelerates from rest according to the force function shown in Fig. 24-18. What is the kinetic energy of the object at

- (a) $s = 2$ m?
- (b) $s = 5$ m?
- (c) $s = 13$ m?
- (d) $s = 15$ m?

The change in kinetic energy is equal to the work done. With a constant force, the work done in any distance interval is simply the product, force × distance. When the force varies with distance, the work can be found by calculating the area under the force vs. distance graph.

In the graph below the area under the curve of Figure 24-18 has been divided into easily calculable segments.

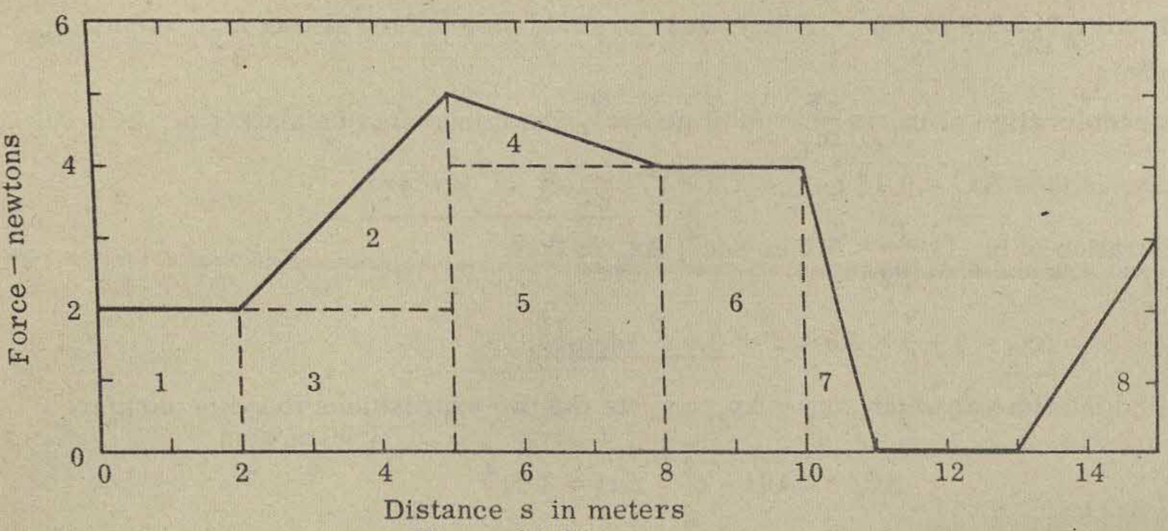


Figure 24-18

$A_1 = 2 \times 2 = 4$ joules

$A_2 = \frac{1}{2} 3 \times 3 = 4.5$ joules

$A_3 = 2 \times 3 = 6$ joules

$A_4 = \frac{1}{2} \times 1 \times 3 = 1.5$ joules

$A_5 = 3 \times 4 = 12$ joules

$A_6 = 4 \times 2 = 8$ joules

$A_7 = \frac{1}{2} \times 4 \times 1 = 2$ joules

$A_8 = \frac{1}{2} \times 2 \times 4 = 4$ joules.

a) At s = 2m, $A_{total} = A_1 = \underline{4 \text{ joules}} = E_k$.

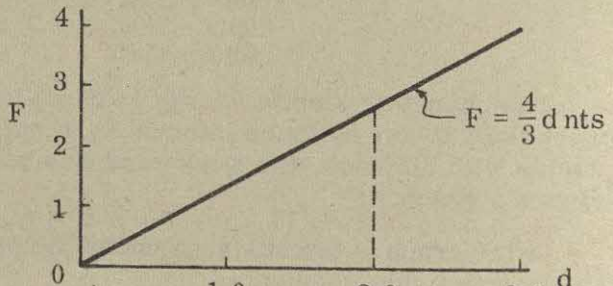
b) At s = 5m, $A_{total} = A_1 + A_2 + A_3 = \underline{14.5 \text{ joules}} = E_k$, or $E_k = 15$ joules, to two significant figures.

c) At s = 13m, $A_{total} = 14.5 + A_4 + A_5 + A_6 + A_7 = \underline{38 \text{ joules}}$.

d) At s = 15m, $A_{total} = 38 + A_8 = \underline{42 \text{ joules}} = E_k$.

PROBLEM 18

A 10.0-kg mass moves 2.00 meters against a retarding force that increases linearly by 4.00 newtons for every 3.00 meters the mass moves (see Fig. 24–19). If the force is zero at the beginning, how much kinetic energy is lost?



The energy lost by the 10.0-kg mass is measured by the work it does. The work done by the mass in pushing against a retarding force that changes with distance can be found from the area under the graph of force vs. distance. Between 0 and 2.00 meters, $E_k = W = 1/2 \times 2.67$ newtons × 2.00 meters = $\underline{2.67 \text{ newton-meters or joules}}$.

PROBLEM 19

Two 3.0-kg bodies interact. At a given moment the first body is moving to the right at 0.50 m/sec and the second body is moving to the right at 0.30 m/sec, and so at that instant the speed of approach of one body as measured from the other is 0.20 m/sec.

If the force of interaction is repulsive and equal to 0.10 newton, at what rate is the total kinetic energy decreasing at that time?

In a short time interval Δt, the first body moves a distance $\Delta x_1 = v_1 \Delta t$ while the second body moves a distance $\Delta x_2 = v_2 \Delta t$. The force of repulsion acts opposite to the direction of travel of the first body, but in the direction of travel of the second. Hence, during the given time interval, the kinetic energy of the first body decreases by $F\Delta x_1$, that of the second body increases by $F\Delta x_2$. Therefore the decrease in total kinetic energy is:

$$= F\Delta x_1 - F\Delta x_2 = \Delta E_k$$

and the rate at which kinetic energy is decreasing is $\Delta E_k/\Delta t$ or:

$$\frac{\Delta E_k}{\Delta t} = \frac{F\Delta x_1 - F\Delta x_2}{\Delta t} = F\left(\frac{\Delta x_1}{\Delta t} - \frac{\Delta x_2}{\Delta t}\right)$$

$$= F(v_1 - v_2) = 0.10 \text{ newton} \times (0.50 - 0.30 \text{ m/sec})$$

$$= \underline{0.020 \text{ joules/sec}}.$$

(Students may wonder what happens to the "lost" kinetic energy. It has, of course, been stored as potential energy of the two-body system. This subject will be discussed in the next chapter.)

PROBLEM 20

A 1.5-kg body is at rest. It is "hit" head on by a body of mass 0.50 kg moving with a speed of 0.20 m/sec. The interaction force depends only on the separation of the two bodies.

(a) What is the final velocity of each body?

(b) In what direction does each move after the interaction?

This problem can be worked out from scratch using the approach described in Section 24-8. For students who do not go through that development, it serves as an exercise in the application of the equations that are derived in that section.

a) Taking m_1 as the moving 0.50-kg body, and m_2 as the 1.5-kg body that is "struck":

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 = 2 \times \frac{0.50 \text{ kg}}{2.0 \text{ kg}} \times 0.20 \text{ m/sec} = \underline{0.10 \text{ m/sec}}.$$

Similarly, $$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{0.50 - 1.5 \text{ kg}}{2.0 \text{ kg}} \times 0.20 \text{ m/sec} = \underline{-0.10 \text{ m/sec}}.$$

b) The significance of the positive sign in the answer for v_2' is that m_2, the second body (the struck one), moves in the same direction the incident ball did before the collision. The significance of the negative sign in the answer for v_1' is that m_1, the incident ball (which is lighter), bounces back in the collision so that its final velocity is in the opposite direction from its velocity before the collision.

PROBLEM 21

A moving electron "hits" a stationary proton head-on. The forces between the two depend on the separation only. (The mass of a proton is 1840 times the mass of an electron.) What fraction of the electron's kinetic energy is transferred to the proton?

This problem, like Problem 20, illustrates a collision where a light body strikes a heavy one, except in this case, the light body is much lighter than the heavy one. All we need to know about the masses is their ratio, since we can write

$$v_2{}' = \frac{2m_1}{m_1 + m_2} v_1 = \frac{2}{1 + \frac{m_2}{m_1}} v_1, \quad \text{and } v_1{}' = \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}}.$$

In this problem, m_1 is the electron, so $\frac{m_2}{m_1} = 1840$, and

$$v_2' = \frac{2}{1841} v_1 = 1.086 \times 10^{-3} v_1$$

$$v_1' = \frac{1 - 1840}{1 + 1840} v_1 = -\frac{1839}{1841} v_1 = -.999\, v_1 .$$

Hence the electron recoils with .999 of its incident speed and the proton acquired a forward speed of about 1/1000 that of the incident electron.

The initial electron kinetic energy was $(E_k)_e = \frac{1}{2} m_1 v_1^2$. The final proton kinetic energy is $(E_k')_p = \frac{1}{2} m_2 v_2'^2 = \frac{1}{2} 1840\, m_1 \left(\frac{2}{1841}\right)^2 v_1^2$.

The ratio $$\frac{(E_k')_p}{(E_k)_e} = \frac{\frac{1}{2}\, 1840\, m_1 \left(\frac{2}{1841}\right)^2 v_1^2}{\frac{1}{2} m_1 v_1^2} = 1840 \left(\frac{2}{1841}\right)^2 = \frac{4}{1842} = .00217 = .217\%.$$

That is, the proton recoils with about 0.2% of the initial electron kinetic energy.

PROBLEM 22

An object of mass $m_1 = 2.0$ kg moving with a kinetic energy of 1.0 joule collides head-on with an object of mass m_2 at rest.

Assuming the force of interaction depends only on the separation, calculate the kinetic energy transferred if

(a) $m_2 = .01$ kg.
(b) $m_2 = 2.0$ kg.
(c) $m_2 = 400$ kg.

What is the ratio of m_1 and m_2 when the transfer of kinetic energy is

(d) a maximum?
(e) very small?

Note: To see this relationship more clearly, you may wish to find the energy transferred for additional values of m_2, and make a graph.

The kinetic energy of m_2 after the collision is:

$$\frac{1}{2} m_2 v_2'^2 = \frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_1^2 = \frac{1}{2} m_1 v_1^2 \frac{4 m_1 m_2}{(m_1 + m_2)^2} = \frac{1}{2} m_1 v_1^2 \frac{4\frac{m_2}{m_1}}{\left(1 + \frac{m_2}{m_1}\right)^2}$$

$$= \frac{1}{2} m_1 v_1^2 \frac{4r}{(1 + r)^2},$$

where $\frac{1}{2} m_1 v_1^2$ is simply the initial kinetic energy of m_1, 1 joule in this problem, and

$r = m_2/m_1$, the mass ratio.

a) $r = \frac{.01}{2} = .005$, and $\frac{4r}{(1+r)^2} = \frac{.020}{1.01} = .0198$

$$(E_k')_2 = 0.02 \times 1 \text{ joule} = \underline{.02 \text{ joule}}.$$

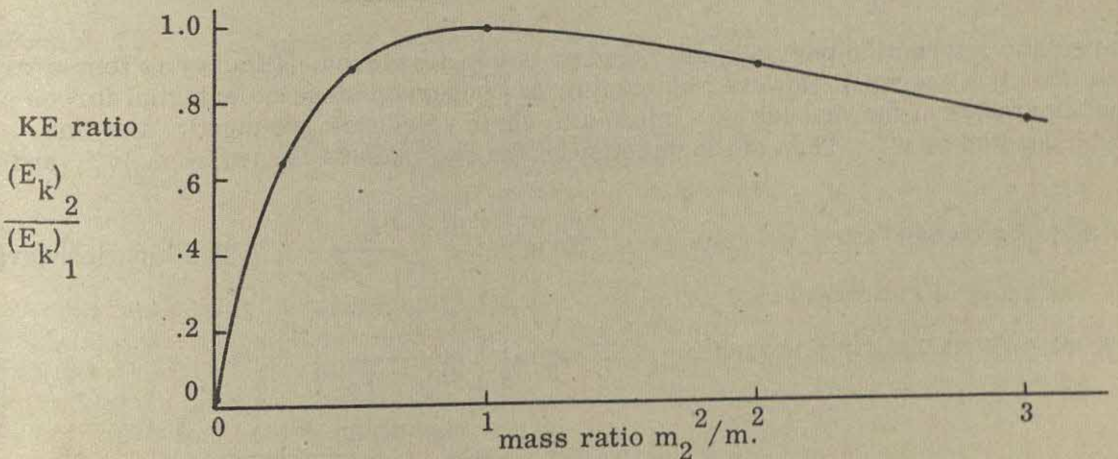
b) $r = \frac{2.0}{2} = 1$, and $\frac{4r}{(1+r)^2} = \frac{4}{4} = 1$

$$(E_k')_2 = \underline{1 \text{ joule}}.$$

c) $r = \frac{400}{2} = 200$, and $\frac{4r}{(1+r)^2} = \frac{800}{40201} = .02$

$$(E_k')_2 = .02 \times 1 \text{ joule} = \underline{0.02 \text{ joule}}.$$

d) The graph of the function $\frac{4r}{(1+r)^2} = \frac{(E_k')_2}{(E_k)_1}$ is given below, showing a maximum at $\underline{m_2/m_1 = r = 1}$.



e) The function has very small values both for very small and for very large r. (The zero value for $r = 0$ is easy to see analytically. The approach to zero for very large r can be understood as follows: When r is very large $1 + r \approx r$ and the function becomes:

$$\frac{4r}{(1+r)^2} = \frac{4r}{r^2} = \frac{4}{r},$$

and $\frac{4}{r}$ goes to zero as r goes to infinity.)

PROBLEM 23

A 5.0-kg body is at rest. A 10-kg body approaches it with a velocity of 0.20 m/sec. The interaction force is zero when the separation is greater than 0.10 meter and is 4.0 newtons when the separation is less than this distance. (Note that some of the answers in this problem are easier to get if you use the conservation of momentum.)

(a) What is the kinetic energy of the masses before the interaction?

(b) What will be the kinetic energy of each mass after the interaction is complete?

(c) What will be the kinetic energy of each mass when the separation is at a minimum? Recall that the speeds at minimum separation are equal.

(d) What is the minimum separation? (Your answer to part (c) tells you the net loss of kinetic energy of the two masses at minimum separation.)

a) $(E_k)_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2} \times 10\ (0.20)^2 = \underline{0.20 \text{ joule.}}$

$(E_k)_2 = \frac{1}{2}m_2v_2^2 = \underline{0.}$

b) $(E_k')_2 = (E_k)_1 \times \frac{4r}{(1+r)^2}$ (See Guide solution to Problem 22.)

$r = \frac{5}{10} = \frac{1}{2}$, and $\frac{4r}{(1+r)^2} = \frac{8}{9}$

$(E_k')_2 = \frac{8}{9} \times 0.20 \text{ joule} = \underline{.18 \text{ joule.}}$

Since kinetic energy must be the same in the initial and final states (when the interaction force is zero);

$$(E_k')_1 = (E_k)_i - 0.18 = 0.20 - 0.18 = \underline{0.02 \text{ joule}}.$$

c) During the interaction period kinetic energy is not conserved. (Energy is temporarily stored as potential energy.) However momentum is conserved since no external forces act. When the separation of the masses is a minimum, their velocities are equal. Let this velocity be denoted as v''. Then the momentum of the two masses is: $(m_1 + m_2)v''$, and is equal to m_1v_1 by conservation of momentum. Hence $v'' = \frac{m_1}{m_1 + m_2}v_1$. The kinetic energy of the 10-kg mass at this time is then $\frac{1}{2}m_1v''^2 = \frac{1}{2}m_1\left(\frac{m_1}{m_1+m_2}\right)^2 v_1^2$

$$= \left(\frac{1}{2}m_1v_1^2\right)\left(\frac{m_1}{m_1+m_2}\right)^2$$

$$= .20 \times \frac{100}{225} = \underline{.089 \text{ joule}}.$$

The kinetic energy of the 5.0-kg mass is similarly

$$E_{k_2} = \frac{1}{2}m_2v''^2 = \left(\frac{1}{2}m_1v_1^2\right)\frac{m_1m_2}{(m_1+m_2)^2} = 0.20 \times \frac{50}{225} = \underline{0.044 \text{ joule.}}$$

The total kinetic energy is .089 + .044 = .133 joule, which is less than the original kinetic energy for the reason noted above. Since the original kinetic energy was 0.2 joule, the net loss of kinetic energy is 0.067 joule.

d) Now that we know the kinetic energy gained by the 5.0-kg mass and the kinetic energy lost by the 10-kg mass, we know the net kinetic energy lost by the pair and therefore the work done against the interaction force:

$$F\Delta s = -(\Delta E_k)_1 + (\Delta E_k)_2$$

$$\Delta s = \frac{-(\Delta E_k)_1 + (\Delta E_k)_2}{F} = \frac{-0.111 + 0.044}{4.0} = -\frac{0.067}{4.0}.$$ (The negative sign for $(\Delta E_k)_1$ indicates its decrease in kinetic energy.)

$$\Delta s = -0.017\,\text{m}.$$

(Here the minus sign indicates that the distance of separation has decreased by 0.017 m.) Since $\Delta s = -0.017$ m, the minimum separation is $0.10 - 0.017 = \underline{0.083\,\text{m}}$.

PROBLEM 24

An antiaircraft shell has a kinetic energy E_K and a momentum $\vec{p}$. Just then it explodes. What can you say about:

(a) the momentum of the pieces?

(b) the kinetic energy of the pieces?

a) Since no external force acts on the shell, the total momentum of the shell must remain unchanged by the explosion. The vector sum of the momenta of all of the pieces is therefore equal to the momentum of the shell just before it exploded.

b) The total kinetic energy of the pieces must equal the kinetic energy of the shell plus the work done (the energy transferred into kinetic energy of the pieces) in the explosion, and so it is greater than the initial kinetic energy of the shell above.

PROBLEM 25

(a) In Fig. 24-6 make the necessary measurements and calculate the ratio of the kinetic energy before and after impact of the billiard ball marked with the dot.

(b) What fraction of the initial energy of the dotted ball is gained by the undotted ball?

(c) What is the ratio of the total kinetic energy before the collision to the total kinetic energy after collision?

a) From measuring distances on the figure (it will be hard to convert to actual distances because the scale is indistinct; conversion is not necessary in any case), the dotted ball originally moved 1.09 cm/flash. After the collision the dotted ball moved 0.51 cm/flash. Since the kinetic energy is proportional to v^2, the ratio of the kinetic energies of the dotted ball after and before the impact is

$$\frac{0.51^2}{1.09^2} = \underline{0.22}.$$

b) After the collision the undotted ball moves with a speed of 0.91 cm/flash. Since kinetic energy is proportional to v^2, and since the balls have equal masses, the fraction of the initial energy of the dotted ball gained by the undotted ball is

$$\frac{0.91^2}{1.09^2} = \underline{0.70}.$$

c) After the collision the sum of the kinetic energies of the two balls is $0.22 + 0.70 = 0.92$ of the initial kinetic energy. 8% of the energy must have gone into heat. Thus, the kinetic energy before the collision is $\underline{1.09}$ of the kinetic energy after the collision.

PROBLEM 26

A 3-kg ball with a speed of 6 m/sec strikes another 3-kg ball moving 6 m/sec in the opposite direction. After collision each ball has a velocity equal in magnitude but opposite in direction to its initial velocity. Is this an elastic collision?

This collision must have been elastic since the kinetic energy of each ball is the same after the collision as before. If any kinetic energy had been lost, the collision would have been inelastic.

PROBLEM 27

In one of the experiments that led to the determination of the mass of the neutron, Chadwick measured the velocity of protons that had been hit head-on by neutrons. The speed of the protons was 3.3×10^7 m/sec.

(a) What was the speed of the neutrons before and after collision with the protons?

(b) Chadwick also measured the speed of nitrogen atoms hit head-on by the neutrons. What was it?

(c) What was the velocity of the neutrons after each kind of collision?

a) This is the case of a head-on collision of one object with another of equal mass, initially at rest. The kinetic energy of the projectile is entirely transmitted to the target. The projectile is brought to rest. Analytically:

$$v_1 = \frac{m_1 + m_2}{2m_1} v_2' = v_2' = \underline{3.3 \times 10^7 \text{ m/sec}}.$$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \underline{0}.$$

b) $m_2 = 14 m_1$

$$v_2' = \frac{2 m_1}{m_1 + m_2} v_1 = \frac{2 m_1}{m_1 + 14 m_1} v_1 = \frac{2 \times 3.3 \times 10^7}{15} = \underline{4.4 \times 10^6 \text{ m/sec}}.$$

c) From part (a), after colliding head-on with a proton, the neutron's speed will be $\underline{0}$. After colliding head-on with nitrogen the neutron will have a velocity;

$$v_1' = \frac{m_1 - 14 m_1}{m_1 + 14 m_1} \times 3.3 \times 10^7 = \underline{-2.9 \times 10^7 \text{ m/sec}}.$$

(i.e., after colliding with nitrogen, the neutrons bounce backward.)

PROBLEM 28

An empty freight car *A* of mass 2.5×10^4 kg (25 tons) coasts along a horizontal track at 2.0 m/sec until it couples to a stationary car *B* of mass 5.0×10^4 kg. There is little friction, and the brakes are off.

(a) What is the initial momentum of car *A*?

(b) What is the speed of the two cars as they move along the track after the interaction?

(c) What is the total kinetic energy before and after the impact? Is this an example of an elastic collision?

$v_1 = 2.0$ m/sec, $m_1 = 2.5 \times 10^4$ kg.

$v_2 = 0$, $m_2 = 5.0 \times 10^4$ kg.

a) $p_1 = m_1 v_1 = \underline{5.0 \times 10^4 \text{ kg m/sec}}.$

b) $p' = p_1, \therefore (m_1 + m_2) v' = m_1 v_1$

$$v' = \frac{m_1 v_1}{m_1 + m_2} = \frac{5.0 \times 10^4}{7.5 \times 10^4} = \underline{0.67\ \text{m/sec.}}$$

c) E_k (before) $= \frac{1}{2} m_1 v_1^2 = \underline{5.0 \times 10^4 \text{ joules}}$.

E_k (after) $= \frac{1}{2}(m_1 + m_2) v'^2 = \underline{1.7 \times 10^4 \text{ joules}}$.

3.3×10^4 joules were transferred in the collision into other forms of energy, therefore the collision must have been inelastic.

PROBLEM 29

Rub the palms of your hands together briskly. Do you do any work? What happens to the energy?

Work is done when you rub your hands together because each arm exerts a force on its hand in the direction of motion. The energy expended by the muscles goes into heating the hands.

PROBLEM 30

I push a finger steadily across a glass table top so that it moves at a constant speed of 3 cm/sec under a 1-newton horizontal force.

(a) What work is done in 1 sec by the force my arm exerts on the finger? What energy does it transfer into the finger each second?

(b) How large and in what direction is the force of friction on my finger? What work is done by the force of friction on my finger each second? In what direction does it transfer energy — in or out of my finger?

(c) What is the net force on the finger? What work does the net force do per second on my moving finger? What happens to the kinetic energy of the finger?

(d) What is the force exerted horizontally on the table by my finger? How far does the table move? What work on the table does this force do each second? In which direction does it transfer energy — in or out of the table?

(e) What happens in the region where finger and table are in contact?

a) The finger is pushed across the table top 3 cms in 1 second, exerting a 1-newton horizontal force, so the work done (and the energy transferred) is 1 newton × 0.03 m = 0.03 joules per second.

b) Since the finger moves at constant speed, the frictional force on the finger is equal in magnitude but opposite in direction to the 1-newton force applied by the arm. Therefore work is done (and energy is transferred) out of the finger at exactly the same rate as in part (a), 0.03 joules per second.

c) The net force on the finger is zero because it moves at constant speed. The net force thus does no work, and the kinetic energy of the finger stays constant.

d) The force exerted by the finger on the table is the same as by the arm on the finger, but the table, as a whole, does not move. Consequently, work is not done by this force in the usual meaning, and the kinetic energy of the table as a whole does not increase. The energy transferred out of the finger, as noticed in part (b), therefore must go into something else. It goes into the region where the finger and the table are in contact and

e) we find heat there.

Note: The point to be emphasized in this problem is that work is done by the arm in pushing the finger along the table. No change in kinetic energy occurs. What has happened to the energy that has been transferred from the arm by the mechanical work it does? The only other change which occurs in the objects with which the problem is concerned is the heating of finger and table. The student is then led to suspect that this is another form of energy which must eventually be fitted into the over all picture. This is analogous to the implicit suggestion of the idea of potential energy in the problems of elastic interactions within which kinetic energy is found not to be conserved.

There are some difficult points in the complete understanding of this problem. The above discussion should suffice, but students may be puzzled by the intricacies of frictional forces. While some of these details are discussed below, it is to be hoped that you can avoid them.

The finger gets hot. Yet in the answers to (a) and (b) it is stated that energy is being transferred out of the finger as fast as it is being transferred in. This seems to be inconsistent with the principle of energy conservation. The explanation lies in the fact that frictional forces are very complicated when viewed in detail. The interacting spots of the surfaces are not rigidly attached to the bodies that are sliding over one another. Although, as stated in Section 9 of the text, that the work done at each interacting spot on the center of mass will be

$$W_a = F_{ax}x, \quad W_b = F_{bx}x \ldots,$$

this is not necessarily the complete story. The trouble is that the force components F_{ax}, F_{bx}, etc. do not usually act on the same point of the body and relative motions of the points of application are possible. Thus while the body as a whole may move a distance, x, the points of application of the many small contributing forces which total up to the "frictional force" may move through quite different distances.

Consider a "simple" macroscopic example. A violinist draws his bow across a violin string. In doing this he moves it through a distance L, applying a force F_v. He has then done an amount of work, $W_v = F_vL$. The motion is opposed by the frictional force associated with the bow sliding across the string. This force must be equal and opposite to F_v, since the bow does not accelerate. However this frictional force is really the time-average of a varying force between bow and string. In a simple model, the bow can be thought of as pushing the string forward until the elastic restoring force of the string exceeds the force of static friction of string on bow. The string then starts to slide backwards while the bow continues to move forwards. The relationship between force × distance and force × time becomes very complicated. It is no longer obvious that the sum of the small contributing local (F_iX_i) will be the same as F_vx.

In the case of the finger on the table, this identity must break down locally, (in the region of the contact), because energy in the form of heat accumulates in the finger. Then, since mechanical work is the only important mode of energy transfer in this problem, the negative work done by the frictional force on the finger in the region of the interaction must be less than the positive work done by the arm in pushing the finger along.

Actually, since work is a scalar and force a vector, the total work done by a number of forces is best thought of as the sum of the individual works, not as the vector sum of the forces × the displacement distance. The latter is satisfactory when you are interested only in center of mass motion and kinetic energy, but consider the example of a coil spring held between the jaws of a pair of pliers. Squeeze the pliers. A force +F acts at one end of the spring through a distance +x. A force, -F, acts on the other end of the spring pushing it through a displacement -x. The net force acting on the spring is zero. Therefore the acceleration is zero and the change in bulk kinetic energy is zero. Yet the work done on the spring is

$$(+F)(+x) + (-F)(-x) = 2Fx.$$

It is not desirable to go into details of the microscopic picture at this stage of the development. Heat will be studied microscopically in Chapter 7. At that time, this problem might be discussed again.

PROBLEM 31

A 100-gm light bulb dropped from a high tower reaches a velocity of 20 m/sec after falling 100 m. About how much energy has been transferred to the air?

The downward gravitational force does an amount of work equal to the force × the distance the bulb drops, 0.1 kg × 9.8 nt/kg × 100 m. equals 98 joules. However, after the bulb has fallen 100 m, its kinetic energy is

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1\,\text{kg} \times (20\,\text{m/sec})^2 = 20\ \text{joules}.$$

The difference between the work done on the bulb and its kinetic energy represents work done by the bulb against the force of air friction. It amounts to 78 joules and goes into warming the air and the bulb.

PROBLEM 32

A force of 3.0 newtons and a force of 4.0 newtons are applied simultaneously to a 4.0-kg mass initially at rest. The two forces act at an angle of 90 degrees with each other for 2.0 sec.

(a) What is the net force?

(b) Use the net force to calculate the total work done.

(c) What is the work done by each force separately? How does the sum of these works compare with the answer to (b)?

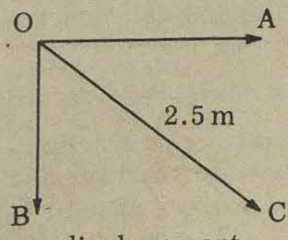
a) The net force is 5.0 newtons at an angle of 37° measured from the direction of the 4.0-newton force toward the 3.0-newton force. (The vector triangle is a 3-4-5 right triangle.)

b) The acceleration of the 4.0-kg mass is $1.25\,\text{m/sec}^2$. During the 2.0 seconds that the forces act, the mass goes a distance:

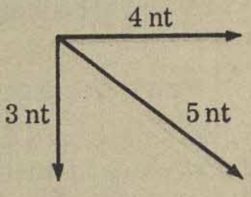
$$s = \frac{1}{2}at^2 = \frac{1.25 \times 4}{2} = 2.5\,\text{m}.$$

The work done by the net force is 5.0 newtons × 2.5 meters = 12.5 joules.

c) The vector diagram of displacements is similar to that of the forces which act.



displacement



Thus, when the mass moves 2.5 meters from O to C, its component in the direction (OA) of the 4.0-newton force is 4/5 of OC. Similarly, the mass moves 3/5 of 2.5 meters in the direction OB, the direction of the 3.0-newton force. Therefore the work done by the 4.0 and 3.0-newton forces is:

$$W_4 = 4.0\text{nt} \times \frac{4}{5} \times 2.5\,\text{m} = 8\ \text{joules}.$$

$$W_3 = 3.0\text{nt} \times \frac{3}{5} \times 2.5\,\text{m} = 4.5\ \text{joules}.$$

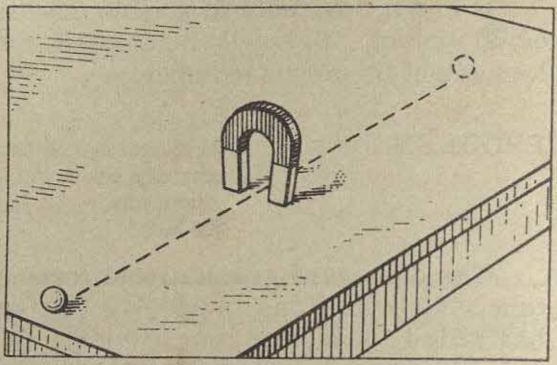
$W_3 + W_4 = 8 + 4.5 = 12.5$ joules, the same amount of work that is accomplished by the net force.

PROBLEM 33

A horseshoe magnet of mass *m* stands on end on a frictionless table. A steel ball bearing of mass *m* is rolled toward the magnet from far away with velocity *v* and goes through the magnet and far beyond. Assume the force of attraction *F* changes with distance and is the same in front and behind the magnet. (See Fig. 24–20.)

(a) What is the final velocity of the ball?

(b) What is the final velocity of the magnet?



This problem involves an elastic collision, although the students may not recognize it as a collision. Furthermore, this collision is novel in that the forces are attractive. It is only because the ball can scoot between the legs of the magnet that the ball can go away from the collision instead of sticking to the magnet. Why is it elastic? Because the force between the two objects depends only on the separation distance.

Both the magnet and the ball are acted on by the magnetic force, so both are accelerated toward each other; the magnet accelerates from rest, and the ball accelerates from its initial velocity. As the ball and magnet move closer together, the force (and the accelerations) increase steadily until the ball passes through the magnet. The force then rapidly goes through zero and reverses sign. As the ball recedes from the magnet, the force on both ball and magnet is a decelerating force. At every value of separation distance, the decelerating force is equal in magnitude but opposite in direction to the accelerating force which acted when the ball and magnet approached each other. At a very large separation distance the magnet will have been brought to rest. To conserve momentum and energy, the ball must then have been slowed to its initial velocity.

* * *

Note. In the course of this interaction, as the ball starts from point A and goes through the magnet to point B, the magnet is first accelerated toward A and then brought to rest. During this process, in accordance with conservation of momentum, the ball must have been accelerated toward B to a speed greater than its initial speed, v_o, and then decelerated until, far away from the magnet its speed again was v_o. Altogether, the effect of the magnet is to decrease the time it takes the ball to get from A to B. This is done by using the magnet as "rocket fuel" of a temporary nature. The ball thrusts the magnet back and thus gains speed. Then it "reaches back" and stops the magnet, thus losing its increment of speed. The center of mass motion is constant throughout – the backward magnet momentum being counterbalanced by the extra forward momentum of the ball.

Kinetic energy is conserved over all; i.e., initial E_k = final E_k, but kinetic energy is not conserved during the interaction. In this problem (attractive force) the kinetic energy is greatest when the ball is passing under the magnet, since at that time both ball and magnet are traveling at their maximum speeds. Students may wonder how energy is conserved. The answer, of course, is that the "magnetic spring" has been allowed to contract. There is energy stored in a magnetic field and this stored energy is less, when the ball is near the magnet, than it is when the ball is far away. The energy stored in the magnetic field is transformed into kinetic energy as the ball approaches the magnet, and as the ball pulls away from the magnet the extra kinetic energy thus gained is transformed back into energy stored in the magnetic field. The energy stored in the magnetic field is a form of potential energy. Potential energy is the subject of the next chapter. You might ask students what happens to the magnet. Is it in the same place at the end of the interaction as it was at the beginning?

PROBLEM 34

A two-stage rocket is moving with velocity v in a region where gravity is negligible. It consists of a final stage, or head, of mass m, and a tail of mass $2m$ which includes an explosive charge to separate the head. The charge is carefully designed so that after separation the tail has the least possible energy. (Assume that the gases from the explosion stay with the tail.)

(a) What is the least possible energy the tail can have?

(b) In this case, how much energy has the head got after the explosion? Give the answer in terms of m and v.

(c) How much kinetic energy did the *head* have before the explosion?

(d) How much energy has the head gained?

(e) How much energy is supplied by the fuel in the explosion?

(f) Why is the answer to (e) different from the answer to (d)?

a) Since kinetic energy is always a positive quantity, its least possible value is zero.

b) After the explosion, by conservation of momentum the initial momentum of the whole rocket, $p = 3\,mv$, is then equal to the final momentum of the head, $p' = mv'$. Solving this equality for v, we find:

$$p = p'$$

$$3\,mv = mv'$$

$$v' = 3\,v.$$

The kinetic energy of the head after the explosion is:

$$E_k = \frac{1}{2} \times m \times (3\,v)^2 = \underline{4.5\,mv^2}.$$

c) Before the explosion, the head had a kinetic energy of $\underline{\frac{1}{2} mv^2}$.

d) The head has gained $\underline{4\,mv^2}$ of kinetic energy.

e) In the explosion process, the tail lost mv^2 of kinetic energy, while the head gained $4\,mv^2$. The difference of $\underline{3\,mv^2}$ had to be supplied by the explosive charge.

f) The difference between the answer to (e) and the answer to (d) lies in the kinetic energy transferred from the tail of the rocket to the head during the explosion.

Chapter 25 – Potential Energy

CHAPTER SUMMARY

In this chapter, the concept of energy is broadened to include potential energy. The exchange of energy between the two forms, kinetic and potential, is studied; the law of conservation of mechanical energy for "conservative" forces is formulated.

Sections 1 and 2 The elastic collision is reexamined, noting that the kinetic energy which "disappears" during the interaction may be thought of as stored. An expression for the potential energy of a stretched or compressed spring is developed. These new ideas are generalized to an arbitrary two-body interaction.

Sections 3 through 5 The results of Sections 1 and 2 are extended in a consideration of the potential energy in a gravitational field. The potential energies of bodies near the earth's surface are considered. The criteria for "free" and "bound" states under a gravitational attraction are developed.

Section 6 The earlier conclusions are generalized to show that the total mechanical energy is conserved in many complex systems.

SCHEDULING CHAPTER 25

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part III.

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2	3	0	-	2	0	-
Secs. 3, 4, 5	3	2	III-12 III-13	2	1	III-12
Sec. 6	1	0	-	0	0	-

RELATED MATERIALS FOR CHAPTER 25

Laboratory. Experiment III-12, Changes in Potential Energy, employs a mass suspended on a coil spring to examine the transfer of energy from gravitational potential energy to the potential energy of a stretched spring. This experiment is best done after Section 3 is discussed.

Experiment III-13, The Energy of a Simple Pendulum, uses a simple pendulum and the ticker-tape timer to provide data for the kinetic and potential energies at various points in the pendulum's swing. Adding the curves for potential and kinetic energy shows the near-constancy of the total energy in the system. This experiment is of slightly lower priority than III-12. It can be done after Section 3 is discussed.

See the yellow pages for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Problems 23 and 24 might be assigned as a pair.

Section	Easy	Medium	Hard	Class Discussion
1	2, 12*	1, 3*, 5, 6, 7, 8*, 10	4, 11	4, 9, 10
2		13	14	13
3	15, 19, 20	17	16*, 18, 19	15, 16, 19, 20
4	23	22, 25, 27	21, 24	23
5	26			26

Film. If not used earlier, "Work and Energy", which was described in the summary of Chapter 24, can be used in the course of this chapter.

Section 1 — The Spring Bumper

PURPOSE To introduce the concept of potential energy and its use; to set the stage for cases of more general forces by treating in detail a familiar example, the spring.

CONTENT a. Mechanical energy is conserved whenever the force between two bodies depends only on their separation.

b. In a good (elastic) spring, the force required to hold the spring compressed a given amount is always the same, independent of the manner in which the spring is compressed or of the number of times it is compressed. When this is the case, both the work required to compress the spring and the energy which can be realized when the spring is released are also always the same and independent of how the spring reached its given compression.

c. The potential energy of a spring, U, is given by the area under the curve of force versus compression.

d. $\frac{1}{2}mv^2 + U = E$ = constant in an elastic interaction.

e. When the force is proportional to the displacement, $F = kx$, the potential energy is $U = \frac{1}{2}kx^2$, and $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$. This relation enables us to solve for the velocity at any point in an elastic interaction.

EMPHASIS An understanding of this section is basic to comprehension of the rest of the chapter.

COMMENTS The details of the non-elastic spring should not be stressed but it will be wise to make sure that students understand what property is crucial. For the spring to be elastic it must be able to do as much work when released as was done on the spring in compressing it. Students should know that they can compute the potential energy of a spring from the area under the force-compression curve. They are likely to memorize $U = 1/2\,kx^2$ automatically, but the important thing is to understand where it comes from. The key objective is to attain a familiarity with energy conservation and transfer. This can be gained through discussing a number of examples in which the form of the energy (kinetic or potential) is followed, and the signs of the transfer recognized. In this section try to avoid the abstraction of gravitational potential energy or that of any other "action at a distance" force field.

* * *

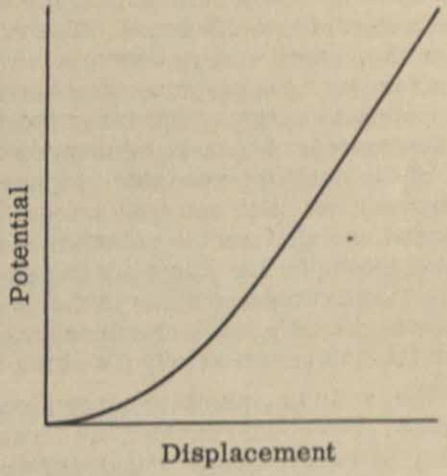
You can discuss many different kinds of interaction between a mass and a spring. For example, let the mass strike the spring, but catch the spring with a ratchet device at the moment of maximum compression. Then the mass stays at rest. If you slowly release the spring, the potential energy of the spring does work, transferring energy to your hand. (How does the work your hand does in releasing the spring compare with the original kinetic energy of the mass? It is exactly the negative since you "absorb" the potential energy of the spring; it is minus, because in releasing the spring, your hand moves in the opposite direction from that of the force it exerts on the spring.)

You can let one mass strike and compress the spring, and then after you have caught the spring as before, substitute a different mass. Follow the acceleration of this second mass in detail and show that the final kinetic energy is still the same.

Another variation: Suppose the mass hits the spring and rebounds, but leaves the spring vibrating. How does the kinetic energy of the mass compare with its value before the collision? It is less by just the energy left in the spring. You might ask the class whether the energy left in the spring is potential or kinetic. [It goes back and forth between the two forms, just as the energy of a mass oscillating on a massless spring goes back and forth from kinetic energy (when the spring is unstretched and the mass is going its fastest) to potential energy (when the mass is momentarily at rest and the spring is either compressed or extended the maximum amount).]

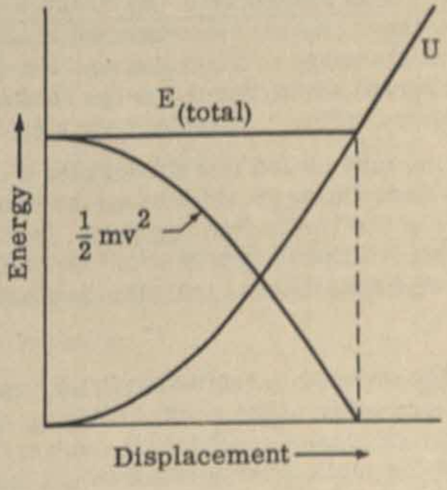
* * *

Figure 25-6 may warrant a little discussion. The graph is a plot of the potential energy of a rather peculiar spring as a function of compression. You might ask the class to consider the same type of plot for the ideal spring whose F vs. x graph is shown in Figure 25-5. For this spring, we have $F = kx$ and $U = 1/2\,kx^2$ as shown. Thus the potential energy graph would resemble Figure 25-4 as shown at the right.



If we wish to represent all energies involved in the motion of a mass, m, as it compresses this spring, we can plot the three curves together on this graph. The potential energy curve will be the same as that plotted above. The total energy, $E = 1/2\,mv^2 + U$, must be a constant as the spring is elastic. When $U = 0$ (at $x = 0$), $E = 1/2\,mv^2$. Thus the horizontal straight line which represents E is drawn at the level of the value of the kinetic energy when the compression is zero. The kinetic energy is then given at each point by the difference between the horizontal line depicting E and the curve depicting U, $1/2\,mv^2 = E - U$.

Since a negative value of $1/2\,mv^2$ has no meaning, we draw the line representing E only out to the point at which it intersects U. At that point, $1/2\,mv^2 = 0$, and therefore $v = 0$.



Section 2 – Potential Energy of Two Interacting Bodies

PURPOSE To generalize the previous section; to deal with various kinds of forces in preparation for gravitational potential energy.

CONTENT a. With forces which depend only on separation, the kinetic energy which "disappears" during the course of a collision is stored as potential energy and can be recovered.

b. The potential energy depends only on the separation and not on the details of the collision.

c. Potential energy is stored "in the force field" – the imaginary spring.

EMPHASIS This section should be developed with strong reference back to Section 1. It may be useful to make some reference ahead to Sections 3 and 4, since gravitational forces are the only specific examples of an action-at-a-distance force that have been considered in detail thus far.

COMMENTS Consideration is given to the potential energy of a system of a pair of bodies which interact at a distance. The mathematical aspects were worked out in Chapter 24, where the kinetic energy during a collision was followed in detail. For action-at-a-distance forces, in apparent contrast with the spring, there is no visible storage place for the potential energy. This may trouble some students. All that is required for introducing the concept of potential energy is the fact that some kinetic energy disappears at one part of the collision and later reappears. One might choose to say "the kinetic energy is destroyed, but later an equal amount is created" or "the kinetic energy is converted to potential energy, and the potential energy is then reconverted to kinetic energy". The second viewpoint has some advantage in that it emphasizes the fact that the total work which the particles could do is the same at any time during the collision, whether we let it appear directly as kinetic energy, or whether we reach in, grab the particles, and make them separate slowly (in which case the energy appears as work against us).

Where is the potential energy located? For the collision of Section 1 we say "in the spring". We could consider the force between two bodies which interact at a distance as having its source in an "imaginary spring" as suggested in the text. Actually, there is no need to "locate" the potential energy, no matter how natural that desire may be. All that is necessary for the concept of potential energy to be useful is that we be able to use it to predict the work we can realize (or must supply) when bodies go from one separation to another. In fact, the separation of the bodies gives us as clear an indication of the potential energy of a system as does the extension of a spring. As explained in the text, even a real spring must, on the atomic level, be understood in terms of action-at-a-distance forces, "imaginary springs"

One may be led into difficulties if he tries too hard to locate the potential energy, perhaps associating it with one or the other of the two bodies. It should be considered a property of the two bodies together, as a single system. The work needed to bring up B while holding A fixed is identical to that of bringing up A while holding B fixed, assuming only that we bring them to the same final separation in the two cases.

* * *

The several paragraphs (Col. 2, page 411) beginning with "To understand more fully. . . ", give a preview of the next chapter on heat and kinetic theory, and begin to give an atomic picture of loss of mechanical energy. The statements are quite brief, indicating that this is not the main treatment. Such a treatment will come in Chapter 26, Section 3. The statements in this chapter should be treated simply as indicative of details that will be carried further.

Section 3 — Gravitational Potential Energy Near the Surface of the Earth
Section 4 — Gravitational Potential Energy in General
Section 5 — Escape Energy, Escape Velocity, and Binding Energy of Satellites

PURPOSE To apply the energy concept to man's most familiar force field — the gravitational force.

CONTENT a. When a mass, m, moves in the gravitational field of the earth, the accompanying motion of the earth may be ignored if m is small compared to the mass of the earth.

b. Near the earth's surface, when changes in height are small compared to the radial distance from the center of the earth, the potential energy of position is mgd, where d is the vertical displacement of the mass, m, from some arbitrary level of zero potential energy.

c. When changes in displacement are large relative to radial distance from the earth's center, the potential energy of a mass, m, located a distance, r, from the earth's center is $U(r) = U(\infty) - \frac{GMm}{r}$, where $U(\infty)$ is the arbitrary value (usually taken to be zero) assigned to the potential energy at infinite distance.

d. The concept of potential energy in a gravitational field is applied to motion of a satellite, enabling us to compare the kinetic energy required for escape with that required for orbit, and introducing the idea of binding energy. The total energy, $E = 1/2\ mv^2 + U$, remains constant.

EMPHASIS These sections should be treated thoroughly not only because of the interest inherent in the problem itself, but also because familiarity with the potential energy in an inverse square force field will again be required in Part IV in the study of the Coulomb field and the nuclear atom.

DEVELOPMENT The starting point for the development is the kinematic relationship, $v^2 \infty 2gd$ for falling objects at the earth's surface. This was derived in Chapter 5. Students should be reminded that g is not exactly constant. It is only constant insofar as the gravitational force is constant over the range of displacement, and this is true only if the distance, d, is negligible compared to the radius of the earth. Multiplying both sides of the above by m/2 gives the kinetic energy of a falling object in terms of the height through which it has fallen. This motivates the assignment of a potential energy to a mass in the earth's gravitational field. This must be done in such a way that a change in height equal to d gives a change in potential energy equal to mgd. The equation $U(h) = U(h_o) + mg(h - h_o)$ satisfies this requirement. One can regard the arbitrary constant $U(h_o)$ as a means for simplifying the potential energy term in any particular problem. One frequently chooses $U(h_o) = 0$ where h_o is the origin of the height-coordinate in the problem in question. The choice of $U(h_o)$ does not affect the physics of the problem, since we always deal with changes of potential energy.

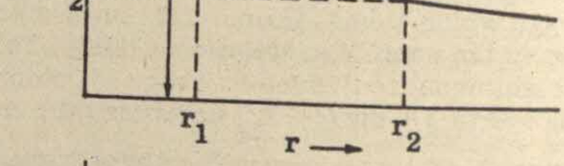
The next step is the recasting of the theme of Section 24-4 (work and kinetic energy) in terms of potential energy and kinetic energy. The gain in kinetic energy, as an object falls, is equal to the decrease in potential energy. The summary of experimental observations which thus far has been couched in the language of Newton's law can now be restated as the conservation of mechanical energy. The law applies to the total system, earth plus mass m.

One should not lose sight of the fact that potential energy applies to the system of interacting objects. At the outset it is proved that, in most of the situations involving the motion of a mass in the field of the earth, the kinetic energy of the earth can be neglected. For this reason one is apt to think of the mass as having the potential energy, but actually the potential energy belongs to the system rather than separately to either of the partners of the system. One may think of potential energy in a gravitational field as residing in the stretch of an imaginary gravitational spring. One end of this "spring" is fastened to the earth. If one loses sight of the fact that potential energy resides in the two-body system, he can get into such trouble as concluding that when a man raises an object from the floor to a table no work is done on the object. There are two forces acting on the object; a man's pull and the pull of gravity. These are equal; there is no acceleration, and the net work done on the object is zero (i.e., there is no net change in the kinetic energy of the object). Work has been done, and energy has been transferred from the man to the system of the earth and object. The energy transferred can best be viewed as the result of the combined push of the feet on the floor and pull of the hand on the object. These, together, stretch the "gravitational spring" and thus store energy in it.

* * *

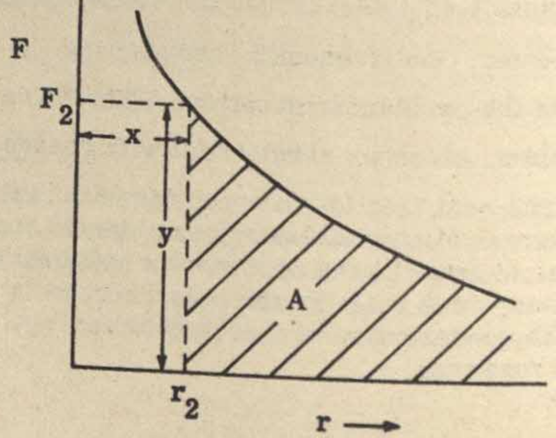
When the displacement of an object is large compared to the radial distance to the center of the earth it is necessary to take proper account of the variation of force over the range of the displacement. We know the gravitational force to be of the form $F = \frac{GMm}{r^2}$. We know that over small displacements (constant force) the gravitational potential energy of an object can be approximated as $U(h) = U(h_o) + mg(h - h_o)$. What is an exact expression for the potential energy, good for any radial distance r (greater than the earth's radius)? This problem can be solved exactly, using integral calculus. The text verifies the validity of the chosen potential energy function by showing that the inverse square force function can be derived from it.

Another useful approach is the use of the technique of scaling. Consider a graph of force vs. radial distance. F_1 is the attractive force at a distance r_1, and F_2 is the force at a distance r_2. Let the ordinate distance corresponding to F_1 newtons be y meters, and let the abscissa distance corresponding to r_1 meters be x meters.



Then the area, xy meters^2, corresponds to F_1r_1 joules.

Now draw a second graph to such a scale that the force F_2 is represented by y meters on the ordinate scale and the distance r_2 is represented by x meters on the abscissa scale. Such a graph is shown on the right.



On this second graph, the work required to move the mass from $r = r_2$ to $r = \infty$ is represented by the shaded area, A;

$$W_2 = \frac{A}{xy} F_2 r_2 \text{ joules.}$$

On the first graph, the work required to carry the same mass from $r = r_1$ to $r = -\infty$ is represented by the area to the right of r_1.

As a result of our scaling procedure this area is also A m^2 and therefore $W_1 = \frac{A}{xy}F_1r_1$. The difference in potential energy of the system with mass at $r = \infty$ and with it at $r = r_1$ is equal to the work done in carrying it from $r = r_1$ to $r = \infty$. Thus:

$$U(\infty) - U(r_1) = W_1 = \frac{A}{xy}F_1r_1.$$

Similarly, $$U(\infty) - U(r_2) = W_2 = \frac{A}{xy}F_2r_2, \text{ and}$$

$$\frac{U(\infty) - U(r_1)}{U(\infty) - U(r_2)} = \frac{W_1}{W_2} = \frac{F_1r_1}{F_2r_2} = \frac{\frac{GmM}{r_1^2}r_1}{\frac{GmM}{r_2^2}r_2} = \frac{\frac{1}{r_1}}{\frac{1}{r_2}},$$

which proves that $U_\infty - U_r$ is proportional to $1/r$.

To find the constant of proportionality, the method used in the text can be employed. A slightly different wording may be used to bring out at the same time the connection between this exact expression for potential energy and the approximate one that we use at the earth's surface when we ignore the change in gravitational force with distance from the center of the earth.

The increase in potential energy when an object of mass, m, moves from the surface of the earth ($r = r_o$) to a height, h, above the surface of the earth ($r_1 = r_o + h$) is:

$$U(r_1) - U(r_o) = \left(U(\infty) - \frac{C}{r_1}\right) - \left(U(\infty) - \frac{C}{r_o}\right) = \frac{C}{r_o} - \frac{C}{r_o + h} = \frac{Ch}{r_o(r_o + h)}.$$

In Section 25-3 we found:

$$U(r_1) - U(r_o) = mgh = \frac{GMm}{r_o^2}h.$$

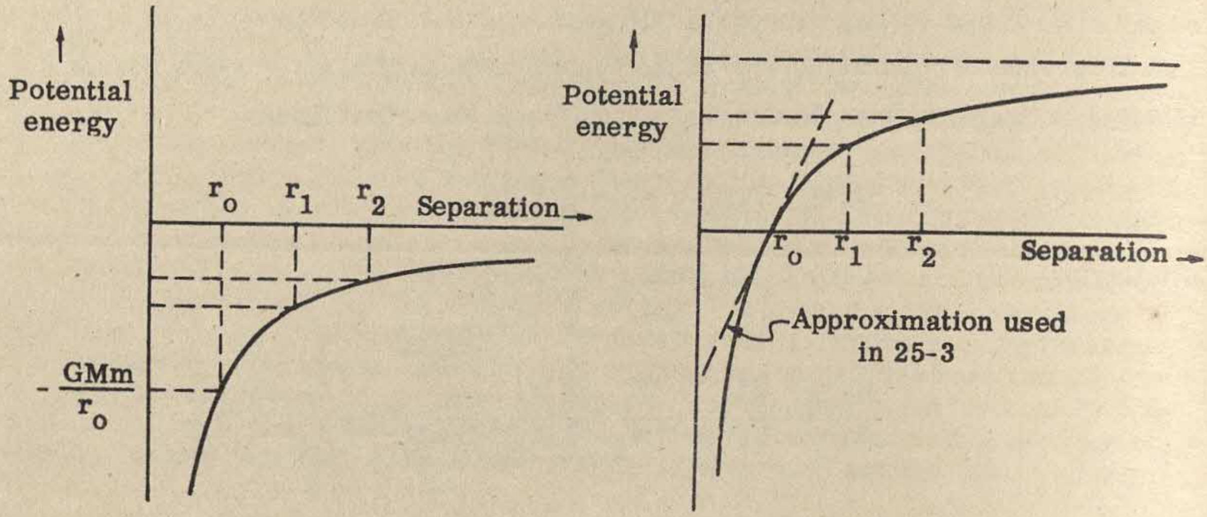
We see that the two results agree provided:

(1) h is very small compared to the radius of the earth r_o so that $r_o^2 + r_oh \approx r_o^2$

(2) C has the value GMm.

The first requirement comes in because in Section 25-3 we ignored the variation of the force $\frac{GMm}{r^2}$ with r and used the approximation $\frac{GMm}{r^2} \rightarrow \frac{GMm}{r_o^2} = mg$.

It cannot be emphasized too strongly that changing the potential energy by either adding or subtracting a constant makes no difference in any physical result. For example, in Section 25-4 the choice $U(\infty) = 0$ is made, while in Section 25-3, the choice $U(r_o) = 0$ was made. The latter is equivalent to $U(\infty) = +\frac{GMm}{r_o}$. It may be worthwhile to graph the two choices side by side and show that the shift between the two figures does not matter.



The increase in potential energy from r_1 to r_2 is the same in both cases. We note incidentally that in the approximation made in Section 25-3 we replaced the curve $\left[0 = \frac{GMm}{r_o} - \frac{GMm}{r}\right]$ with the tangent line $\left[U = \frac{GMm}{r_o^2}(r - r_o)\right]$

For an extension of this discussion of the relationship between potential energy and fields of force, see Appendix 8 to the Guide for this volume. There the case of a compressed spring is discussed.

COMMENTS In using the energy concept we ignore many of the details of the motion, since we do not concern ourselves with interaction forces and accelerations of various parts of the system with respect to one another. However, in ignoring these details we gain in simplicity and in our understanding of fundamentals. The use of the conservation laws is often called an integral approach to problems of mechanics. It relates final conditions to initial conditions independent of the interaction that is involved. Thus, for example, with no precise knowledge of the nuclear force law, we can still make a great many exact deductions about the effects of a given interaction.

* * *

When the horizontal component of velocity is constant, as in projectile motion, it is a great temptation to think of the energy associated with one component as distinct from another. For example, the total energy is:

$$E = E_k + U_h = \frac{1}{2}mv^2 + mgy = \frac{1}{2}m(v_x^2 + v_y^2) + mgy.$$

In projectile motion v_x is constant, so we may write

$$\frac{1}{2}mv_y^2 + mgy = E - \frac{1}{2}mv_x^2 = E',$$

where E' is a new constant. The maximum height to which the projectile rises is found by noting that at the top of the flight $v_y = 0$, so that $y_{max} = E'/mg = (E - 1/2\ mv_x^2)/mg$.

It is correct to speak of the contribution of the horizontal component of the velocity to the kinetic energy but it is incorrect to think of this part of the energy as the "horizontal component of energy". Energy is a scalar and has no associated direction. It is conserved in toto but the "parts" associated with horizontal or vertical components of the motion are not separately conserved. To see this, consider the example of a ball rolling from rest down an inclined plane. Although the initial useful potential energy is all due to the vertical separation of ball and earth, $\frac{1}{2}mv_x^2$ is not a constant.

* * *

To appreciate the power of the information obtained from the conservation of energy, one should ask the question: Does the escape velocity depend on the direction in which the rocket is fired? Using the principle of conservation of energy we see immediately that it does not for the case of a gravitational interaction. Had we attempted to answer the question by using Newton's law directly, the problem would have been much harder to solve. We would have found eventually that three types of orbits are possible: elliptical (including circular as a special case), parabolic, and hyperbolic. (Straight line motion is a special case of parabolic or hyperbolic motion.) Elliptical orbits correspond to rockets that do not escape (satellites), parabolic orbits to rockets that just escape, and hyperbolic orbits to rockets that escape with energy to spare. The energy argument completely bypasses a fairly involved discussion. Naturally, we do not have quite as much information if we use conservation of energy alone; we know the speed of the satellite at large separation, but we know nothing about the actual orbit. [In the above discussion, as in the text, the escape velocities referred to are relative to the center of the earth, not relative to the surface of the earth. We can take advantage of the eastward motion of the rocket launching path (≈ 0.5 km/sec at the equator) to save fuel.]

The escape energy calculated in the text gives the kinetic energy that a rocket must have (outside the atmosphere) to escape. The fuel requirements are of course, much larger; in a typical rocket most of the work done by the rocket engine ends up as kinetic energy of the exhaust. If this point comes up, Problem 27 can be used for illustration.

* * *

The concept of binding energy is one which is often used in discussing atoms (Section 34-6). The example given here, in which we calculate the energy which has to be added to a bound satellite moving in a circular orbit around the sun has a very close counterpart in the hydrogen atom, in which an electron moves in a circular orbit around a proton.

* * *

Appendix 9 of the Guide for this volume gives the calculations of the escape velocities and binding energies that are mentioned in Section 4 of the text.

Section 6 – Conservation of Mechanical Energy

PURPOSE To point out that the concepts of potential and kinetic energy can be extended to more complex mechanical systems, including those in which we do not have detailed knowledge of the nature of the interaction forces. Furthermore, we believe that mechanical energy is always conserved for systems in which other forms of energy play no part.

CONTENT a. Conservation of energy is discussed in terms of a simple toy that nevertheless is a more complex system than any previously considered.

b. The plausibility of general conservation of mechanical energy is discussed.

EMPHASIS This reaction is a summary and extension of the ideas that have already been presented. A brief treatment will suffice. The energy concept will be extended further in the next chapter where heat will be considered.

DEVELOPMENT The first paragraph of this section defines the interaction force between two masses as Newtonian provided: (1) that the masses can be regarded as points, (2) that the force on one mass is equal and opposite to the force on the other, and (3) that the direction of the force is along the line joining the two mass points.

The question is then asked: Can the ideas of kinetic and potential energies be used in systems in which the mass cannot be regarded as being concentrated at a point and/or the forces of interaction are not Newtonian? An example of a complex system (the toy merry-go-round) is then presented. A plausibility argument shows that the conservation of mechanical energy (except for frictional effects) still seems to be a valid concept.

You can point out to students that the descending mass exerts a Newtonian force on the string and that this force can be considered to be transmitted undiminished around the pulley to the surface of the shaft. But the mass of the merry-go-round does not appear to be concentrated at the axis of rotation because the inertial properties of a rotating mass depend markedly on where the mass is located relative to the axis. Students probably have had some experience with this effect (a whirling ice skater speeding up as he brings his arms in close to the axis of rotation, the heavy wheel in gyroscope and momentum toys) but they probably have not had an opportunity to compare the rotational inertial effects of two wheels of the same mass but with different mass distributions.

To compare two wheels that have the same mass you can make the following simple calculation. Consider a bicycle wheel which is idealized as having all its mass at the rim. Replace the merry-go-round with this wheel, and now suppose it is turning N times a second at a certain instant. The mass M of the wheel is distributed uniformly around the rim at a distance of R meters from the axis of rotation. The speed of a point on the rim is then $2\pi R \times N$ m/sec because in one second a point will have gone N times around the circumference, that is, $2\pi R$ meters in one turn or $2\pi R N$ meters each second. Now consider the mass of the rim to be made up of 1000 pieces (or any number you wish). Then the mass of each piece is M/1000, and each piece is so small that it is practically a point mass. Then the kinetic energy of each piece is

$$\frac{1}{2}(\text{mass})(\text{speed})^2 = \frac{1}{2}\frac{M}{1000}(2\pi R N)^2 .$$

Since kinetic energy is a scalar quantity, the total kinetic energy of the wheel is the sum of the energy of each piece. These energies are all equal, and there are 1000 pieces so that the total kinetic energy of the wheel is

$$\frac{1}{2}M(2\pi R N)^2 \text{ or } \frac{1}{2}MR^2(2\pi N)^2 .$$

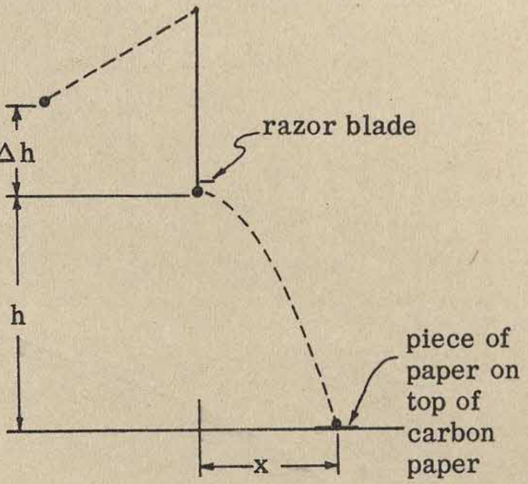
This expression shows that two wheels each of the same mass M and each turning at N revolutions per second can have markedly different kinetic energies depending on whether the mass is close to or far from the axis of rotation.

When the mass has descended a given distance, the gravitational interaction force has been allowed to do work on the system (the potential energy of the system has therefore decreased), and the kinetic energy has increased to a given value. This value is the same whenever the descending mass has fallen through the same vertical distance from rest. A given kinetic energy of rotation of the wheel can be realized in two contrasting ways. If R of the wheel is large, then N must be small; and if R is small, then N must be large. But N is the rate that the wheel is turning, and so it is a direct measure (via the shaft radius r) of the speed of the falling mass; the speed is $2\pi R N$ m/sec. This means that after the mass has fallen a given distance it will be moving more or less rapidly according to whether R is small or large. Thus a wheel of given mass will accelerate more slowly if R is large than it will if R is small. A wheel with large R is harder to start turning than one of the same mass with small R.

DEMONSTRATION An interesting experiment that you can do as a classroom demonstration shows nicely the conversion of potential energy into kinetic energy and involves some rather nice, simple techniques. This demonstration is a variation in technique of Experiment III-13, The Energy of a Simple Pendulum. If students do this experiment, this demonstration either should not be done, or done after the laboratory experiment as the basis for a review of the ideas of the chapter.

A measurable amount of potential energy is stored in a pendulum bob by swinging it a known distance up from its equilibrium position. The bob is released. At the bottom of its swing the original increment of potential energy will have been completely converted into kinetic energy, $\frac{1}{2}mv^2$. The motion is entirely horizontal. If we could measure this horizontal speed, we could calculate the kinetic energy and check the energy balance. In order to measure this speed, the bob can be hung from a light thread, and a razor blade can be carefully and rigidly mounted so as to cut the thread just when it reaches the vertical. Then the speed can be calculated by measuring the horizontal distance traveled by the bob before hitting the floor. This distance must be divided by the time of flight which is determined, by calculation, from the known height of the bob above the floor when in the equilibrium position.

$$U = mg\Delta h$$
$$E_k = \frac{1}{2}mv^2$$
$$v = \frac{x}{t}$$
$$h = \frac{1}{2}gt^2,$$
$$\therefore \quad t = \frac{2h}{g}.$$



It is advisable to try this out before doing it in class. The main source of error lies in the energy that is dissipated in cutting the thread. Be sure that the razor blade is rigidly mounted and use as heavy a bob as possible. In class you might make several measurements using different values of Δh.

Chapter 25 – Potential Energy

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

The assignment of Problem 24 might be accompanied by the assignment of Problem 23.

Section	Easy	Medium	Hard	Class Discussion
1	2, 12*	1, 3*, 5, 6, 7, 8*, 10	4, 11	4, 9, 10
2		13	14	13
3	15, 19, 20	17	16*, 18, 19	15, 16, 19, 20
4	23	22, 25, 27	21, 24	23
5	26			26

SHORT ANSWERS

1. (a) Yes. Force of interaction depends only on separation distance.
   (b) 18 joules.
   (c) 0.15 m.
   (d) 1/2.

2. (a) See graph on page 25-15.
   (b) 0.5 joule, 5.5 joules
   (c) See discussion on page 25-15.

3. (a) 0.5 joule.
   (b) 0.5 joule.
   (c) 0.3 joule.

4. See discussion on page 25-16.

5. (a) 180 joules.
   (b) Heat.
   (c) Graph (a) is the best fit. See discussion on page 25-17.

6. (a) 100 nt/m.
   (b) $U = 50x^2$ joules.

7. (a) See graph on page 25-19.
   (b) Not quite linear.

8. (a) 1:2.
   (b) 4 times as high.

9. See discussion on page 25-20.

10. See discussion on page 25-21.

11. (a) 1.0 m/sec.
    (b) 0.8 m/sec.

12. (a) 1.00 kg-m/sec.
    (b) 0.500 kg-m/sec,
    2.00 kg-m/sec,
    4.00 kg-m/sec.
    (c) 1.00 joule.

13. (a) 1.5 joules.
    (b) 0.4 joule.
    (c) 1.1 joules.
    (d) 1.1 joules.
    (e) 0.4 joule.

14. 18 joules, 9 joules.

15. (a) Vertical height = 0.20 m, slant height = 0.29 m; moves as though rebounding from a collision.
    (b) Oscillatory; equal heights right and left; 1.2 joules.
    (c) -0.5 joule.

16. (a) 0.15 above equilibrium (0.3 m above starting point).
    (b) ± 1.1 m/sec.
    (c) No change.

17. (a) 79.2 joules.
    (b) 79.2 joules.
    (c) 22.3 m/sec.

18. (a) Very nearly 1.
    (b) $E_k'(m_2)$ is negligible compared to $E_k'(m_1)$.
    (c) Yes.

19. See discussion on page 25-30.
20. See discussion on page 25-31.
21. See discussion on page 25-32.
22. (a) 4.4×10^{9} joules.
    (b) 3.9×10^{28} joules.
23. Increases.
24. (a) 0.575×10^{-18} joule.
    (b) 1.13×10^{6} m/sec.
25. 2.3×10^{-18} joules.
26. See discussion on page 25-36.
27. 0.0037 for an exhaust velocity of 2 km/sec.

COMMENTS AND SOLUTIONS

PROBLEM 1

A mass of 4.0 kg sliding with a speed of 3.0 m/sec on a frictionless horizontal table collides with a queer kind of spring bumper. The bumper exerts a constant force of 120 newtons on the mass as it moves in (compressing the spring) and the same force on the way out until the spring is back where it was.

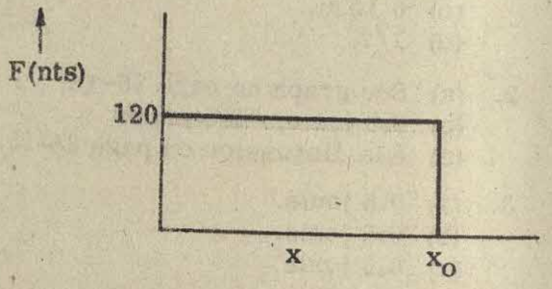
(a) Is this an elastic collision? How do you know?

(b) What is the kinetic energy at the beginning of the interaction?

(c) How much is the spring compressed?

(d) What is the ratio of kinetic energy to potential energy when the spring has been compressed 10 cm?

We can make a simple plot of the force exerted by the bumper versus distance of compression:



a) The collision is elastic, because the force depends only on the distance. A step function is the simplest form of force-distance relationship, but it is a form not usually encountered. Whatever work is done on the spring by an approaching object, FΔx, is done by the spring as the object recedes. Thus no kinetic energy is lost since, in the over all interaction, no energy is transferred to the spring.

b) The kinetic energy at the beginning of the interaction is

$$\left(\frac{1}{2}mv^2\right)_{\text{initial}} = \frac{1}{2} \times 4.0\text{ kg} \times \left(3.0\,\frac{\text{m}}{\text{sec}}\right)^2 = \underline{18\text{ joules}}.$$

c) The compression of the "spring" will continue until there is no more energy of motion – that is, no more kinetic energy. The kinetic energy is reduced to zero when the speed of the mass is zero. The original kinetic energy of the mass then has been transferred to the spring as potential energy. The work done in compressing this spring is the constant force, F, multiplied by the compression distance, Δx. This work done on the spring is stored as potential energy, so at the instant that the mass stops moving the initial kinetic energy of (b) is equal to the stored potential energy (120 nt) × (x meters)

$$120x\text{ nt-m} = 18\text{ joules}$$

$$x = \frac{18\text{ m}}{120} = \underline{0.15\text{ m}}.$$

d) When the spring has been compressed 10 cm, the potential energy has increased from zero to 120 × 0.10 joules = 12 joules. This increase has been at the expense of kinetic energy of motion (the mass has slowed down), and the mass has lost 12 joules of kinetic energy. It still has 18 - 12 = 6 joules of kinetic energy. The ratio of the kinetic to the potential energy is 6 to 12, i.e., there is half as much kinetic energy as potential energy at this instant in the interaction.

PROBLEM 2

A 3.0-kg mass moving with a speed of 2.0 m/sec collides with a spring bumper which exerts a force $F = 100x$, where F is the force in newtons and x is the compression in meters.

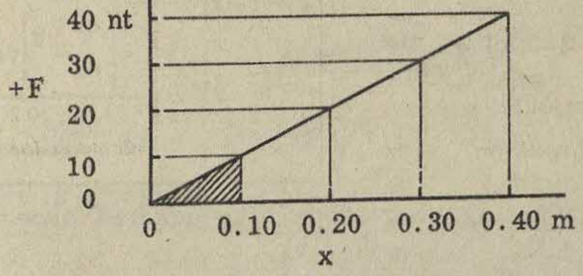
(a) Draw a graph of F versus x from $x = 0$ to $x = 0.40$ m.

(b) What is the potential energy stored in the spring when $x = 0.10$ m? What is the kinetic energy of the mass at this point?

(c) What will happen if the spring is compressed 0.10 m by hand and then the 3-kg mass is placed in contact with the spring and the hand is removed?

This problem is similar to Problem 1. The principle of conservation of energy is used to obtain the kinetic and potential energies at an intermediate stage of the interaction.

a)



b) The work done by an external agent in compressing the spring a distance of 0.10 m is the shaded area under the force-compression curve shown in part (a). The work done by the mass in moving from x = 0 to x = 0.10 m is therefore:

$$W = \frac{1}{2} \times 10 \text{ newtons} \times 0.10 \text{ m} = 0.5 \text{ joule.}$$

The potential energy stored in the spring is the same as the amount of work done on the spring, 0.5 joule.

The initial kinetic energy of the moving mass was:

$$E_k = \frac{1}{2} \times 3.0 \text{ kg} \times (2.0 \text{ m/sec})^2 = 6.0 \text{ joules.}$$

The increase in potential energy occurs at the expense of the kinetic energy of the mass. Therefore the kinetic energy must decrease by the 0.5 joule that the potential energy increased. Hence, at x = 0.10 m, the kinetic energy of the mass is 6.0 - 0.5 = 5.5 joules.

c) The work done in compressing the spring 0.10 m is the same whether it is done by a moving mass or by hand:

$$W = \frac{1}{2} \times 10 \text{ newtons} \times 0.10 \text{ m} = 0.5 \text{ joule.}$$

At this point the potential energy of the spring has increased from 0 to 0.5 joule. This energy is available to accelerate the mass when the hand is removed. While the resultant acceleration is caused by a non-constant force, solving the motion problem is easy with conservation of energy. When the spring has returned to its normal position, the interaction ends and the transfer of potential energy to kinetic energy is complete. Hence, at this point the kinetic energy $= \frac{1}{2}mv^2$ = 0.5 joule.

PROBLEM 3

The force-compression curve of a spring is shown in Fig. 25-16.

(a) How much work is done in compressing the spring 0.3 m?

(b) What is the potential energy of the spring when compressed this amount?

(c) Place a 2-kg mass at rest against the spring when it is compressed 0.3 m. Let go. What is the kinetic energy of the mass as it passes the point where the spring is compressed 0.2 m?

This problem is conceptually the same as Problem 2, but involves a nonlinear force curve.

a) The work done in compressing the spring is the area under the force-distance curve. In this case, between 0 and 0.3 m, the total area under the curve is the sum of the area of a triangle and the area of a trapezoid, these figures being the two parts of the area if it is cut vertically at 0.2 m.

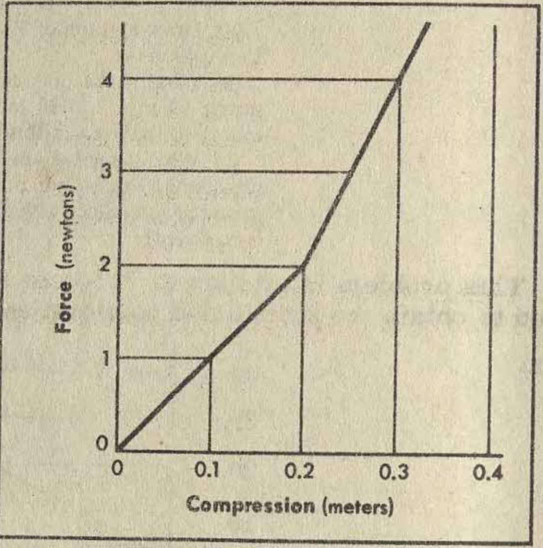
$$\text{Area of triangle} = \frac{1}{2}(2\text{ newton})(0.2\text{ m})$$

$$= 0.2\text{ newton-meter or joule.}$$

$$\text{Area of trapezoid} = \frac{1}{2}(2\text{ nt} + 4\text{ nt})(.1\text{ m})$$

$$= 0.3\text{ nt-m or joule.}$$

Sum = 0.5 joule = work of compression.



b) The potential energy of the spring = work done by the applied force = 0.5 joule (from above).

c) The gain in kinetic energy is equal to the loss of potential energy and this is the area under the force-distance curve from x = 0.3 to x = 0.2 m. This area is just the area of the trapezoid which was found in part (a) to be 0.3 joule. Therefore,

$$\frac{1}{2}mv^2 = \underline{0.3\text{ joule}} = \text{gain in kinetic energy.}$$

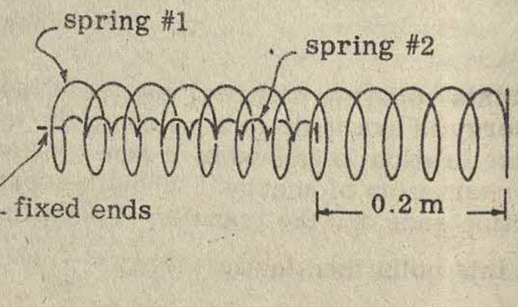
PROBLEM 4

Pretend you are given two springs of different sizes. How would you set them up so that the force-compression curve of the system looks like Fig. 25-16?

This problem is a challenge to the student with a flair for inventing mechanical devices.

In order to get a change of slope at 0.2 meters in the force-compression curve, there must be an accompanying change in the force constant of the spring at the 0.2-meter compression point. There are a number of ways to accomplish this. One way is to set up two springs so that the movable end of one spring is 0.2 meters behind the movable end of the other. If the springs have different diameters, one can be placed inside the other as shown in the sketch.

As an object pushes in from the right, it first comes in contact with spring #1. It is subject to the force $F_1 = k_1x$ until it has compressed spring #1 a distance 0.2 meters. From the graph we see that when $F_1 = 2$ newtons, x = 0.2 meters, therefore $k_1 = 10$ nts/m.



At the point $x = 0.2$ m, the object comes in contact with spring #2. From there on it is subject to the force $F = F_1 + F_2 = k_1x + k_2(x - 0.2) = (k_1 + k_2)(x = 0.2) + 0.2k_1 = 0.2k_1 + (k_1 + k_2)\Delta x$, where Δx represents the distance of compression beyond the point where $x = 0.2$ meters. From the graph, the slope of the straight line for $x > 0.2$ is $\Delta F/\Delta x = \frac{4 - 2}{0.3 - 0.2} = 20$ newtons/meter $= k_1 + k_2$. Therefore k_2 also has the value 10 newtons/meter.

PROBLEM 5

A 10-kg mass moving with a speed of 10 m/sec strikes a spring, compressing it a distance of 0.20 m. The mass rebounds with a speed of 8.0 m/sec.

(a) What is the loss in kinetic energy of the mass?

(b) What happens to this lost energy?

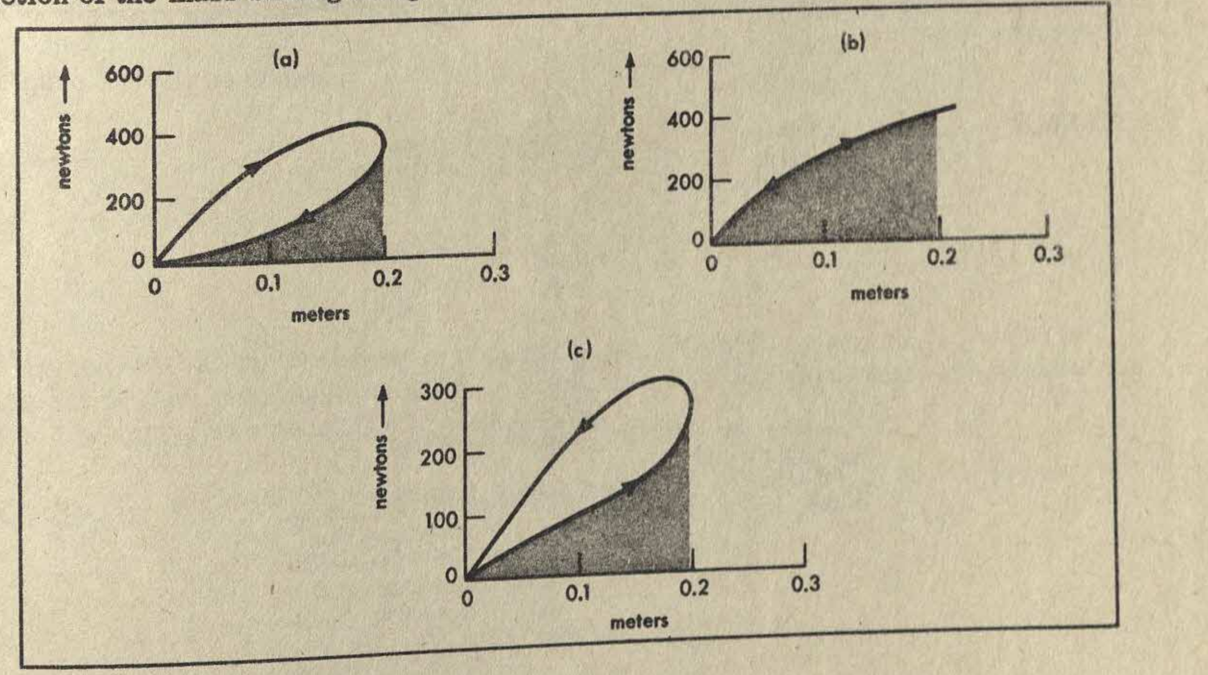
(c) Which of the force-compression curves in Fig. 25-17 do you think is most likely correct for this spring?

a) $(E_k)_i = \frac{1}{2}mv_i^2 = \frac{1}{2}10\,(10)^2 = 500$ newton-meters.

$(E_k)_f = \frac{1}{2}mv_f^2 = \frac{1}{2}10\,(8.0)^2 = 320$ newton-meters.

$\Delta E_k = 500 - 320 = $ 180 newton-meters or joules.

b) As the mass is brought to rest, it compresses the spring, doing work equal to its loss in kinetic energy (500 joules). On the way out, the spring transfers back to the mass only 320 joules of work. While we cannot say definitely what happened to the "lost" energy, one possibility is that mechanical energy was transformed into heat during the bending and unbending of the spring. Another possibility is that heat is generated through friction of the mass sliding along some surface.



c) Qualitatively, graph (a) can account for the data; the area under the upper curve (compression varying from 0 to 0.2 m) is greater than the area under the lower curve (compression varying from 0.2m to 0m). The first area is equal to the work done on the spring in compressing it, the second area is equal to the work given back by the spring in expanding back to its normal length. Quantitatively, the graph does not fit the problem situation. The area under the upper curve should equal the work required to stop the mass (500 joules), and the area under the lower curve should equal the work required to accelerate the mass from rest to 8.0 m/sec (320 joules). In graph (a), the first area is about 50 joules and the second area is about 30 joules. The force should be increased by a factor of 10 to obtain quantitative agreement. Curve (b) represents a case in which work put in = energy transferred out. Curve (c) represents a case for which more energy is taken out than is put in. Neither of these situations corresponds with the facts of this problem.

PROBLEM 6

A linear elastic spring is compressed 0.2 m by a force of 20 newtons.

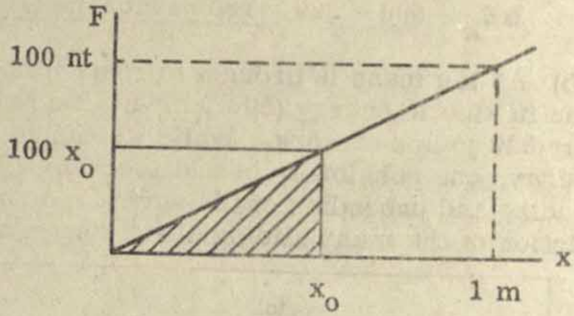
(a) What is the force constant *k* (or force-compression ratio) of the spring?

(b) What is the equation for the potential energy stored by the spring as a function of its compression?

a) The spring is linear and elastic. Therefore, F = kx relates the magnitude of the force to the amount of compression. Rewriting,

$$k = F/x;\ \text{in particular,}\ k = \frac{20\,\text{nt}}{0.2\,\text{m}} = \underline{100\,\text{nt/m}}.$$

b) If this relation is plotted, we see from F = 100 x that the area of the triangle of base x_o is $\frac{1}{2} \times x_o \times 100\,x_o = 50 \times x_o^2$ joules. This is the work that must be done to compress the spring and therefore is also the potential energy. Hence, $\underline{U = 50\,x^2}$ where x is the compression measured in meters, and U is given in joules.



PROBLEM 7

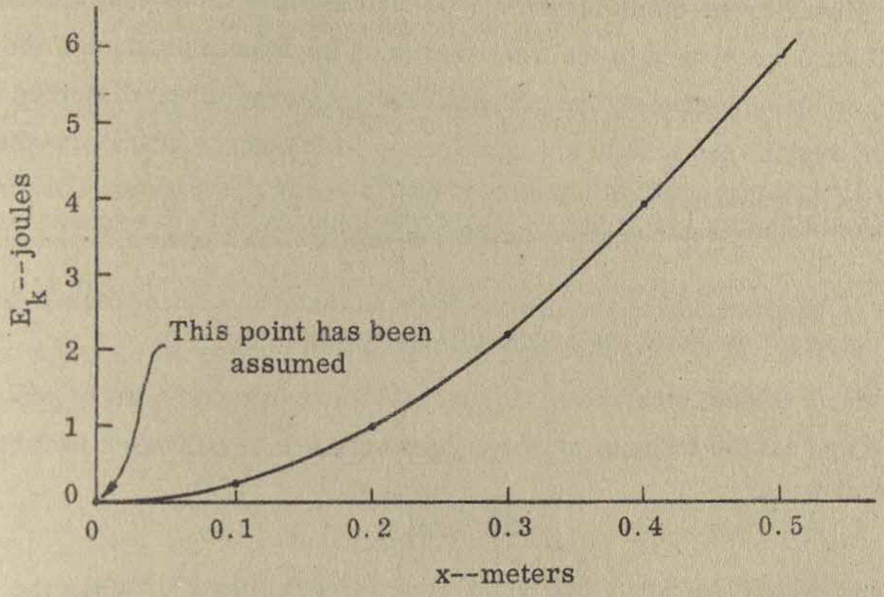
The following table gives the kinetic energy E_K of a 2.0-kg mass when it leaves a spring compressed different distances *x*.

x Meters	E_K Joules
0.1	0.25
0.2	1.00
0.3	2.25
0.4	4.00
0.5	6.00

(a) Plot the potential energy of the spring against the compression *x*.

(b) Does the spring exert a linear restoring force?

a)



E_k, the final kinetic energy of a mass as it leaves the spring, is plotted against the compression distance, x, before the mass was released. If we assume that no losses occur as the spring expands, then the initial potential energy of the spring is exactly equal to E_k, the final kinetic energy of the mass. In this case, the above plot is also the plot of the potential energy. (If there were frictional losses, the problem would be indeterminate.)

b) As was seen in Problem 6, the potential energy of a spring varies as the square of the compression, x. If $F = kx$, the potential energy $= E_k = \frac{1}{2}kx^2$, and $\frac{2E_k}{x^2} = k$.

Let us then compute $\frac{2E_k}{x^2}$. We expect that if the spring exerts a linear restoring force this quantity will be constant.

x	x^2	E_k	$2E_k/x^2$
0.1	0.01	0.25	50
0.2	0.04	1.00	50
0.3	0.09	2.25	50
0.4	0.16	4.00	50
0.5	0.25	6.00	48

We see that the spring exerts a linear restoring force for the greater part of its range but the last point deviates slightly. This may indicate that, at a compression of 0.5 m, the spring was permanently deformed.

PROBLEM 8

Assume an archer's bow acts like a linear spring.

(a) Compare the initiai velocity of an arrow shot when the bow string is pulled back 10 cm with the initial velocity of the arrow when the string is pulled back 20 cm?

(b) How many times higher can the archer shoot in the second case?

a) As the archer pulls back on the bow string he exerts a force of magnitude $F = kx$. This gives a linear force-displacement curve and the area under that straight line is $A = \frac{1}{2}kx^2$. This corresponds to the work that must be done in producing the displacement x. It therefore also represents the potential energy stored in the distorted system and hence also represents the kinetic energy, $\frac{1}{2}mv^2$, which can be transferred to an arrow when the string is released.

$$W = U = E_k = \frac{1}{2}mv^2 = \frac{1}{2}kx^2.$$

Therefore v is proportional to the initial displacement. If x is increased from 10 cm to 20 cm, the resultant speed of the arrow will be doubled.

b) If the arrow is shot straight up, its initial kinetic energy, $\frac{1}{2}mv^2$, will decrease as work is done against the force of gravity. The measure of this work is force × distance = mgh, therefore

$$\frac{1}{2}kx^2 = mgh_{max}.$$

Thus the height which the arrow can reach is proportional to x^2. When the archer doubles his initial draw-back he will quadruple the height which will be attained by the arrow.

PROBLEM 9

Fig. 25-18 shows a roller skate with a mass *M* mounted on it by four hack-saw blades attached to a base. The mass *M* is equal to the mass of the roller skate plus the mass of the base. If you hold the roller skate and pull the mass *M* out to the left, and then let the whole system go, the mass moves to the right and the skate to the left. Be prepared to describe the further motion, indicating the form of the energy at various stages.

For the solution to this problem we shall assume that all of the motions are frictionless and that the entire system is at rest at the instant the displaced mass is released.

Since no external forces act on the skate-mass system, conservation of momentum tells us that the center of mass must always remain at rest. The motion of the system will consist of back and forth oscillations of its top and bottom halves, one always moving in the opposite direction from the other. The initial potential energy stored in the distorted blades is transformed into kinetic energy of the motion of the parts. When the blades are vertical, the potential energy is zero and all of the energy resides in the equal speeds of the two masses, $E_K = 2 \times \frac{1}{2}mv^2$. When the blades are at the extremes of their distortion, the motion of the masses is zero and all of the energy of the system is now in the form of potential energy.

If we describe the motion more accurately we can take into account the vertical motion of the upper mass as the blades sweep it through an arc. This up-down motion "stretches" and "relaxes" the "spring" of the gravitational field. However, in the undistorted position of the hack-saw blades where the kinetic energy is a maximum, the gravitational potential energy is also a maximum. Thus the useful energy of the system is never entirely kinetic. There is always some available potential energy stored either in the spring or in the gravitational field.

If we take friction into account, energy will gradually be transformed into heat, and the motion will stop. Insofar as the friction is rolling friction on a surface, an external force will act on the skate-mass system and its center of mass, also, will describe gradually diminishing back and forth oscillations instead of remaining stationary as described.

PROBLEM 10 What will happen if the 3.0-kg mass in Problem 2 hooks itself onto the spring and remains fastened to the spring?

If the mass remains attached to the spring it will oscillate back and forth in simple harmonic motion.

We can obtain the amplitude of the oscillation by applying conservation of energy. Just as the mass reaches the spring (and hooks on), its energy is simply the kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}(3.0)(2.0)^2 \frac{\text{kg}-\text{m}^2}{\text{sec}^2} = 6.0$ joules. At the maximum compression (or extension) of the spring, the energy is wholly potential, $U = \frac{1}{2}kx^2 = 6.0$ joules. With $k = 100 \frac{\text{newton}}{\text{meter}}$, we get:

$$x^2 = \frac{2U}{k} = 2 \times \frac{6.0 \text{ newton-meter}}{100 \frac{\text{newton}}{\text{meter}}} = 0.12\,\text{m}^2.$$

$$x = \pm \sqrt{0.12\,\text{m}^2} = \pm\, 0.35 \text{ meters.}$$

The mass oscillates back and forth from 0.35 meters to the right, to 0.35 meters to the left of the equilibrium position. We can also obtain the period of the oscillatory motion. Using the result of Section 2:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3.0\text{ kg}}{100\text{ nt-m}}} = 1.1 \text{ seconds.}$$

PROBLEM 11 A 0.50-kg mass free to slide on a table is attached to a linear spring with a force-extension ratio *k* of 50 newton/m. The mass is pulled back and released so that it vibrates with an amplitude of 0.10 m.

(a) What is its maximum velocity?

(b) What is its velocity when it is 0.06 m away from its equilibrium position?

a) In this case of a mass vibrating at the end of a spring we must deal with both potential and kinetic energy. Since the total energy, E, is a constant, we may write:

$$E = E_k + U.$$

The potential energy of a spring compressed a distance x is $\frac{1}{2}kx^2$, where k is the force to extension ratio. The kinetic energy is $\frac{1}{2}mv^2$. Hence,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

At the point where the mass stops and reverses its motion, the kinetic energy is zero because the velocity is zero, and the potential energy is a maximum because the mass has reached its maximum displacement, x_o, from the equilibrium position. Hence, when $x = +x_o$ or when $x = -x_o$, the kinetic energy, E_k, is zero, and the total energy is given by:

$$E = \frac{1}{2}kx_o^2.$$

Similarly, when $x = 0$, the potential energy is zero, and v is at its maximum value, v_o. At this point the total energy is given by:

$$E = \frac{1}{2}mv_o^2.$$

Then, since E is a constant, $\frac{1}{2}mv_o^2 = \frac{1}{2}kx_o^2$, and

$$v_o = x_o\sqrt{\frac{k}{m}}.$$

Substituting $x_o = 0.10$ m, $k = 50$ nt/m, and $m = 0.50$ kg,

$$v_o = 0.10\sqrt{\frac{50}{0.50}} = 0.10 \times \sqrt{100} = \underline{1.0\text{ m/sec.}}$$

b) In general,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_o^2.$$

Therefore,

$$v^2 = \frac{kx_o^2 - kx^2}{m}, \text{ and}$$

$$v = \sqrt{\frac{k(x_o^2 - x^2)}{m}}.$$

Substituting for $x = 0.06$ m, and the other given values,

$$v = \sqrt{\frac{k(x_o^2 - x^2)}{m}} = \sqrt{\frac{k(x_o - x)(x_o + x)}{m}} = \sqrt{\frac{50\,(0.10 - 0.06)(0.10 + 0.06)}{0.50}}$$

$$= \sqrt{100 \times 0.04 \times 0.16} = 10 \times 0.2 \times 0.4 = \underline{0.8\text{ m/sec}}.$$

PROBLEM 12

A spring bumper with restoring force $F = 200x$ where F is in newtons and x in meters is compressed 0.100 meter. A 0.500-kg mass is placed next to the end of the spring and the whole thing let go.

(a) With what momentum will the mass leave the spring?

(b) We do the same thing with masses 0.125 kg, 2.00 kg, and 8.00 kg. What is the momentum of each as it leaves the spring?

(c) What is the energy of each as it leaves the spring?

a) At the point of maximum compression, all of the energy is potential, $U = \frac{1}{2}kx_o^2$. When the spring is released the mass is accelerated until the spring reaches the equilibrium position. At this point all of the energy is kinetic, and the mass leaves the spring. Then $E = E_{K_o} = \frac{1}{2}mv_o^2$. Since the total energy is constant,

$$\frac{1}{2}mv_o^2 = \frac{1}{2}kx_o^2, \text{ and}$$

$$v = x_o\sqrt{\frac{k}{m}}.$$

The momentum of the 0.500-kg mass is then

$$mv_o = x_o\sqrt{km}$$

$$= 0.100 \times \sqrt{200\text{ nt/m} \times 0.500\text{ kg}}$$

$$= \underline{1.00\text{ kg-m/sec.}}$$

b) For a 0.125-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 0.125} = \underline{0.500\ \text{kg-m/sec.}}$$

For a 2.00-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 2.00} = \underline{2.00\ \text{kg-m/sec.}}$$

For an 8.00-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 8.00} = \underline{4.00\ \text{kg-m/sec.}}$$

c) Since the original potential energy is the same no matter what mass is put in motion by the spring, the kinetic energy is the same for each mass:

$$E_{K_o} = \frac{1}{2}kx_o^2 = \frac{1}{2} \times 200 \times 0.100^2 = \underline{1.00\ \text{joule}}.$$

Thus, when a fixed amount of energy is transferred to objects of different mass, the resulting momenta are not the same, but vary in proportion to the square root of the mass.

PROBLEM 13

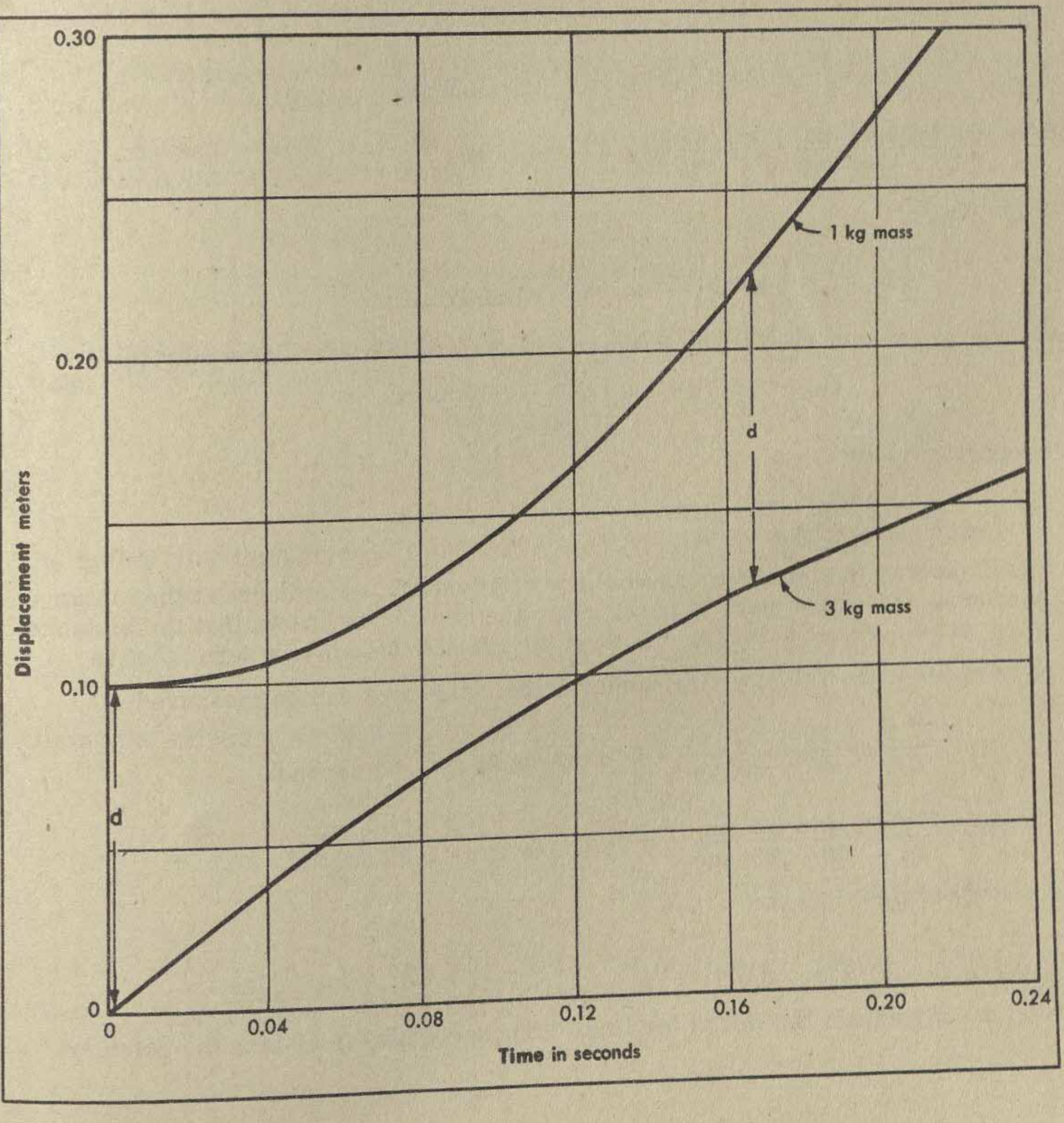


Fig. 25–19 is the graph of the displacements as a function of time of two interacting masses $m_1 = 3$ kg and $m_2 = 1$ kg.

(a) What is the initial kinetic energy of m_1?
(b) What is the final kinetic energy of m_1?
(c) What is the final kinetic energy of m_2?
(d) What is the minimum total kinetic energy?
(e) What is the maximum potential energy?

To find the various kinetic energies we can measure velocities (slopes) of the appropriate displacement vs. time curve at the times of interest. To do this, draw tangent lines to the displacement vs. time curves and calculate the ratio $\Delta s/\Delta t$ for the tangent line.

a) $v_1 = \frac{\Delta s}{\Delta t} = \frac{(0.2 - 0.0)\text{ m}}{(0.2 - 0.0)\text{ sec}} = 1.0\text{ m/sec}$, at $t = 0$ seconds.

The initial E_k of m_1 is:

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}(3\text{ kg})\ (1.0\text{ m/sec})^2 = \underline{1.5\text{ joules}}.$$

b) $v_1 = \frac{\Delta s}{\Delta t} = \frac{0.14 - 0.04}{0.2 - 0.0}\frac{\text{m}}{\text{sec}} = 0.5\frac{\text{m}}{\text{sec}}$ at $t > 0.2$ second.

The final E_k of m_1 is:

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}(3\text{ kg})(0.5\text{ m/sec})^2 = \underline{0.4\text{ joule}}.$$

c) $v_2 = \frac{\Delta s}{\Delta t} = \frac{0.28 - 0.04}{0.20 - 0.04}\frac{\text{m}}{\text{sec}} = 1.5\frac{\text{m}}{\text{sec}}$ at $t > 0.2$ second.

The final E_k of m_2 is:

$$\frac{1}{2}m_2 v_2^2 = \frac{1}{2}(1\text{ kg})(1.5\text{ m/sec})^2 = \underline{1.1\text{ joules}}.$$

We note that energy is conserved in the interaction. Before the interaction started:

$$(E_k)_1 + (E_k)_2 = 1.5 + 0.0 = 1.5\text{ joules}.$$

After the interaction is over:

$$(E_k)_1 + (E_k)_2 = 0.4 + 1.1 = 1.5\text{ joules}.$$

d) The kinetic energy is a minimum and the potential energy a maximum at the distance of closest approach. Moving a piece of paper along the two curves shows that the distance $s_2 - s_1$ is least at $t = 0.09$ seconds. At this time the relative velocity is zero, that is, both masses have the same velocity. As a check, the two slopes can be measured:

$$v_1 = \frac{\Delta s_1}{\Delta t} = \frac{0.22 - .07}{.20 - .00}\frac{\text{m}}{\text{sec}} = 0.75\text{ m/sec at } t = 0.09\text{ second}.$$

$$v_2 = \frac{\Delta s_2}{\Delta t} = \frac{.16 - .01}{.20 - .00}\frac{\text{m}}{\text{sec}} = 0.75\text{ m/sec at } t = 0.09\text{ second}.$$

The kinetic energy at this time is:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(3 + 1)(0.75)^2 = \underline{1.1\text{ joules}}.$$

e) The total energy equals the initial (or final) kinetic energy, if we take the potential energy to be zero at separation greater than the range d of the interaction. The total energy is therefore 1.5 joules. At t = 0.09 second, the kinetic energy was 1.1 joules. Thus the potential energy at this time must have been:

$$U = E - E_k = 1.5 \text{ joules} - 1.1 \text{ joules} = \underline{0.4 \text{ joule}}.$$

Note. Two curves similar to those in Figure 25-19 can be obtained by using the soft spring bumpers with the carts in the arrangement described in Experiment III-14.

PROBLEM 14

Two masses $m_1 = 1.0$ kg and $m_2 = 2.0$ kg are pushed together despite an elastic interaction force until their potential energy is 27 joules greater than it is when they are beyond the distance at which they interact. If they are let go, what will be the final kinetic energy of each?

Conservation of energy indicates that the final kinetic energy must equal the initial potential energy:

$$(E_k)_{\text{initial}} + (U)_{\text{initial}} = (E_k)_{\text{final}} + (U)_{\text{final}}$$

$$0 + 27 \text{ joules} = (E_k)_{\text{final}} + 0.$$

Therefore, $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = 27$ joules.

To determine how the kinetic energy is divided between the two masses we must use the principle of momentum conservation. Since no other forces act on m_1 and m_2 except the forces of mutual interaction, the total momentum does not change. Since both masses are initially at rest, the momentum is zero initially and thereafter:

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

(the minus sign signifies that they fly off in opposite directions). Knowing the equality in magnitude of the momenta enables us to find the ratio of kinetic energies:

$$\frac{(E_k)_1}{(E_k)_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}.$$

By multiplying the numerator and the denominator by m_1m_2, this becomes

$$\frac{m_2}{m_1}\frac{(m_1v_1)^2}{(m_2v_2)^2} = \frac{m_2}{m_1}.$$

Using the values of m_1 and m_2 we have:

$$\frac{(E_k)_1}{(E_k)_2} = \frac{m_2}{m_1} = \frac{2.0}{1.0} = 2.0$$

$$E_k = (E_k)_1 + (E_k)_1 = 2(E_k)_2 + (E_k)_2 = 3(E_k)_2.$$

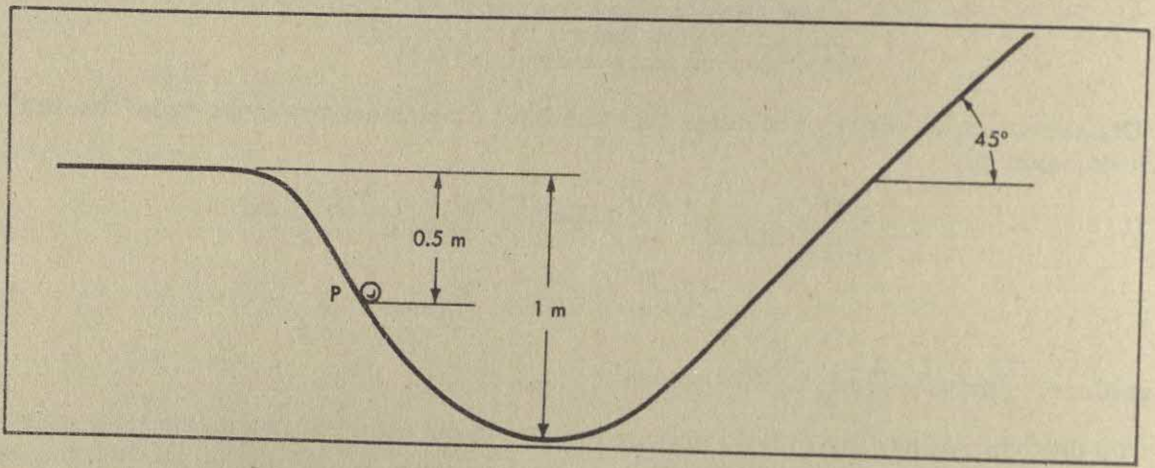
Thus $(E_k)_2 = \frac{27 \text{ joules}}{3} = \underline{9 \text{ joules}}$, and $(E_k)_1 = 27 \text{ joules} - 9 \text{ joules} = \underline{18 \text{ joules.}}$

PROBLEM 15

(a) A ball of mass 0.25 kg is thrown to the right at a speed of 2.0 meters per second. It starts along the frictionless surface at the left of Fig. 25-20. How high up the slope at the right will it go before coming momentarily to rest? What kind of motion will it perform?

(b) If the ball is released from rest at point *P*, what kind of motion will it perform? How high up the slope on the right will it rise? What is the binding energy (the extra energy needed to make the ball escape from the well in the center of the figure)?

(c) What is the binding energy in part (a)?



a) The initial E_k is $\frac{1}{2}mv^2 = \frac{1}{2}(0.25\,\text{kg})(2.0\frac{\text{m}}{\text{sec}})^2 = 0.50$ joules. The ball gains kinetic energy (and loses potential) as it drops into the valley, but then loses kinetic energy as it rises up the other side, until at the instant it reaches the original elevation it will again have 0.50 joules E_k. It continues to rise to a vertical height h such that mgh = 0.50 joules.

Thus, $h = \frac{E_k}{mg} = \frac{0.50 \text{ joules}}{(0.25\,\text{kg})(9.8\frac{\text{nt}}{\text{kg}})} = \underline{0.20 \text{ meters}}$. The slant height $= \frac{h}{\sin 45°} = \underline{0.29\text{ m}}$.

The ball can be thought of as rebounding from an interaction; it comes in from the left, dips, rolls up the hill, and them reverses, retracing its path and going off to the left with the same speed it had initially.

b) The motion will be oscillatory. It will rise as high on the right as it was initially on the left. To escape from the valley it would need additional energy in the amount mgh, where h is the additional height it must rise, 0.5 meters. The binding energy is then $0.25 \times 9.8 \times 0.5 = \underline{1.2 \text{ joules}}$.

c) The binding energy in part (a) is negative because the particle has kinetic energy to spare when it is free of the valley. One must take away 0.50 joule of energy to leave the ball just free of the well (with no kinetic energy on the horizontal sufrace). Thus the binding energy in (a) is $\underline{-0.5 \text{ joules}}$.

PROBLEM 16

A linear spring whose force-extension ratio *k* is 40 newton/m hangs vertically, supporting a 0.80-kg mass at rest. The mass is then pulled down a distance of 0.15 m.

(a) How high will it rise?

(b) What will be its maximum velocity?

(c) How would the answers to (a) and (b) differ if the experiment were done on the moon?

This problem is straightforward if it is recognized that the constant force of gravity (mg downward) on the hanging mass has no effect beyond that of shifting the equilibrium position. This is easy to see if we consider first the forces acting on the mass when it is in the equilibrium position. The resultant force is zero; the downward pull of gravity is balanced by an upward pull by the spring of the same magnitude. What is the resultant force on the mass at other positions? The gravitational force does not change, but the spring force changes. If the mass is above the equilibrium position the spring extension is smaller, the spring force is smaller and the resultant force is down, back toward the equilibrium position. Similarly, if the mass is below the equilibrium position, the extension of the spring is larger, the upward pull by the spring is larger and the resultant force is up, again back toward the equilibrium position. The resultant forces are just kx (where x is the displacement from equilibrium) in a direction opposite to the displacement. At equilibrium the spring extension, s_o, is such that $ks_o = mg$. For a displacement x, below the equilibrium position, the spring extension is $s_o + x$, the spring pull is $k(s_o + x) = ks_o + kx = mg + kx$ and the resultant force is kx (up).

Since the net spring force at any x is kx, the potential energy will be $1/2\,kx^2$, (the potential energy being taken as 0 at equilibrium, $x = 0$).

This problem is similar to Experiment III-12. The yellow pages for that experiment contain further commentary.

After these preliminaries, the questions can be answered readily.

a) Since the force is linear we have simple harmonic motion, with the extreme points of the motion at equal distances below and above the equilibrium position. Therefore the mass will rise 0.15 meters above the equilibrium position, (0.30 m above the release point).

b) As the mass passes through the equilibrium position ($x = 0$) its potential energy will be a minimum ($1/2\,k \times 0^2 = 0$) and its kinetic energy will equal the total energy:

$$\frac{1}{2}mv^2 = E = \frac{1}{2}kx_o^{\;2}.$$

$$v^2 = \frac{k}{m}x_o^{\;2}, \text{ and}$$

$$v = \pm\sqrt{\frac{k}{m}}\,(x_o) = \pm\sqrt{\frac{40\text{ nt-m}}{0.80\text{ kg}}}\,(-0.15\text{ meters}) = \underline{\pm\, 1.1\text{ m/sec.}}$$

The ± sign simply reminds us that the speed of the mass is alternately upward and downward.

c) Since both answers are independent of the gravitational field, the same answers are obtained on the earth, on the moon or anywhere else. The only effect of changing g is to change the extension of the spring at equilibrium, $s_o = \frac{mg}{k}$.

PROBLEM 17

A 0.200-kg stone is thrown upward from a point 20 meters above the earth's surface at an angle of 60 degrees with the horizontal and with a speed of 20.0 m/sec.

(a) What is its total energy?

(b) What will be its total energy when it is 15.0 m above the earth's surface?

(c) What will be its speed 15.0 m above the earth?

To compute the total energy of the stone we must first select a convenient zero of potential energy. For convenience we choose the surface of the earth.

a) The initial kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}(0.200)(20.0)^2 = 40.0$ joules. The initial potential energy, mgh, is $mgh = 0.200 \times 9.8 \times 20.0 = 39.2$ joules.

The total energy is the sum of the kinetic and potential energy at any instant:

$$39.2 + 40.0 = \underline{79.2 \text{ joules total energy}}.$$

b) Since the total energy in this system is conserved (we neglect air resistance) the total energy is 79.2 joules at all points on the path.

c) At 15.0 m above the earth's surface the potential energy $mgh = 0.200 \times 9.8 \times 15.0 = 29.4$ joules. Therefore using the conservation of energy the kinetic energy, $\frac{1}{2}mv^2$, is given by:

$$\frac{1}{2}mv^2 + 29.4 = 79.2, \text{ or } \frac{1}{2}mv^2 = 49.8 \text{ joules},$$

$$v^2 = \frac{2 \times 49.8}{m} = \frac{2}{0.20} \times 49.8$$

$$v = \underline{22.3 \text{ m/sec}}.$$

PROBLEM 18

A mass m_1 moving with speed v_1 hits a mass m_2 head on. The mass m_2 is originally at rest, and the collision is elastic. If m_2 equals 10^{24} m_1, estimate the ratio of:

(a) the initial kinetic energy of m_1 to the final kinetic energy of m_1.

(b) the final kinetic energy of m_2 to the initial kinetic energy of m_1. A mass colliding elastically with a spring bumper solidly fastened to the earth loses practically no kinetic energy as a result of the interaction.

(c) Do your answers to (a) and (b) agree with this statement? Be prepared to discuss this.

Note: For parts (a) and (b), use the equations

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

for the final speeds of m_1 and m_2.

The solution can be estimated without using the formulas in the note:

a) As suggested in the last sentence of part (b), the stipulation of this problem is analogous to the situation of a ball bouncing on the earth. The earth's recoil is negligible. Thus in an elastic collision m_1 will have equal initial and final kinetic energies.

b) The final kinetic energy of m_2 will be negligible.

c) Yes.

In order to calculate the answers, numbers can be substituted in the equations that are given.

(a) $$v'_1 = \frac{m_1 - 10^{24}m_1}{m_1 + 10^{24}m_1}v_1 = \frac{1 - 10^{24}}{1 + 10^{24}}v_1 = -\frac{10^{24}(1 - 10^{-24})}{10^{24}(1 + 10^{-24})}v_1.$$

$$\frac{(E_k)_1}{(E'_k)_1} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1v_1'^2} = \frac{v_1^2}{v_1'^2} = \frac{(1 + a)^2}{(1 - a)^2}, \text{ where } a = 10^{-24}.$$

Now $(1 + a)^2 = 1 + 2a + a^2$. Since a is much less than 1, the three terms on the right contribute in various approximations to the term on the left. That is, a first approximation is

$$(1 + a)^2 = 1.$$

A more accurate approximation is

$$(1 + a)^2 = 1 + 2a = 1 + 2 \times 10^{-24},$$

while the exact answer is

$$(1 + a)^2 = 1 + 2a + a^2 = 1 + (2 \times 10^{-24}) + 10^{-48}.$$

The three terms $(1,\ 2 \times 10^{-24},\ 10^{-48})$ are progressively smaller, the ratio of successive terms being only about 1 part in 10^{24}.

Likewise $(1 - a)^2 \approx 1 - 2a = 1 - 2 \times 10^{-24}$.

We now approximate $\frac{1}{1 - 2a}$ by $1 + 2a$.

The validity of this approximation is seen by simply performing long division:

$$\begin{array}{r l}
 & 1 + 2a + 4a^2 + \dots \\
1 - 2a \,\big) & 1 \\
 & \underline{1 - 2a} \\
 & \quad 2a \\
 & \quad \underline{2a - 4a^2} \\
 & \qquad 4a^2 \\
 & \qquad \underline{4a^2 - 8a^2} \\
 & \qquad\quad 8a^2 \\
 & \qquad\quad + \dots
\end{array}$$

The remaining terms involve higher and higher powers of a. In the spirit of the approximation of neglecting a^2 we have, then

$$\frac{1}{1 - 2a} = 1 + 2a.$$

Thus,

$$\frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1v_1'^2} = \frac{(1 + a)^2}{(1 - a)^2} \approx \frac{1 + 2a}{1 - 2a} \approx (1 + 2a)^2 \approx 1 + 4a,$$

$$\frac{(E_k)_1}{(E_k')_1} \approx 1 + 4 \times 10^{-24}, \text{ which is very close to unity.}$$

(b) We can compute the ratio of the final kinetic energy of m_2 to the initial kinetic energy of m_1 using the principle of energy conservation.

In part (a), we have obtained:

$$(E_k)_1 = (E'_k)_1 \, (1 + 4 \times 10^{-24}).$$

But by conservation of energy:

$$(E'_k)_1 + (E'_k)_2 = (E_k)_1 ,$$

$$\text{or} \quad (E'_k)_2 = (E_k)_1 - (E'_k)_1 = (E_k)_1 - \frac{(E_k)_1}{1 + 4 \times 10^{-24}},$$

$$\text{or} \quad (E'_k)_2 = (E_k)_1 \, [1 - (1 - 4 \times 10^{-24})].$$

Therefore $\dfrac{(E'_k)_2}{(E_k)_1} = 4 \times 10^{-24}.$

An alternate method is:

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 .$$

Therefore:

$$\frac{\frac{1}{2} m_2 v_2'^2}{\frac{1}{2} m_1 v_1^2} = \frac{m_2}{m_1} \left(\frac{2m_1}{m_1 + m_2} \right)^2 = \frac{4m_1}{m_2 \left(1 + \dfrac{m_1}{m_2} \right)^2}$$

$$= \frac{4 \times 10^{-24}}{\left(1 + 10^{-24}\right)^2} \approx 4 \times 10^{-24}.$$

c) Yes, since the mass of the earth is 6×10^{24} kg, much larger than a typical mass used in the laboratory.

PROBLEM 19

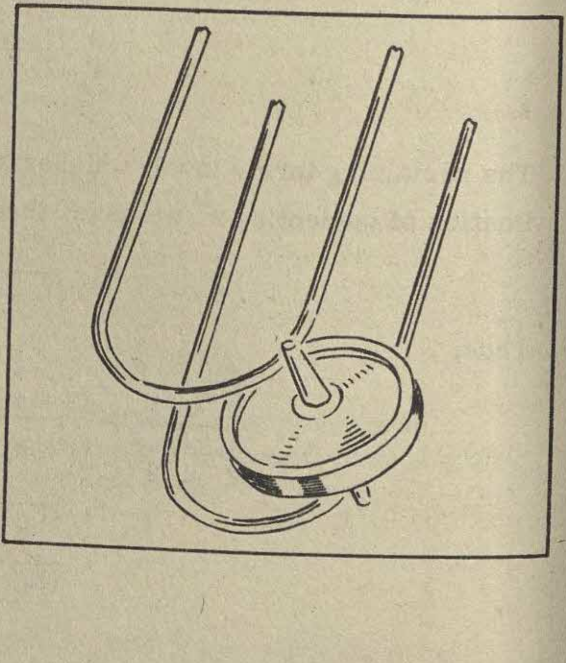
Fig. 25–21 illustrates a very common toy consisting of a wheel and shaft held to a support magnetically. When the toy is held upright, the rotor will move down one side starting from rest, round the curve at the bottom, and climb up the other side.

(a) If you have such a toy, find how much energy is lost from mechanical energy (the sum of gravitational potential energy plus kinetic energy) in one period of the motion.

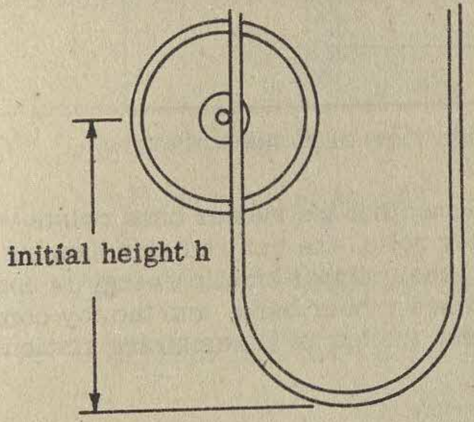
(b) The energy lost is what fraction of the maximum kinetic energy during the motion?

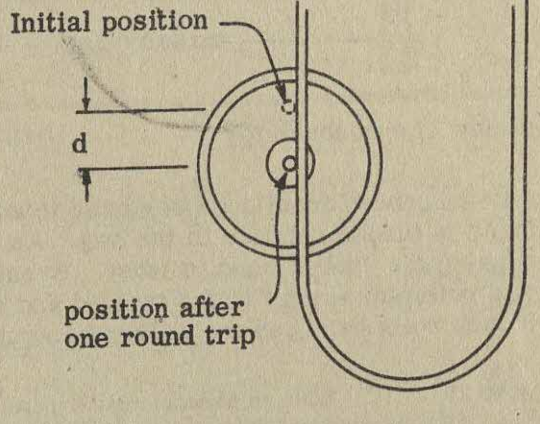
(c) Into what form of energy does the lost energy probably go?

Note: A "period" of the motion is the time it takes the rotor to go completely around the support and return to its initial position.



To compute the fraction of the maximum kinetic energy represented by the energy lost, we consider Fig. (a). Starting at rest from the top, the rotor would acquire its maximum kinetic energy at the bottom, and, if there were no losses, the kinetic energy at the bottom would be equal to the initial potential energy, mgh. The maximum kinetic energy with losses will be a little less. As a rough approximation we shall simply use mgh.





If, on completing one round trip, the rotor's highest point is a distance, d, below its starting point, it has lost energy mgd. Therefore the ratio of the energy loss to the initially available energy is:

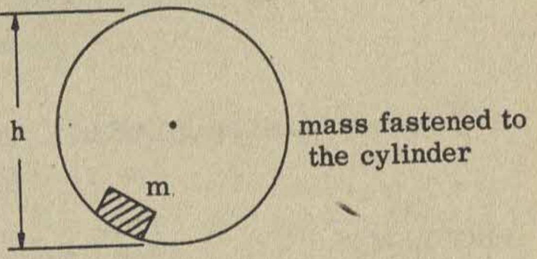
$$\frac{mgd}{mgh} = \frac{d}{h}.$$

In this example the lost mechanical energy will appear as a temperature rise. The loss results from friction and from electric currents set in circulation by the rolling magnet.

PROBLEM 20

Can you design a simple come-back toy? The outside is a cylindrical container. When the container is rolled away from you on a table, it will come to rest in a short distance, reverse its direction of motion, and come back to you.

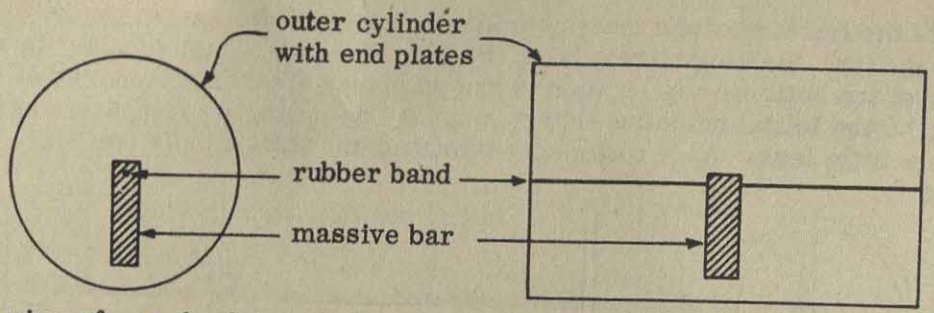
The toy, to come back, must have a device which enables it to build up potential energy as the cylinder rolls away.



If a mass is attached to the inside of the cylinder, (see figure), the potential energy has a maximum when the mass is at the top and a minimum when it is at the bottom. The toy is much like a pendulum which can swing through the full 360°.

If the cylinder is rolled so that, when the mass is at the bottom, the kinetic energy is less than mgh, it will roll until the kinetic energy has gone entirely into gravitational potential energy of the raised mass. Then it will roll back. Its motion will be oscillitory. If the initial kinetic energy is larger than mgh, the cylinder will never come to rest, but its motion will be somewhat jerky.

The toy described above cannot roll farther than one half its circumference and still come back. To make a device that will return after many revolutions, we must make the potential energy change more slowly with the angle of rotation. One way to do this is shown in the figure on the following page.



End view of comeback toy

Side view of comeback toy

The bar is suspended from a rubber band in such a way that the rubber band twists when the cylinder is turned relative to the bar. As the toy rolls, the bar remains hanging downward, causing the rubber band to twist. Eventually the original kinetic energy is converted entirely to potential energy of the twisted and stretched rubber band, and the toy comes to rest. It then rolls back, as the rubber band unwinds, the bar still remaining stationary.

PROBLEM 21

When an object of mass m moves from r to r' in the earth's gravitational field the potential energy changes by $\Delta U = -GmM\left(\frac{1}{r'} - \frac{1}{r}\right)$, where M is the mass of the earth.

Show that, if the object moves away from the earth a distance $\Delta r = (r' - r)$ which is very small compared to its distance from the earth's center, the above expression reduces to

$$\Delta U = mg\Delta r.$$

$$\Delta U = -GmM \quad \frac{1}{r'} - \frac{1}{r} \quad = \frac{+GmM\,(r' - r)}{rr'}.$$

Substituting $r' = r + \Delta r$, we get:

$$\Delta U = \frac{GmM\Delta r}{r(r + \Delta r)} = \frac{GmM\Delta r}{r^2\,(1 + \frac{\Delta r}{r})}.$$

Then, since $\frac{GM}{r^2} = g$, we have:

$$\Delta U = \frac{mg\Delta r}{1 + \frac{\Delta r}{r}}.$$

If $\frac{\Delta r}{r}$ is very small compared to 1, this is the same as

$$\Delta U = mg\Delta r.$$

PROBLEM 22

Find the binding energy, to two significant figures, of:

(a) a 70-kg man to the earth.
(b) the moon to the earth.

a) The basic relation needed is found is Section 4. The binding energy, E_B, is:

$$E_B = \frac{GmM}{r} - \frac{1}{2}mv^2.$$

The first term represents the potential energy we must supply to move m from r to infinity. However, if m has a velocity v when at the initial distance r, we can make use of the kinetic energy as a head start. With an initial velocity, and if we are content to bring the mass at the great distance to rest, then the full $\frac{GmM}{r}$ is not required to separate a mass from its system. We can save $1/2\ mv^2$.

A man at rest on the surface of the earth has some useful speed (and kinetic energy) due to the rotation of the earth. (A satellite must be fired from west to east to make use of this kinetic energy.) If the man is located near the equator,

$$v = \frac{2\pi \times 6.38 \times 10^6 \text{ meters}}{8.64 \times 10^4 \text{ sec}} = 4.64 \times 10^2 \text{ m/sec.}$$

Using this v, the kinetic energy can be calculated:

$$\frac{1}{2}mv^2 = \frac{70 \times (4.64 \times 10^2)^2}{2} = 7.5 \times 10^6 \text{ joules.}$$

The required increase in potential energy is:

$$\frac{GmM}{r} = \frac{(6.67 \times 10^{-11}) \times 70 \times 5.98 \times 10^{24}}{6.38 \times 10^6} = 4.4 \times 10^9 \text{ joules.}$$

From the above, it is apparent that, in this case, the kinetic energy is negligible compared to the required change in potential energy.

$$E_B = \frac{GmM}{r} - \frac{1}{2}mv^2 = 4.4 \times 10^9 \text{ joules} - 7.5 \times 10^6 \text{ joules}$$

$$\approx \underline{4.4 \times 10^9 \text{ joules.}}$$

b) To find the binding energy of the moon we must compute both $\frac{GmM}{r}$ (where m is now the mass of the moon, and r is the radius of the moon's orbit about the earth, and also the moon's kinetic energy, $\frac{1}{2}mv^2$. (The kinetic energy will prove to be a significant fraction.)

The moon travels a path of length $2\pi r$ in a time T, its period. Therefore

$$v = \frac{2\pi r}{T} \text{ and } E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{4\pi^2 r^2}{T^2} = \frac{2\pi^2 mr^2}{T^2}.$$

From page 334 of the text we have: $m = 7.34 \times 10^{22}$ kg, $r = 3.8 \times 10^8$ meters, and $T = 2.36 \times 10^6$ seconds. We can then find the kinetic energy:

$$E_k = \frac{2\pi^2 mr^2}{T^2} = 2\pi^2 \times 7.34 \times 10^{22} \text{ kg} \times \left(\frac{3.8 \times 10^8 \text{ m}}{2.36 \times 10^6 \text{ m}}\right)^2 = 3.8 \times 10^{28} \text{ joules.}$$

The potential energy, $\frac{GmM}{r}$, is found using $G = 6.67 \times 10^{-11}\ \frac{\text{nt-m}^2}{\text{kg}^2}$, $M = 5.98 \times 10^{24}$ kg, and $r = 3.8 \times 10^8$ meters.

$$U = \frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22} \times 5.98 \times 10^{24}}{3.8 \times 10^8} = 7.7 \times 10^{28} \text{ joules.}$$

The binding energy is therefore:

$$E_B = \frac{GmM}{r} - \frac{1}{2}mv^2 = (7.7 - 3.8) \times 10^{28} \text{ joules} = \underline{3.9 \times 10^{28} \text{ joules}}.$$

(In detail $\frac{GmM}{r} = mv^2$; $\therefore$ the kinetic energy supplies just 1/2 the needed change in potential energy — we must supply the other half.)

PROBLEM 23

The force field between a pair of protons is repulsive. Does the potential energy increase or decrease as a pair of protons are brought together?

Since the force between the protons is repulsive, they resist being pushed together, or fly apart if released from rest. When accelerating apart, they are changing potential energy to kinetic energy. There is therefore more potential energy when they are compressed and less as they go apart.

Another way to look at the situation is to note that we transfer energy to the protons in order to push the protons together. Consequently their potential energy is increased when they are brought closer together. The situation is analogous to that of a compressed spring.

PROBLEM 24

The force of repulsion in newtons between two electrons is given by $F = \frac{2.30 \times 10^{-28}}{x^2}$ where x is the separation between them in meters. If each of the electrons has a mass of 9.00×10^{-31} kg and they are held 1.00×10^{-10} m apart and then released, what will be:

(a) the kinetic energy of each when they are 2.00×10^{-10} m apart?

(b) the velocity of each?

This problem may be a bit easier if it is preceded by Problem 23.

The force, F, between two electrons is given by

$$F = \frac{2.30 \times 10^{-28}}{x^2};$$

hence, this is an inverse square law and depends on the separation in the same way as does the gravitational force. The fact that this is a force of repulsion means however, that, in any relative displacement, the change in potential energy is the negative of what it would be if the force were one of attraction.

If the force were one of attraction, by analogy to the expressions for gravitational attraction, the potential energy of the particles at a separation, x, would be

$$U = -\frac{2.30 \times 10^{-28}}{x},$$

where we take the potential energy to be zero when the particles are a great distance apart. Since the force is one of repulsion we need the negative of this result to get the actual potential energy at separation x:

$$U \text{ at separation } x = +\frac{2.30 \times 10^{-28}}{x}.$$

As x increases, the potential energy decreases, as discussed in Problem 23.

a) Since the electrons are released from rest, they move in opposite directions; their velocities being equal and opposite to conserve the total momentum. Therefore their kinetic energies are equal. Denoting the mass of either particle by m and the magnitude of the velocity of either by v, $2 \times (1/2\ mv^2) + U = \text{constant (the total energy)} = \frac{2.30 \times 10^{-28}}{1.00 \times 10^{-10}}$. (Since when $x = 1.00 \times 10^{-10}$ m, $v = 0$.)

The increase in kinetic energy $2\left(\frac{1}{2}mv^2\right)$ = decrease in potential energy. At a separation of 2.00×10^{-10} m, the potential energy, U, is given by:

$$U = \frac{2.30 \times 10^{-28}}{2.00 \times 10^{-10}}.$$

Therefore, $$2 \times \left(\frac{1}{2} mv^2\right) = \frac{2.30 \times 10^{-28}}{1.00 \times 10^{-10}} - \frac{2.30 \times 10^{-28}}{2.00 \times 10^{-10}},$$

or, $$\frac{1}{2} mv^2 = \underline{0.575 \times 10^{-18} \text{ joules.}}$$

b) Since $m = 9.00 \times 10^{-31} = 0.900 \times 10^{-30}$ kg,

$$v = \sqrt{\frac{2}{m} \times 0.575 \times 10^{-18}} = \sqrt{\frac{2 \times .575 \times 10^{-18}}{0.900 \times 10^{-30}}} = \underline{1.13 \times 10^6 \text{ m/sec.}}$$

PROBLEM 25

What is the binding energy of the proton and electron in a hydrogen atom if the proton and electron are 0.50×10^{-10} m apart and the force of attraction between them is given by

$$F = \frac{2.3 \times 10^{-28}}{r^2}$$

newtons, when *r* is in meters?

Remember, the electron will not stand still long with a force on it. Assume it is moving in a circle around the proton.

In Section 25-4 the potential energy corresponding to an attractive force $\frac{GMm}{r^2}$ was shown to be $U = -\frac{GMm}{r}$, if the potential energy is taken to be zero at infinite separation. The force of attraction between an electron and a proton is $\frac{k}{r^2}$. The corresponding potential energy is therefore $U = \frac{-k}{r}$. The total energy is then:

$$E = \frac{-k}{r} + \frac{1}{2} mv^2,$$

where m is the mass of the electron and v its speed.

Since the mass of the proton is much greater than that of an electron, we can (to a good approximation) neglect its motion just as we neglect the motion of the sun due to the force of the planets. For uniform circular motion, the acceleration toward the center (i.e., the acceleration of the electron toward the proton) is $\frac{v^2}{r}$. Equating ma to force then gives:

$$\frac{mv^2}{r} = \frac{k}{r^2}, \text{or } mv^2 = \frac{k}{r}.$$

Thus the total energy is:

$$E = \frac{-k}{r} + \frac{1}{2} \times \frac{k}{r} = -\frac{k}{2r}.$$

If the electron were pulled just free of the proton the total energy of the system would be 0 (0 kinetic energy is infinite separation, where U = 0 also). Therefore to free the electron we would need to add an amount of energy:

$$E_B = \frac{k}{2r} = \frac{2.3 \times 10^{-28}}{2 \times 0.50 \times 10^{-10}} = \underline{2.3 \times 10^{-18} \text{ joules.}}$$

PROBLEM 26

A drain pipe pointing slightly downhill sticks out from a retaining wall at the side of the road. A ball thrown up the pipe comes back with a greater speed than that with which it was thrown. If this happened to you, would you be surprised? What would you suspect?

Whether or not one would be surprised no doubt depends on whether he realized that the returning ball has gained in total energy (the potential energy is the same as when it was thrown since it is at the same place, but the kinetic energy is larger). Let's hope the students would be surprised! The most probable explanation would be that someone had caught it at the other end of the pipe and had thrown it back down.

PROBLEM 27

With about what fraction of the original mass can a rocket using chemical fuel "escape" from the earth?

In Section 25-5, the excape velocity was calculated as 11.2 km/sec. The amount of fuel required depends very strongly on the exhaust velocity. If we assume an exhaust velocity, $v_e = 2$ km/sec, the required rocket velocity can be expressed as $\frac{11.2}{2} v_e = 5.6 v_e$. Figure 23-15b (page 375) was obtained using conservation of momentum. From this figure we find that the ratio of final mass to original mass is about 0.0037 for $\frac{v}{v_e} = 5.6$. (Note that $0.0037 = e^{-5.6}$.) We can see the significance of improved fuels by noting the effect of a 10% increase in exhaust velocity. For $v_e = 2.2$ km/sec, $v/v_e = 11.2/2.2 = 5.1$, and the mass ratio increases to 0.0061, almost twice as much pay load as with the slightly lower exhaust velocity. Doubling the exhaust velocity would give a ratio of about 0.06, an improvement by a factor of about 16!

Chapter 26 — Heat, Molecular Motion, and Conservation of Energy

This chapter serves a two-fold purpose. First it provides a major link in the energy conservation chain. In any experiment involving mechanical interactions, a transfer of energy is almost always accompanied by the conversion of some mechanical energy into heat. In developing the concept of heat as a form of energy, the text accomplishes a second, equally important, objective. Chapter 26 provides a natural transition from the problems of mechanics to the ideas of atomic structure in Part IV. This bridge is provided by treating heat from the point of view of kinetic theory.

Sections 1 and 2 discuss the motion of the molecules of a perfect gas. Relations are developed between the momentum and kinetic energy of individual molecules on the one hand, and the pressure and temperature of the gas as a whole, on the other. The sections present the kinetic theory view of gas pressure, and point out the proportionality of absolute temperature and mean kinetic energy of center of mass motion of the molecules. The concept of thermal energy is developed, and a distinction is drawn between total thermal energy and kinetic energy of center of mass motion.

Sections 3 through 7 tie up the remaining loose ends. Mechanisms for conversion of bulk kinetic energy to random molecular kinetic energy are discussed. Experiments which establish the mechanical equivalent of heat are described. Heat transfer and calorimetry are described briefly. The chapter closes with a summary of the breadth and scope of energy conservation.

SCHEDULING CHAPTER 26

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part III.

	15-week schedule for Part III			9-week schedule for Part III		
Subject	Class Period	Lab Period	Exp't.	Class Period	Lab Period	Exp't
Secs. [1], [2]	4	0	-	2	0	-
Secs. [3], 4, 5, 6, 7	4	1	III-14	2	0	-

Laboratory. Experiment III-14, A Head-on Collision, uses the roller skate carts with spring loaded bumpers and the ticker-tape timer to analyze energy changes in a collision. By using less elastic bumpers, increased loss of kinetic energy is shown. This experiment can be done at any point in the discussion of Chapter 26.

Home, Desk and Lab. A number of the problems for this chapter deal qualitatively with energy transformations in systems such as gasoline engines or machine guns. Discussion of some of these problems will be an effective way to drive home the concept of heat as kinetic energy of molecular motion. If you are pressed for time, discussion of such problems can carry the principal load in developing this chapter. Problems which are well suited to this purpose are numbers 1, 10, 21, and 22. Problems 7 and 8, involving the behavior of an ideal gas, and Problems 16 and 19, concerned with the mechanical equivalent of heat, are interesting if you have enough time.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages.

Sections	Easy	Medium	Hard	Class Discussion	Home Projects
1	-	1*	2	2	
2	4*	3, 5**, 6*	7, 8	4	
3	9	10*		10	
4	11, 16				12, 13, 14, 15
5, 6	17, 18	19, 20			
7	16	10, 21*, 25	22*, 23, 24	10, 21, 22	15, 26, 27

** Students will find Problem 5 easier if it is preceded by Problem 4.

Films. "Mechanical and Thermal Energy", by Jerrold R. Zacharias of the Massachusetts Institute of Technology. This film views energy conservation microscopically, relating random molecular kinetic energy to the macroscopically observed variables. Using models, this film shows the interconnection between energy of bulk motion and thermal energy of random motion, how random motion can average out to a smooth effect, and how thermal conduction occurs. This film is best scheduled with Sections 3 through 7. Running time: 22 minutes.

"Conservation of Energy" shows the kind of "energy accounting" that can be done at a power plant by tracing the gross energy changes from the chemical potential energy of coal, through thermal and mechanical energy to electrical output. This film can be scheduled with the latter parts of the chapter. Running time: 27 minutes.

Section 1 – Gas Pressure
Section 2 – Temperature and Molecular Kinetic Energy; Thermal Energy

PURPOSE 1) To develop further the atomic model of matter through the study of the kinetic theory.

2) To introduce the idea that energy resides in the random motion of atoms, and to identify this as thermal energy (heat).

CONTENT a. The force exerted by a gas on the walls of its container can be understood as the average effect of many very small impacts of the gas molecules.

b. The force exerted per unit area, the pressure, is related to the mean kinetic energy of the molecules by $P = \frac{2}{3} E_K \frac{N}{V}$ where P is the pressure, N is the number of molecules in a volume, V, of gas, and E_K is the mean kinetic energy of center of mass motion of a single molecule.

c. E_K is directly proportional to the absolute temperature of the gas: $E_K = \frac{3}{2} k\ T$; $k = 1.37 \times 10^{-23}$ joules/molecule –°K.

d. A one mole sample of any monatomic molecular gas (He, A, Ne, etc.) rises 1°K in temperature when 12.4 joules of energy is fed into it. Gases constituted of more complicated molecules (H_2, O_2, N_2, etc) may require more energy input to produce the same temperature change, since some energy may go into rotation and vibration.

e. This close relationship between random molecular kinetic energy and temperature (and therefore heat) supplies one more link in the chain of energy conservation arguments.

EMPHASIS While these sections contain many new ideas, some rather subtle, the time that you spend on this material can be adjusted to bring your over all schedule into line with your requirements for the remainder of the course. Minimally, students should get some feeling for the possibility of conversion of "organized" kinetic energy into the kinetic energy of random molecular motion.

COMMENTS Some students may see that the derivation given in the text is highly simplified. They may note that it implicitly assumes that all particles or molecules travel with the same speed.

Even if all molecules have the same speed initially, they eventually collide with one another. Even if these collisions are elastic, they produce shifts in speed which would eventually lead to a broad spectrum of speeds. When two such objects collide elastically while their center of mass is at rest with respect to the laboratory system, no change in speeds results in the laboratory system. When their center of mass is in motion (e.g., collisions at right angles) changes in speeds result. A molecule moving with speed 3v would have nine times the kinetic energy of one moving with speed v, but would possess only three times the momentum of the slower molecule. Thus the relation between pressure and kinetic energy must somehow depend on the actual distribution of speeds of various molecules. A rigorous proof is possible (one which takes into account the actual distribution of velocities), and it gives the same result as has been simply derived in this section. The actual distribution of velocities can be shown to be simply the one that is statistically most probable. It is called Maxwell's distribution.

* * *

In these sections we are at the very heart of the kinetic theory of gases, one of the great achievements in physics. We show (or at least present plausibility arguments in lieu of more complicated but exact proofs) that the bulk thermal properties of matter can be described exactly in terms of the dynamics of the motion of the individual molecules comprising a substance. Yet some students may feel that, after all, we have defined the equivalence of temperature and kinetic energy by our theory — so why should it be surprising that they turn out to be equivalent! The answer is that there is no quantity in dynamics which corresponds explicitly to the temperature. The parameters of mechanics are mass, force, speed, acceleration, momentum, energy, etc. The remarkable accomplishment of the kinetic theory is that it identifies such a complicated macroscopic property as temperature with so simple a microscopic dynamical property as the kinetic energy of motion of the centers of mass of the molecules.

* * *

Of course, the whole of kinetic theory would be purely formalistic if it were not possible to confirm the model by direct observation. There are several easily verifiable predictions which have been found to be true. Some of these have been discussed earlier, in Part I, Chapter 9, but might well be reviewed in the present context. For example, from the simple relation $kT = \frac{2}{3}\frac{(mv)^2}{2}$, we can conclude:

1) That the average speed of motion of molecules of mass m at absolute temperature T is $\bar{v} = \left(\frac{3kT}{m}\right)^{1/2}$. Typically, for hydrogen (H_2, $m \approx 3 \times 10^{-27}$ kg) at room temperature ($T \approx 300°$ K), $v \approx 2 \times 10^3$ m/sec, a bullet-like speed which can easily be measured by letting molecules escape one by one into a vacuum system.

2) For a collection of molecules of different masses, all at the same temperature, the relative average speeds will be inversely proportional to the square roots of the masses — a fact which is verifiable by measuring the relative rates of diffusion through a porous membrane (this is the basis for the separation of uranium isotopes by the gaseous diffusion process used at the great plants at Oak Ridge).

The words "ideal gas" recur frequently in discussions of the kinetic theory. An "ideal" gas is one in which the molecules are approximated to be geometrical points which interact only through very short-range forces, by colliding with one another in the manner of perfectly elastic spheres. This is our basic model for the kinetic theory, and is approximated by simple gases at low pressure.

* * *

The kinetic theory has quite broad applicability. Not only can we explain the thermal properties of simple gases, but we can also understand why "nonsimple" substances like polyatomic gases and liquids and solids exhibit different thermal behavior. We can account for the differences quantitatively for the polyatomic gases. However, for condensed systems, we are limited by the extent of our knowledge of the actual forces of interaction between the molecules.

Of course, if molecules actually did not interact through long-range forces, we would never see matter in the condensed liquid and solid states. Thus we know that the "ideal" gas model is obviously only an approximation. One of the most striking pieces of evidence of the presence of long-range forces between molecules is the drop in temperature of a gas which is suddenly allowed to expand freely by passing through a nozzle into a large chamber. This phenomenon is usually called the Joule-Thompson effect, and is the basis for most refrigeration machines. In a free expansion, no work is done by the gas, and there would be no energy transfer involved unless there were some potential energy associated with the average separation of the molecules. Thus if there were no long-range interactions between the molecules, their average kinetic energy would not be affected by the distance between them, and there would be no temperature changes associated with a free expansion. (See Appendix 10 at the end of this volume for a discussion of the energy inherent in polyatomic molecular systems. See also HDL Problem 24.)

QUIZ PROBLEMS 1. A box contains a mixture of one mole each of oxygen (molecular weight 32) and argon (molecular weight 40). What is the ratio of the average speed of the oxygen molecules to that of the argon?

$$\frac{1}{2}m_o v_o^2 = \frac{1}{2}m_a v_a^2; \text{ therefore, } v_o/v_a = (m_a/m_o)^{1/2} = (40/32)^{1/2} = 1.12.$$

2. If the box in Problem 1 is at a temperature of 273° K and the gas is at a pressure of 1 atmosphere,

(a) What is the volume of the box?
(b) What is the average kinetic energy of the center of mass motion of the oxygen molecules?
(c) What is the average kinetic energy of the center of mass motion of the argon motion?

a. One mole of any gas, if contained in a volume of $2.24 \times 10^{-2} m^3$ at a temperature of 273° K, will exert a pressure of 1 atmosphere. Inasmuch as this problem specifies 1 mole each of two gases, the pressure in a $2.24 \times 10^{-2} m^3$ container at 273° K would be 2 atmospheres. To reduce it to one atmosphere, the volume would have to be doubled to $4.48 \times 10^{-2} m^3$.

b. $E_K = 12.4 \times 273 = 3.39 \times 10^3$ joules.

c. $E_K = 3.39 \times 10^3$ joules. The center of mass kinetic energy is the same for all molecules at the same temperature. Oxygen molecules could have, in addition, energy of rotation, vibration etc.

3. A box contains a fixed volume of helium at 0° C at a pressure of 2 atmospheres. The box is dropped into boiling water (temperature 100° C). If the box does not change in volume, what is the average kinetic energy of the gas per unit volume? (Take 1 at-

mosphere = 10^5 nt/m^2.)

$$P_1/P_2 = T_1/T_2, \text{ or } 2/P_2 = 273/373. \quad P_2 = \frac{746}{273} = 2.7 \text{ atmospheres;}$$

$$E_K = 373 \times 12.4 = 4.62 \times 10^3 \text{ joules/mole.}$$

The density of the gas does not change during the heating since the volume of the box and the quantity of gas remain constant. Originally the pressure at 273° K was 2 atmospheres. Thus there were 2 mole/$2.24 \times 10^{-2} \text{m}^3$. Then the kinetic energy per unit volume would be

$$E_k \times \frac{\text{moles}}{\text{volume}} = 4.62 \times 10^3 \times \frac{2}{2.24 \times 10^{-2}} = 4.12 \times 10^5 \text{ joules/m}^3.$$

4. 4×10^{24} gas molecules are in a box at a temperature of 50°K. They exert a pressure of 0.03 atmospheres. What is the volume of the box?

$$PV = NkT$$

$$V = \frac{NkT}{P} = \frac{4 \times 10^{24} \text{ molecules} \times 1.37 \times 10^{-23} \frac{\text{newton-meter}}{\text{molecule-°K}} \times 50°\text{K} \times 10^{-5} \frac{\text{atm-m}^2}{\text{newton}}}{0.3 \text{ atm}}$$

$$= 0.09 \text{ m}^3.$$

Section 3 – Mechanical Energy of Bulk Motion and Internal Energy
Section 4 – The Equivalence of Mechanical and Thermal Energy
Section 5 – Heat Flow
Section 6 – Quantitative Relation of Energy Dissipation and Temperature Rise
Section 7 – Conservation of Energy

PURPOSE These sections amplify the principle of equivalence of mechanical and thermal energy.

EMPHASIS These topics are all relevant and interesting, but the main points involved in energy conservation have been made previously. Therefore the time spent on this material is strongly dependent on your over all progress through your schedule for the course.

LABORATORY Experiment III-14 is designed to give students a feel for situations in which some mechanical kinetic energy is converted into heat.

FILMS "Mechanical and Thermal Energy" and "Conservation of Energy" can be used advantageously in connection with work on these sections.

DEVELOPMENT To help give students a good feeling for the magnitude of the mechanical equivalent of heat, you might ask them to work out a few simple energy balance problems. For example, "How far can a 50-kg man climb on one slice of bread?" Assume there are about 100 Calories or about 4.2×10^5 joules in the piece of bread. Then the man can increase his potential energy by an amount $mgh = 4.2 \times 10^5$ joules "on one slice of bread". Solving, $h = \frac{4.2 \times 10^5}{50 \times 9.8} \approx 850 \text{ m}$, quite a fair climb! (Overweight students may see why it is better not to have eaten that "one slice of bread" than to try to exercise it off!) See Problem 15 in HDL.

Students (at least those who do not exercise violently) may wonder where all the food energy in the daily 2000-odd Calorie intake goes. Fortunately (for the lazy ones), most of it is consumed in just keeping us alive and warm, keeping our heart and lungs going and our brains clicking away. For an average person, this basic amount comes to about 100 watts (10^2 joules/sec or about 8.6×10^6 joules per day). Since this energy is just about that supplied by 2000 Calories per day, we can hold our weight without climbing ten mountains a day.

COMMENTS As shown in the film "Conservation of Energy", many large industries keep a complete energy record. A power plant using river cooling usually knows the input and output temperatures of the cooling water, the temperatures of stack gases, the heating of generators and the energy rating of their fuels. Of course they also keep a strict record of the electrical energy delivered to their customers. If they do not see the film, some students might be interested in visiting a local power plant to see how such an "energy accounting" is carried out.

* * *

To most professional physicists, there is no other concept in physics with a grandeur equal to that of the law of conservation of energy. All interactions in the universe are inter-related through this one quantitative framework. Yet even this law, like all physical laws, rests ultimately on experimental observations. It is possible that over the whole universe, of which we have accurately observed only a tiny fraction, energy is not conserved. While such a possibility cannot be excluded, it should be understood that a valid observation of non-conservation of energy would be more than a ripple on the pond of physics. Our fundamental ideas about space and time are intimately connected with our belief in energy conservation, and a change in this "law" would cause an enormous revolution in physics. (See Appendices 11, 12, and 13 at the back of this volume of the Guide.)

QUIZ PROBLEMS 1. If the energy content of a slice of bread (100 Calories) is converted to heat and used to raise the temperature of 2 liters of water from an initial temperature of 10° C, what will be the final temperature of the water?

1 Calorie will raise the temperature of 1 liter of water 1° C. Therefore 100 Calories will raise the temperature of 2 liters of water, 50° C. The final temperature will be 60° C.

2. A high school student consumes 2000 Calories of food in one day. If he needed no energy to sustain his body and could convert this entire amount to useful work, how high could he raise a 100-kg mass with this amount of energy?

$$2000 \text{ Calories} = 2 \times 10^6 \text{ calories} = 8.4 \times 10^6 \text{ joules}$$

$$= 8.4 \times 10^6 \text{ kg-m}^2/\text{sec}^2 = 100 \times 9.8 \times h.$$

$$h = 8.6 \times 10^3 \text{ meters.}$$

3. How much energy is released when 100 grams of water cool from 90°C to 25°C? Express your answer in joules.

$$90°\text{C} - 25°\text{C} = 65°\text{C}.$$

$$100\text{gm} \times 65°\text{C} \times 1 \text{ calorie/gm}°\text{C} = 6.5 \times 10^3 \text{ calories}$$

$$= 6.5 \times 10^3 \text{ cal} \times 4.2 \text{ joules/cal}$$

$$= 2.73 \times 10^4 \text{ joules.}$$

4. 2.1×10^3 joules of work are done in heating 500 grams of water. If the initial temperature of the water is 10°C, what is its final temperature?

2.1×10^3 joules $= \frac{2.1}{4.2} \times 10^3$ calories $= 5 \times 10^2$ calories which will raise the temperature of 500 g of water 1°C. $T_f = 11°\text{C}$.

5. A 20-kg mass traveling at a speed of 10 m/sec strikes a 5-kg mass which was originally stationary. The two masses stick together after the collision.

(a) What is the final speed of the combination?
(b) What was the initial kinetic energy of the system?
(c) What is the final total energy of the system?

(d) How many calories of heat were produced in the collision?

a. $p_i = 20\text{ kg} \times 10\text{ m/sec} = 200\text{ kg-m/sec}$.

$p_f = 200\text{ kg-m/sec} = 25\text{ kg} \times v_f,\ v_f = 8\text{ m/sec}$.

b. $(E_K)_i = \frac{1}{2} \times 20 \times (10)^2 = 10^3$ joules.

c. $(E)_f = 10^3$ joules, by energy conservation.

d. $(E_K)_f = \frac{1}{2} 25 \times (8)^2 = 800$ joules.

Heat $= (E)_f - (E_K)_f = 200$ joules $= \frac{200}{4.2} = 48$ calories.

6. A hot water heater breaks. A man wishes to heat his bath water from 25°C to 35°C. The tub contains 30 kg of water. How long will it take him to heat the tub if he uses a canoe paddle which he moves through the water at a rate of one meter per second while exerting an average force of 30 newtons? Assume all the work he does goes into heating the water.

Work $= 30\text{ nt} \times 1\text{ m/sec} \times T\text{ (secs)} = 30\,T$ joules.

Heat $= (35°\text{ C} - 25°\text{ C})\, 30 \times 10^3 = 3 \times 10^5$ calories $= 12.6 \times 10^5$ joules.

12.6×10^5 joules $= 30\,T$ joules.

$T = 4.2 \times 10^4$ seconds (about 12 hours).

Chapter 26 – Heat, Molecular Motion, and Conservation of Energy

For Home, Desk and Lab – Answers to Problems

A number of the problems for this chapter deal qualitatively with energy transformations in systems such as gasoline engines or machine guns. Discussion of some of these problems will be an effective way of getting students to understand the concept of heat as kinetic energy of molecular motion. Problems which are well suited to this purpose are numbers 1, 4, 10, 21 and 22. Problems 7 and 8, involving the behavior of an ideal gas, and Problems 16 and 19, concerned with the mechanical equivalent of heat, are interesting if you have enough time. Students will find Problem 5 easier if it is preceded by Problem 4.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Sections	Easy	Medium	Hard	Class Discussion	Home Projects
1		1*	2	2	
2	4*	3*, 5, 6*	7, 8	4	
3	9	10*		10	
4	11, 16				12, 13, 14, 15
5, 6	17, 18	19, 20			
7	16	10, 21*, 25	22* 23, 24	10, 21, 22	15, 26, 27

SHORT ANSWERS

1. (a) 20 newtons.
   (b) 1.8×10^5 joules.
   (c) 10 joules/meter.
2. (a) Less (halved).
   (b) Undefined.
3. (a) 6.15×10^{-21} joules.
   (b) 20.6 joules.
4. See discussion on page 26-12.
5. (a) 25 joules.
   (b) 275° K.
6. Increases by 2%.
7. (a) $V_2 = 3.06 \times 10^{-2}\ m^3$.
   (b) 3.12×10^3 joules.
   (c) 4.4×10^4 joules.
   (d) 1/14.
8. (a) 4.8×10^2 m/sec.
   (b) 1.9×10^3 m/sec.
   (c) Nearly the same.
   (d) 4.8×10^2 m/sec.
   (e) The same.
9. (a) See discussion on page 26-16.
   (b) (i) 450 joules.
   (ii) 2.25 joules.
   (iii) 447.75 joules.
   (c) 99.5%.
10. See discussion on page 26-16.
11. 0.38° C.
12. A home project.
13. A home project.
14. A home project.
15. A home project.
16. (a) 600 newtons.
    (b) See discussion on page 26-19.
    (c) See discussion on page 26-19.
17. (a) 820 m/sec.
    (b) None.
    (c) See discussion on page 26-20.
18. 2.95 gm.

19. (a) 2×10^6 joules.
    (b) 8×10^6 joules.
    (c) 4×10^6 calories.
    (d) See discussion on page 26-21.
20. (a) 35° C.
    (b) 35° C.
    (c) Lower than 35° C.
    (d) See discussion on page 26-22.
21. (a) Chemical potential energy.
    (b) See discussion on page 26-22.
    (c) See discussion on page 26-23.
22. See discussion on page 26-23.
23. See discussion on page 26-24.
24. See discussion on page 26-25.
25. (a) -70 m/sec.
    (b) 9×10^5 joules;
    2.7×10^7 joules;
    0.37×10^7 joules.
    (c) See discussion on page 26-27.
26. A home project.
27. A home project.

COMMENTS AND SOLUTIONS

PROBLEM 1

A machine gun fires a stream of 10-gram bullets at the rate of 400 rounds per minute. The bullets, moving at velocity 300 m/sec, hit a wall of solid rock and stop dead. Calculate:

(a) the force on the wall.

(b) the kinetic energy of the bullets arriving at the wall in one minute.

(c) the kinetic energy of the bullets in 1 meter length of the stream as they approach the wall. Compare twice this answer with your answer to (a).

a) Each bullet loses an amount of momentum $mv = 10^{-2}\,\text{kg} \times 300\,\text{m/sec} = 3\,\text{kg-m/sec}$. The rate of change of momentum of the stream of bullets in one second (400 bullets/min = 6.67 bullets/second) is

$$\frac{\Delta p}{\Delta t} = 6.67/\text{sec} \times 3\,\text{kg-m/sec} = 20\,\text{kg-m/sec}^2.$$

The average force is given directly by this rate of change in momentum:

$$F = \frac{\Delta p}{\Delta t} = 20\,\text{kg-m/sec}^2 = \underline{20\text{ newtons}}.$$

b) The kinetic energy of each bullet is

$$\frac{1}{2}mv^2 = \frac{1}{2} \times 10^{-2} \times (300)^2 \text{ joules} = 450 \text{ joules}.$$

In one minute 400 bullets hit the wall. Their total kinetic energy is

$$400 \times 450 \text{ joules} = \underline{1.8 \times 10^5 \text{ joules}}.$$

c) The bullets are fired with a speed of 300 m/sec or 18,000 meters/minute. But, 400 bullets are fired in a one minute interval, therefore the average number of bullets in a one meter interval is

$$\frac{400 \text{ bullets/min}}{18{,}000 \text{ meters/min}} = \frac{1}{45} \text{ bullet/meter}.$$

Since each bullet has a kinetic energy of 450 joules, the average kinetic energy in one meter length is

$$450 \text{ joules/bullet} \times \frac{1}{45} \text{ bullets/meter} = \underline{10 \text{ joules/meter}}.$$

Twice this is 20 joules/meter = 20 newtons (since 1 newton-meter = 1 joule).

* * *

Note. This last observation illustrates the relationship given on p. 427 of the text.

$$P = mv^2 \frac{N}{V}.$$

Multiplying both sides of this equation by A gives

$$PA = F = mv^2 \frac{NA}{V} = mv^2 \frac{NA}{LA} = 2\left(\frac{1}{2} mv^2\right) \frac{N}{L}$$

where L is the length of path in which N particles are to be found.

PROBLEM 2

Suppose there is a tiny sticky area on the wall of a container of gas. Molecules hitting this area stick there permanently.

(a) Is the pressure there greater or less than on the ordinary areas of the wall?

(b) If the sticky region is cut out, leaving a hole, what is the pressure on the hole?

Be prepared to discuss your answers in class.

The two hypothetical situations presented in this problem stimulate thinking of gases in terms of motion of molecules.

a) If the piece of wall under consideration were just like the rest of the wall, the force it would exert on the stream of molecules approaching it would be sufficient to reverse their component of momentum perpendicular to the wall. If it is sticky, so that molecules that strike it stay on its surface, the change in momentum of the molecules is halved, and the force (and consequently the pressure) on the sticky piece of wall will be halved.

b) The phrase, "pressure on the hole" has very little meaning. When we speak of the pressure on a piece of a wall we refer to the force on that piece, divided by the area of the piece. There is obviously no force exerted where there is no wall to push on. Therefore there is no pressure in the sense that we have described it.

(Note: The pressure on the walls of a container depends on the mass and average velocity of the gas molecules, and these quantities are the same throughout the volume of the gas. It is useful to extend our definition of pressure so as to include the concept of the pressure of the gas, a property possessed by a gas throughout its volume. We now imagine a pressure-measuring device to be a small, evacuated box with a movable piece of wall connected to a spring balance. This box is placed at any point in the volume of the gas, and the force on the piston is indicated. This instrument would measure a pressure in the gas at the hole.

PROBLEM 3

In a certain gas $\frac{2}{5}$ of the energy of the molecules is tied up in motion of the atoms around each other and $\frac{3}{5}$ in motion of the centers of mass.

(a) On the average, what is the kinetic energy of the center of mass motion of one such molecule when the temperature is 300°K?

(b) If the temperature is raised 1°C, what energy must be supplied to a mole (0.6025×10^{24} molecules) of the gas?

a) The temperature measures only the average kinetic energy of the center of mass of the molecules. Numerically,

$$E_K = \frac{3}{2}(1.37 \times 10^{-23})T = \underline{6.15 \times 10^{-21} \text{ joules}}.$$

b) We are told that this is only 3/5 of the actual kinetic energy. The other 2/5 exists in the form of rotational kinetic energy. When we raise the temperature of the gas by 1°K, the kinetic energy of the center of mass and the energy of motion about the center of mass will increase together. For the center of mass, $E_K = 3/2\,kT$, and for a 1° temperature rise, $\Delta E_K = \frac{3}{2}k$. The total kinetic energy of one molecule will therefore be increased by

$$\frac{5}{3} \times \frac{3}{2}k = \frac{5}{2}k.$$

To produce a 1°K temperature rise in the 0.6025×10^{24} molecules in a mole of gas, the required increase of energy will be:

$$0.6025 \times 10^{24} \times \frac{5}{2}k \text{ joules} = \underline{20.6 \text{ joules.}}$$

PROBLEM 4

A gas in a cylinder pushes a piston out, increasing its volume by ΔV. The gas exerts a pressure P on the face of the piston, which has an area A. The force exerted by the gas moves the piston a distance Δx, transferring energy $F\Delta x$ to some outside machinery. Show that the work $F\Delta x$ is equal to $P\Delta V$.

Note: This is a very useful expression for work whenever we deal with a gas or a liquid pushing a piston: work equals the pressure times the change in volume.

The force F on the piston is PA where A is the area of the piston. When the piston moves outward through a distance Δx, the work done is FΔx (the force times the distance through which the force moves). Now FΔx = PAΔx, and from the geometry of the piston, AΔx is the volume swept out by the end of the piston; it is the increase in volume of the gas. Thus,

$$W = F\Delta x = P\Delta V.$$

(Note: Where did the energy come from? This depends on what else happened to the gas while the piston moved. If the cylinder was completely insulated from other sources of heat (i. e., energy) then the energy must have come from the gas itself, i. e., from the kinetic energy of the individual molecules. What was the exact mechanism of the energy transfer? When molecules collide with the walls of the cylinder, or with the piston when it is stationary, their perpendicular component of momentum is reversed, the other components are unchanged, and consequently their kinetic energy is unchanged. When molecules collide with the piston moving outward, their perpendicular component of momentum relative to the piston is reversed, but their kinetic energy of motion in the cylinder is reduced on the average. (It is analogous to bouncing a rubber ball off the back of a truck. If the truck is not moving, a ball thrown at it with a speed of 5 m/sec will bounce off with the same speed. However, if the truck is receding at 1 m/sec, then the ball, traveling with a speed of 5 m/sec, approaches the truck at only 4 m/sec. This speed is then reversed. The ball recedes from the truck at 4 m/sec, therefore having a final speed of only 3 m/sec relative to the ground. Measured from the ground, the ball's kinetic energy has decreased considerably as a result of the collision.)

PROBLEM 5

One mole of an ideal monatomic gas (in practice, helium or argon) is placed in a cylinder at temperature 273°K. The gas is at atmospheric pressure, 1.02×10^5 newtons/m². At this pressure and temperature the gas occupies 2.24×10^{-2} m³.

A piston in the cylinder is then pushed in to decrease the volume by 2.45×10^{-4} m³.

(a) How much mechanical work must be done to push the piston in? (Neglect the change in pressure.)

(b) What is the final temperature of the gas if the container is completely insulated? (Remember 12.4 joules of energy raise its temperature one degree.)

a) The amount of work done in pushing the piston in a small distance is $P\Delta V$ where ΔV is the decrease in volume. (The argument is the same as that of Problem 4 except that the work must be done by some external agency.)

$$P\Delta V = 1.02 \times 10^{5} \times 2.45 \times 10^{-4}\,(\text{nt/m}^2)(\text{m}^3) = \underline{25.0 \text{ joules}}.$$

b) If the container is completely isolated, then all of the work done on the gas in compressing it (25.0 joules) must be transformed into kinetic energy of the gas molecules. The molecules are in fact single atoms, with no kinetic energy of rotation so the energy required for a 1°K temperature rise of one mole of the gas is 12.4 joules. Our 25 joules will raise its temperature by $\frac{25.0^\circ}{12.4}$K, i.e., a fraction over 2°K. The final temperature will be $273^\circ + 2^\circ = \underline{275^\circ\text{K}}$.

PROBLEM 6

In Problem 5, by what fraction of its original value does the pressure change?

In Problem 5, we found that the temperature must have changed by 2°K. The equation of state for an ideal gas is $PV = NkT$. Since N is the same in the initial and final states, we can write:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \text{ or}$$

$$\frac{P_2}{P_1} = \frac{V_1}{V_2} \times \frac{T_2}{T_1}.$$

Substituting from Problem 5:

$$\frac{P_2}{P_1} = \frac{2.24 \times 10^{-2}}{2.24 \times 10^{-2} - 2.45 \times 10^{-4}} \times \frac{273 + 2}{273}$$

$$\frac{P_2}{P_1} = \frac{1}{1 - \frac{2.45}{2.24} \times 10^{-2}} \left(1 + \frac{2}{273}\right) \approx 1.02.$$

Hence the pressure increases by about 2% of its original value.

PROBLEM 7

Suppose we take 18 grams of water (one mole) at boiling point and turn it into vapor in a cylinder. The cylinder is closed by a frictionless piston, which is so light that the gas remains at atmospheric pressure at all times.

(a) What volume will the water vapor occupy if it behaves as an ideal gas? (Water vapor is not an ideal gas, but the error due to the assumption will be less than 10 percent.)

(b) What work must be done in pushing the piston out against atmospheric pressure as the whole of the water vaporizes?

(c) Given that each gram of water takes 540 calories to tear its molecules apart into vapor, how much heat is needed to convert the water to vapor *and* push the piston out?

(d) What fraction of the total heat needed is converted to work in pushing the piston out?

a) We assume that water vapor behaves like an ideal gas, i.e., that it satisfies the equation of state $PV = NkT$. 18 grams of water is equal to 1 mole, and under standard conditions would occupy $2.24 \times 10^{-2}\ \text{m}^3$.

$$\frac{P_1V_1}{T_1} = \frac{P_1V_2}{T_2}$$

$$V_2 = V_1\frac{T_2}{T_1} = 2.24 \times 10^{-2} \times \frac{373}{273}$$

$$V_2 = \underline{3.06 \times 10^{-2}\,\mathrm{m}^3}.$$

b) The work done in pushing the piston out through a small volume ΔV was shown in Problem 4 to be $P\Delta V$. Here P is constant throughout the expansion, so the total work done is just P × (total change in volume). Since the initial volume is approximately 18 cm^3 (there are 18 gm of water and its density is 1 $\mathrm{gm/cm}^3$) or $1.8 \times 10^{-5}\,\mathrm{m}^3$, we can neglect the initial volume in comparison with the final volume. Thus the total work done is:

$$P\Delta V = 1.02 \times 10^5 \times 3.06 \times 10^{-2} = \underline{3.12 \times 10^3 \text{ joules}}.$$

c) Each gram of water requires 540 calories or $540 \times 4.2 = 2.27 \times 10^3$ joules to become vapor. The total energy needed to vaporize 18 gm of water is

$$18 \times 2.27 \times 10^3 \text{ joules} = 4.1 \times 10^4 \text{ joules}.$$

The total energy required, including the work done when the vapor expands, is thus

$$(4.1 \times 10^4) + (3.1 \times 10^3) = \underline{4.4 \times 10^4 \text{ joules}}.$$

d) The fraction of the total heat input which is transformed to work is:

$$\frac{3.1 \times 10^3}{4.4 \times 10^4} = \underline{\frac{1}{14}}.$$

The rest of the energy is required to tear the molecules of water apart, i.e., to overcome the very strong attraction between water molecules in the liquid state.

PROBLEM 8

(a) Estimate the speed of oxygen molecules at room temperature from the following data: 32 grams of oxygen at room temperature (20°C) at one atmosphere pressure, 1.02×10^5 newtons/m^2, occupy $2.4 \times 10^{-2}\,\mathrm{m}^3$.

(b) The same volume of hydrogen at the same temperature and pressure weighs only 2 grams. Estimate the average velocity of hydrogen molecules at room temperature.

(c) From your answer to (a), estimate the average velocity of nitrogen molecules at room temperature, *within 10 percent*.

(d) What is the average velocity of air molecules at room temperature correct within 10 percent?

(e) What is the velocity of oxygen molecules at room temperature and two atmospheres pressure?

a) In Section 1, the following equation relating P, N, V and the molecular kinetic energy was obtained:

$$P = \frac{N}{V}\left(\frac{2}{3} \times \frac{1}{2}\,mv^2\right).$$

In this problem we have 32 grams, or one mole, of oxygen; therefore N is Avogadro's number. The above equation can be rearranged to give:

$$v = \left(\frac{3PV}{Nm}\right)^{1/2}$$

We are given that $P = 1$ atmosphere $= 1.02 \times 10^5\ \text{nt/m}^2$, and $V = 2.4 \times 10^{-2}\ \text{m}^3$. The other quantities in the expression can be obtained when we realize that Nm, the number of molecules in a mole of the gas multiplied by the mass of each molecule, is just the mass of a mole of the gas. This is given as $32\ \text{gm} = 3.2 \times 10^{-2}$ kg. The average velocity is then given by:

$$v = \left(\frac{3 \times 1.02 \times 10^5 \times 2.4 \times 10^{-2}}{3.2 \times 10^{-2}}\right)^{1/2} \text{m/sec}$$

$$= \left(2.3 \times 10^5\right)^{1/2} \text{m/sec}$$

$$= \underline{4.8 \times 10^2\ \text{m/sec}}.$$

b) For one mole of hydrogen under the same conditions, we could go through the same procedure. However, it is a little simpler to use a slightly different argument: At the same temperature, the average kinetic energy of translation of hydrogen is the same as that of oxygen:

$$\frac{1}{2} m_H v_H^2 = \frac{1}{2} m_O v_O^2.$$

Therefore,

$$v_H/v_O = \sqrt{m_O/m_H} = \sqrt{\frac{32}{2}} = 4.$$

$$v_H = 4V_O = \underline{1.9 \times 10^3\ \text{m/sec.}}$$

c) The mass of a mole of nitrogen is 28 gm. Using the argument of part (b),

$\dfrac{v_N}{v_O} = (32/28)^{1/2} = 1.07$. Thus to within ten percent they are the same.

d) Air is composed predominantly of oxygen and nitrogen (in the ratio 20%: 80%). Since the velocities of their molecules are the same to within 10%, the average velocity for air molecules is, to this approximation, the same as that obtained in part (a). i.e.,

$$v_{av} = \underline{4.8 \times 10^2\ \text{m/sec}}.$$

e) In part (a) we found the speed of oxygen molecules at room temperature and one atmosphere of pressure. The kinetic energy of the air molecules depends only on T, so the average velocity will be the same even though the pressure is doubled, provided T is the same (in this case it is). In the expression used in part (a), P is doubled, so that (because of the equation of state) V is halved, and PV stays constant.

PROBLEM 9

A large bag of sand is hung from a tree by a long rope. A boy shoots a bullet into the sand bag and the bullet stays in the bag.

(a) Describe the energy changes.

(b) Suppose a 10-gram bullet is moving at 300 m/sec when it hits the bag, and the bag is a 1990-gram bag of sand. Calculate the amount of kinetic energy:

(i) the bullet had originally.

(ii) the bullet and bag have after collision.

(iii) that disappears.

(c) What fraction of the original kinetic energy of the bullet goes into heat?

a) The complications of what happens to the energy in the gun, etc. are discussed in Problem 10. The main parts of the energy chain are:

(i) Kinetic energy of the bullet; this is changed to

(iia) heat of the bullet and the sand surrounding it after it comes to rest, and
(iib) kinetic energy of the bag plus bullet. The bag swings, and we get the cyclic process of kinetic energy becoming gravitational potential energy, and vice versa. Very slowly, because of air friction, this will cease, and the energy will become

(iii) heat energy of the bag, its contents, and the surrounding air.

b) (i) Originally the kinetic energy of the bullet is:

$$\frac{1}{2}mv^2 = \frac{1}{2}\times 0.010 \times (300)^2 = \underline{450 \text{ joules}}.$$

(ii) During the collision of the bullet with the bag, momentum is conserved. (So is energy, as long as we include heat energy, but mechanical energy of bulk motion is not.) If, after the collision, bullet and bag move with velocity v', then

$$mv = (m + M)v'.$$

$$v' = \frac{mv}{m + M} = \frac{0.010 \times 300}{1.990 + 0.010} = 1.50 \text{ m/sec}.$$

The kinetic energy of the bag plus bullet is

$$\frac{1}{2}(m + M)\,v'^2 = \frac{1}{2}\times 2.000 \times 1.50^2 = \underline{2.25 \text{ joules}}.$$

(iii) Before the collision, the bullet had a kinetic energy of translation of 450 joules. After the collision, only 2.25 joules remain in the form of kinetic energy of bulk motion. 447.75 joules have disappeared from view. This energy has gone mainly into heat of the bullet and the sand immediately surrounding it. A little goes into potential energy of separation of the sand particles disturbed by the passage of the bullet. A negligible quantity goes into sound which is quickly dissipated as heat.

c) If we assume that all of the "lost" energy goes into heat, we get for the fractional part of the initial energy that goes into heat:

$$\frac{447.75}{450} = \underline{99.5\%}.$$

PROBLEM 10

A machine gun fires lead bullets at a wall and they stop dead. Describe the changes of energy from the stage when it is in the explosive to the stage when the bullets have been lying on the ground at the bottom of the wall for several hours.

The stages in the "chain" of energy transformations are:

(i) Chemical energy of the gunpowder.

(ii) Kinetic energy of thermal motion of the gases generated in the chemical process.

(iii) Kinetic energy of bulk motion of the bullet, plus that of the recoil of the gun.

(iv) Heat and sound in the air due to the passage of the bullet, and the sound when the bullet strikes the wall.

(v) Heat, and potential energy of deformation of the spent bullet, and of the region of the wall hit.

(vi) Heating of the air in the vicinity as the bullet and wall cool.

We neglect bulk potential energy, assuming that the bullet's path is short, and fairly flat. In that case, and assuming that the bullet falls only a few feet after hitting the wall, it is negligible.

The transformation (i) → (ii) involves the rearrangement of elements in the gunpowder to form more tightly-bound molecules. The potential energy of the whole system is reduced, and the liberated energy appears as thermal motion of these molecules.

In the step (ii) → (iii) the expanding hot gases do work $\int PdV$ on the bullet to give it kinetic energy. Not all of their thermal energy is transmitted, however. When the bullet leaves the end of the barrel, the gases rush out, and mingle with the surrounding air, heating it up as they cool.

The energy transformations of step (iv) reduce the kinetic energy of the bullet as it travels. The bullet traveling through the air generates a shock wave, a very intense pressure pulse. Such a wave passing through the air leaves it at a slightly higher temperature.

The transformation (iv) → (v) is the most sudden. The bullet may be brought to rest in a centimeter, whereas it was accelerated over a distance of almost a meter. The bullet is then markedly deformed, and heat is produced by the violent relative motion of the lead atoms.

All of this energy originated in the chemical process of explosion. You might ask the class to consider where it really came from. You can generate a discussion of arbitrary length on the problems of extracting the elements of gunpowder from the places where they are found. How did they come to be there? Everything can be traced to the sun's energy, which is far enough back for us to go. Problems 12, 17, and 18 give more quantitative practice with the problems of energy transformations, and the mechanical equivalent of heat.

PROBLEM 11

In one of his most famous experiments, Joule churned water with a paddle wheel driven by two loads, each of mass 14 kg, each falling vertically about 2 meters. He had about 7 kg of water to be heated. After each churning he hauled his loads up and let them fall again. What temperature rise would you expect him to find after twenty falls?

Note: this is a reversal of the logic of Joule's great experiment. *You* know that 4.2 joules will always raise the temperature of a gram of water 1°C; Joule was trying to find this out.

In each fall, the loss of potential energy of the two loads is $2\,mgh = 2 \times 14 \times 9.8 \times 2 = 550$ joules. The process is repeated twenty times, so the total loss in potential energy is $20 \times 550 = 1.1 \times 10^4$ joules. Assume that this is all transformed into thermal energy of the water. The amount of energy required to raise the temperature of 7 kg of water by 1° is $7 \times 10^3 \times 4.2 = 2.9 \times 10^4$ joules. Hence 1.1×10^4 joules supplied by the falling loads will raise the temperature by $\dfrac{1.1 \times 10^4}{2.9 \times 10^4} \times 1^\circ = \underline{0.38^\circ\,\mathrm{C}}$.

(Where did the energy come from? It came from the unfortunate laboratory attendant who had to keep lifting the loads after each fall.)

PROBLEM 12

A project: (a) Make or obtain a small spring of steel wire. Stretch it and let it go many times. Hold it to your cheek to feel whether it grows warmer.

(b) Make a similar spring of soft copper wire and repeat the trial.

Note: If you overstretch the spring, just push it back into close coiling.

No appreciable heating of the steel spring will be detected, but the copper "spring" will become noticeably warmer after ten or twenty rapid flexings. To avoid buckling the copper spring, you can wind it around a smooth glass rod, and then flex the spring, keeping it on the rod. By observing that the glass rod does not heat up much, you can verify that the heating of the spring is not caused by the friction of its rubbing on the glass. An even simpler demonstration of the effect is to take a piece of copper wire in your hands and try to break it in two by repeatedly bending it. Be careful when testing the temperature at the break – the copper can get hot enough to burn.

This effect can be explained in terms of work done on the springs. The steel spring is (so far as we are concerned) perfectly elastic. The force you must exert on it depends only on the amount it is stretched. All of the work done on the spring in stretching it reappears as work done by it on you when it is allowed to contract. There is no net work done on it, and no energy is lost. Therefore no heat can be developed. With the copper spring, on the other hand, you stretch it, and it stays there. A force must be exerted on it to compress it again. Thus you perform a net amount of work on it in one cycle, and this must appear as heat. The copper heats up.

PROBLEM 13

A project: Swing a hammer fast and give several violent blows to a small block of lead on a firm anvil of stone or iron. Feel the lead before and after. Make rough estimates of the kinetic energy lost by the hammer and of the thermal energy gained by the lead. (Assume that 1 kg of lead needs about 130 joules to raise its temperature 1°C.)

Ten or so sharp blows with the hammer will produce a distinct rise in temperature (of order 10°). The explanation is similar in principle to that given in Problem 12. The kinetic energy of the hammer (which comes from the student's muscles) is absorbed by the lead, which is deformed. This energy appears as heat.

In making a rough estimate of the kinetic energy lost by the hammer, it is easier to estimate the force exerted on the hammer (if you let it fall, the force will be its weight), thus the work done, and finally its kinetic energy at the end of the swing, rather than estimating the velocity of the hammer directly, and computing $\frac{1}{2}mv^2$. The energy input, when obtained in this way, should agree within a factor of two with the energy inherent in the measured temperature rise.

PROBLEM 14

A project: (a) Obtain an electric egg beater and run it at full rate for 5 minutes in a measured mass of water. (If you put the water in an aluminum pan, you may allow for the heating of the pan also. 1000 grams of aluminum take as much heat as 200 grams of water. An ingenious way to avoid having a container that takes heat is to hold the water in a thin plastic bag like the ones used in a deep-freeze — but this makes a good deal of mess if the egg beater touches the bag.)

(b) Repeat the experiment for twice the time; and again with half or twice the mass of water.

In each case, estimate the heat delivered by the egg beater per minute.

Experiments on the mechanical equivalent of heat are very difficult to do accurately. The expected rise in temperature due to the work done on the water is very small, and may easily be masked by the other ways in which heat can be exchanged with the surroundings. With this particular experimental arrangement, and with water at room temperature (to avoid ordinary heating or cooling due to contact with the air), it is difficult to avoid a drop in temperature. This is due probably to the increased evaporation caused by the agitation of the water. If the container is reasonably well sealed off from circulating air, this effect will be minimized.

PROBLEM 15

A project: Estimate the energy delivered in 5 minutes by a small electric light of known wattage.

(a) Do this experimentally by running the lamp in ink (why ink?). (You can use an automobile headlight bulb connected to a 6-volt battery. Do not use a bulb of higher voltage, because of danger of shock!)

(b) Do this by calculation from other knowledge: note that one kilowatt hour, which is likely to cost you 3 or 4 cents, is worth 3.6×10^6 joules.

a) The lamp is immersed in ink so that most of the radiation will be absorbed rather than transmitted (as it would be with water).

b) With the above arrangement, the calculated rise in temperature agrees with the measured one to better than a factor of two. For a 15-watt lamp, the total energy supplied in 5 minutes is .015 kilowatts × 5/60 hours = 1.25×10^{-3} kilowatt hours. Each kilowatt hour is 3.6×10^6 joules, so the energy expended would be $1.25 \times 10^{-3} \times 3.6 \times 10^6 =$ 4500 joules. If we use about 100 cm^3 of ink, the temperature rise should be about

$$\frac{4500}{100 \times 4.2} \approx 11°\,C.$$

PROBLEM 16

A high-speed swimmer uses 120,000 joules of energy in a half-minute race. Three quarters of the energy is released as waste heat; the rest is dissipated by his hands and legs, by mechanical work.

(a) In 30 seconds he swims 50 meters. Estimate the average force opposing his motion.

(b) Describe the changes of form of energy in the swimming.

(c) Where and in what form is the energy that has been released when he has finished the race?

a) If the average force opposing the swimmer's motion through the water is F newtons, then in swimming 50 meters against this force he does work equal to $Fx = 50\,F$ joules. He uses 120,000 joules, of which only a quarter, i.e., 30,000 joules, goes into useful mechanical work. Hence the average force opposing his motion is:

$$F = 30{,}000 \text{ joules}/50 \text{ m} = \underline{600 \text{ newtons}}.$$

b) Chemical energy of food is turned into mechanical energy by the muscles. The swimmer's kinetic and potential energies remain constant, of course, since his velocity is roughly constant, and he swims on the surface of the water; but his heat energy increases. Hence all of the work he does ends up as heat. Some energy first becomes kinetic energy of turbulent motion of the water through which he passes, and as this gradually dies down, the water is left at a slightly higher temperature.

c) This has already been answered in (b): the energy expended finishes up as heat; the water in the pool is slightly warmer. With a pool of usual dimensions (25 m × 15 m ×

3 m) containing about 10^6 kg of water, the temperature rise due to the addition of 120,000 joules would be

$$\Delta t = \frac{120{,}000}{4.2 \times 10^3 \times 10^6} = 3 \times 10^{-5}\,{}^\circ\mathrm{K}.$$

PROBLEM 17

(a) With what velocity must a 2-kg block of ice at 0°C be thrown against a stone wall so that the entire mass of ice melts upon contact? (1 kg of ice takes 3.36×10^5 joules to melt.)

(b) What effect does changing the mass have on the velocity?

(c) What makes this problem unrealistic?

a) The energy required to melt m kg of ice is 3.36×10^5 m joules. This energy is supplied by the kinetic energy of the ice, so

$$\frac{1}{2} mv^2 = 3.36 \times 10^5 \text{ m joules.}$$

$$\text{Hence, } v = \sqrt{2 \times 3.36 \times 10^5} = \underline{820 \text{ m/sec}}.$$

b) This solution is independent of the mass because the mass enters into the kinetic energy and the heat of fusion in exactly the same way.

c) When jet planes achieve speeds such as we have obtained above, the surface of the plane becomes red hot, due to friction with the air. If our block of ice travels very far before reaching the wall, it will have melted before it gets there. In other words, the energy lost to drag of the air would be an important effect at this speed.

PROBLEM 18

One mole of helium at 24°C is placed in contact with water at 26°C insulated from the rest of the world. The final temperature of both is measured to be 25°C. How much water was there?

One mole of a monatomic gas such as helium requires 12.4 joules of energy to raise the temperature one degree. Hence the helium must have absorbed 12.4 joules of energy from the water. The water cooled one degree. Since 4.2 joules are given up in lowering the temperature of 1 gram of water one degree, the amount of water which will yield 12.4 joules when cooled one degree is

$$\frac{12.4}{4.2} = \underline{2.95\,\text{gm}}.$$

The mass of water was about 3/4 the mass of helium. (One mole of helium is 4 gms.)

PROBLEM 19

A mountain climber can climb about 1500 feet or 500 meters vertical rise per hour.

(a) How much energy does such a climber gain as gravitational potential energy in a five-hour climb up a mountain?

(b) The human body is an inefficient chemical-mechanical machine. At best its muscles deliver only 25 percent of the chemical energy used as useful mechanical energy. The other 75 percent or more is dissipated as waste heat. Assuming this efficiency, how much chemical energy does the climber use in 5 hours?

(c) Assume that, aside from mountain climbing, he needs 2.2×10^6 small calories per 24 hours. How much total energy should he take in in his daily diet if he makes that climb every morning?

(d) If the climber walks down the mountain every afternoon, he loses the potential energy he gained. Why does that not help him to reduce his diet?

a) In five hours, the increase in height of the climber is $5 \times 500\ \text{m} = 2.5 \times 10^3\ \text{m}$. If his mass is, say 80 kg, his increase in gravitational potential energy is

$$mgh = 80 \times 9.8 \times 2.5 \times 10^3\ \text{joules} = \underline{2 \times 10^6\ \text{joules}}.$$

b) The above energy is only a quarter of that expended by the climber. The chemical energy used is therefore

$$4 \times 2 \times 10^6 = \underline{8 \times 10^6\ \text{joules}}.$$

c) In calories, this chemical energy is

$$\frac{8 \times 10^6}{4.2} = 2 \times 10^6\ \text{small calories.}$$

This is about the same as his normal daily calorie requirement, so on his mountain climbing days, he would need approximately $\underline{4 \times 10^6\ \text{calories}}$.

d) The only way that the human engine accepts energy is in chemical form. It is not possible to increase its energy content by doing work on it (as you can on a spring-driven toy by winding it up). It is just not that kind of machine. When you get up in the morning, and feel that you need energy to go through the day's tasks, you can't connect yourself to an electric outlet (as you would if you stored energy like an electric cell). You can't connect yourself to some mechanical device with moving parts in order to obtain energy from the work done on you by the machine. Instead, you have breakfast. Food contains chemical energy, which your body, by complicated processes, can turn into work.

The difference and similarities between the human engine and other kinds of engines can be brought out: even when "idling", the body must be rigidly temperature-controlled, blood must be pumped, etc. An electronic computer uses almost as much energy when idle as it does when solving long problems, and a human being has some of the same characteristics. When "working", energy must be expended in all movements, and even when not doing "work" in the mechanical sense, the body expends energy internally. (Try holding your arm outstretched, unsupported, for a few minutes!)

Discussion of this topic can be overextended to the point where it is not too fruitful. As soon as the major points are made, it will be wise to move on.

PROBLEM 20

Suppose you have two large boxes of negligible mass. Box A contains one mole of hot helium at 60°C, the other box, B, contains one mole of cool argon (also a noble gas) at 10°C.

(a) The two boxes are placed side by side in contact with all their outer surfaces insulated. After some time, both gases are at the same temperature. What is the temperature and why?

(b) Instead of being placed side by side, the two boxes are joined together to make one large box, without change of volume, so that the gases mix. What will be the final temperature of the mixture?

(c) Now suppose that box B contains one mole of cold nitrogen at 10°C instead of the argon. The hot helium and cold nitrogen are allowed to mix as in (b) above. Will the final temperature be the same as in (b) or higher or lower?

(d) Give a clear reason for your answer to (c).

a) Both helium and argon are monatomic gases; therefore each of them requires 12.4 joules of energy to increase the temperature of one mole by one degree. Thus the one will cool by as many degrees as the other is heated. Since they have the same final temperature, this temperature must be just halfway between their initial temperature, i.e., 35°C.

b) In equilibrium, the double box will contain a mixture of the two gases (they are "noble" gases, and do not combine chemically with anything), all at a uniform temperature. This means that all of the molecules (helium and argon) have the same translational kinetic energy on the average. No energy is gained or lost from the system, so the kinetic energy gained by the argon in heating up to the final temperature, T, must be exactly the same as the kinetic energy lost by the helium in cooling down to T. As far as temperature changes are concerned, the situation is identical with that in (a). Therefore the final temperature in this case also will be 35° C.

c) The difference between this case and (b) is that nitrogen is a diatomic gas. When it is heated, some energy must be transferred to the motions of molecules about their centers. As we calculated in Problem 3, it takes 20.5 joules of energy to raise the temperature of one mole of an ideal diatomic gas by one degree.

More energy is required to raise the temperature of the nitrogen to 35° C, than is liberated in cooling the helium to 35° C. Thus when the helium temperature reaches 35° C, the nitrogen temperature will be less than that, and the final equilibrium temperature will be lower than 35° C.

d) The reason has already been given in part (c). The actual final temperature, T, can be obtained by writing the equation for the heat exchange. The energy supplied by the cooling helium is 12.4 (60° − T) joules. For the nitrogen, the increase in energy is 20.5 (T − 10°). Since the energy lost by the helium must be the same as that gained by the nitrogen, and since they end up at the same equilibrium temperature,

$$12.4\,(60^\circ - T) = 20.5\,(T - 10^\circ)$$

$$T = \frac{(12.4 \times 60) + (20.5 \times 10)}{20.5 + 12.4} = 28.8^\circ\,\mathrm{C}.$$

PROBLEM 21

A new small airplane is to be tested. To maintain secrecy, the test is conducted in a large, closed hangar whose walls are completely insulated from the outside world. The airplane is filled up with 10 kg of gasoline, flies around and around the hangar for half an hour, and then lands with its gas tanks empty again.

(a) In what form was the energy that the airplane uses in flying supplied to it originally?

(b) Discuss the changes of energy during the flight.

(c) Where is the energy and in what form, half an hour after the flight?

a) Clearly the gasoline provides the energy. The energy is in the form of chemical potential energy of the gasoline molecules.

b) An internal-combustion engine transforms the chemical energy of gasoline into work (and also waste heat energy). The various forms that the energy takes in the engine are as follows:

(i) Chemical energy of the gasoline-air mixture.

(ii) Thermal energy of the hot, high-pressure gas produced when this mixture burns.

(iii) Mechanical energy of the pistons, which eventually (with a little frictional loss) becomes mechanical energy of the rotating propeller and kinetic energy of turbulence in the "slipstream".

(iv) The next stage involves the mechanism of flight, which is complicated. The essential point is that in order to progress at a constant speed through still air, an airplane must do work at a constant rate. This work becomes kinetic energy of bulk motion of the air the airplane had passes through.

(v) The turbulent motion of the air gradually dies down as the relatively ordered motion of eddies changes into completely random thermal energy of the air molecules, due to air viscosity.

(vi) There is also the exhaust heat from the engine, an unavoidable consequence of trying to change heat energy into mechanical energy.

c) Half an hour after the flight, all of the bulk kinetic energy has been changed to thermal energy of the air in the hangar. The end result is that the air temperature has been increased slightly.

Let us make a rough calculation of the rise in temperature. The energy supplied when 1 kg of gasoline burns is approximately 4×10^7 joules. The total energy supplied is therefore 4×10^8 joules. Suppose that the hangar is 100 m long, 100 m wide and 50 m high. Its volume is thus $5 \times 10^5 \mathrm{m}^3$. Since 1 mole of air at roughly room temperature and at atmospheric pressure occupies $2 \times 10^{-2} \mathrm{m}^3$, there are

$$\frac{5 \times 10^5}{2 \times 10^{-2}} \approx 2 \times 10^7 \text{ moles}$$

of air in the hangar. The molecules of oxygen and nitrogen that constitute air are diatomic, so 20.5 joules are needed to raise the temperature of one mole by one degree. The rise in temperature of all of the air in the hangar is:

$$\frac{4 \times 10^7}{20.5 \times 2 \times 10^7} = 0.1°\mathrm{K}.$$

PROBLEM 22

A rocket is equipped with insulated containers full of very hot gas instead of combustible fuel. The hot gas rushing out of a nozzle drives the rocket forward.

(a) Where does the momentum that the rocket acquires come from?

(b) Where does the rocket's kinetic energy come from?

(c) If the gas is ejected into a box standing on the ground as the rocket starts out, and the temperature of the gas collected in the box is measured after the rocket has left, do you expect that temperature to be higher, lower, or the same as the original temperature of the store of gas in the rocket?

a) The total momentum of the whole system of rocket, fuel and expelled gas is conserved. A rocket acquires a forward momentum by ejecting mass in the backward direction. This is true for any type of rocket. In the present gas-filled rocket, the "fuel" chamber contains many molecules traveling with high momentum in all directions, but the nozzle allows only those traveling in the backward direction to leave the rocket. The momentum of an individual molecule inside the rocket is not constant (there is a continual exchange of momentum between molecules and between molecules and the rocket itself) so that although initially only certain molecules will be traveling in just the right direction to escape through the nozzle, other molecules will very quickly also acquire the correct momentum. This explains why there is a steady stream of gas from the nozzle. It also explains how, from an initial state in which the rocket is at rest and the momenta

41

of the molecules are distributed randomly, it is possible to get to the final state in which the rocket is traveling with a high speed in the forward direction, and the gas molecules are traveling, on the average, in the opposite direction.

b) There has been no change in the total energy of the system. (We assume that the rocket is in outer space, where there will be no air friction or gravitational attraction.) The kinetic energy of the rocket plus the kinetic energy of the backward-traveling molecules must be exactly equal to the initial thermal energy of the gas molecules.

c) The kinetic energy gained by the rocket comes from the gas. In releasing this energy, the gas cools.

To understand from the microscopic point of view how this happens, consider billiard balls bombarding a light, movable wall from both sides. Let five balls be incident toward the left, and six toward the right. Let all balls have exactly the same speed, and let them all strike at exactly the same instant. After the collision, the wall will recoil toward the right. Because of this, the group of five balls will rebound with slightly increased speed toward the right, while the group of six balls will recoil with slightly increased speed toward the left. In the case of the rocket, the front and back walls of the gas cylinder play the parts of the right and left sides of the wall described above. The front and back walls of the gas cylinder are rigidly coupled. The hole in the back wall brings about the inequality in the numbers of collisions. The rocket accelerates forward, therefore the molecules moving backward in the cylinder have, on the average, less energy. It is a sample of these that comes out of the hole.

PROBLEM 23

Imagine you have a light bulb in a perfectly reflecting box which is otherwise empty — a perfect vacuum.

(a) By running the light bulb do you produce
- (i) kinetic energy of bulk motion?
- (ii) potential energy of the bulk matter of the box?
- (iii) thermal energy of internal atomic motion?
- (iv) thermal energy in the form of intermolecular potential energy?
- (v) internal energy in some form?
- (vi) no additional energy in the box?

(b) By running a light bulb, do you produce
- (i) momentum of bulk motion?
- (iii) momentum of thermal motion of atoms?
- (vi) no momentum in the box?

Be prepared to discuss your answer in class.

This problem involves the concept of radiation as a form of energy, subject to the same conservation laws as bulk mechanical energy and thermal energy. An understanding of radiation theory is beyond the scope of this course, but an acceptance of these properties of radiation will give students a more complete picture of the flow of energy, and its transformation, in the solar system as a whole.

a) The system under consideration consists of the heated filament, and whatever else is present in the evacuated space enclosed by the perfectly reflecting walls. Suppose that the system is initially at absolute zero; it contains no energy of any form. Then the current in the filament is switched on. Electrical energy is fed into the filament. Some of it is converted into heat; the filament becomes hot. The filament then emits radiant energy. This radiant energy cannot escape through the walls of the box, but is completely reflected. Some of it will be reabsorbed by the filament. It is all trapped inside the box, so the energy density must be steadily increasing. We can now answer the first parts of the problem.

(i) There is no kinetic energy of bulk motion.

(ii) There is no potential energy of bulk matter.

(iii) The only thermal energy of internal atomic motion is in the bulb itself.

(iv) and (v) There is no place where intermolecular potential energy or any other kind of internal energy can be stored.

(vi) There is additional energy in the box; it is in the form of radiation.

b) The filament and its enclosure make up an isolated system, whose momentum stays constant. If it is zero initially, it must always be zero. The radiation is continually reflected from each of the walls of the enclosure. Thus parts of it are moving in all directions (like the molecules in a gas) and the total momentum averages to zero. Since the momentum of the whole system is also zero, the box and filament must have zero momentum. Thus

(i) There is no momentum of bulk motion.

(iii) The only momentum of thermal motion of atoms is in the filament, and it adds up to zero.

(vi) The radiation in the box is a form of energy, and has momentum associated with it, but the net momentum is zero.

PROBLEM 24

(a) A cylinder containing helium gas is closed with a movable piston which has negligible friction. A man pushes on the piston and drives it in quickly, compressing the helium. The helium heats up. Why does the helium heat up? Discuss the mechanism of the heating up in terms of molecular behavior.

(b) A cylinder with a movable piston contains compressed helium. The piston is released and the helium pushes it out and cools. Explain in terms of molecular behavior how the helium cools.

(c) A large box with a good vacuum in it contains a small bottle of compressed helium. A trigger is arranged to remove the stopper of the helium bottle. When the helium is let out of the bottle no change of temperature is observed after the release is all over. Explain, from the point of view of molecular behavior, why the helium does not change temperature when it expands into the big box.

(d) Although compressed helium shows no final change of temperature, some other gases show a noticeable cooling after they have expanded into vacuum from high compression. What does that tell you about these other compressed gases?

a) Since work was done in pushing the piston into the cylinder against the pressure of the gas, the same amount of energy must appear somewhere else. The only place it can appear is as in an increase in thermal energy of the gas, i.e., as a rise in temperature.

In greater detail, when the piston is at rest, the gas atoms continually collide with it and bounce off with the same kinetic energy as they had before they hit. (The component of momentum perpendicular to the piston is reversed, and the other components are unchanged.) When the piston is moving into the gas, the atoms hit a surface moving towards them. They recoil with increased kinetic energy (as does a baseball from a moving bat). That is, on the average, we are increasing the kinetic energy of the molecules, i.e., the temperature rises.

b) The argument is similar to that of part (a). Essentially, the atoms are colliding with a piston which is retreating from them. Since their momentum relative to the piston is reversed on impact, their momentum relative to the container is decreased. Thus, the thermal energy of the gas molecules decreases. On the other hand, work is done by the gas on the mechanism connected to the piston. It can be shown that the decrease in thermal energy is exactly the same as the work done by the expanding gas.

c) This process is called "free expansion". The very important difference between it and the changes in volume produced by movement of the piston is that now the atoms never collide with a moving wall which could produce a change in their kinetic energy. The atoms will rush out of the opening in the bottle and will continue to travel until they meet the stationary wall of the box. Their kinetic energies will not change, and consequently the temperature will not change.

d) In our previous arguments, we have neglected potential energy. For helium this is justified, since two helium atoms are very much like billiard balls; they only interact when they are almost touching. (They do not attract each other enough to form a molecule.) However there are other gases in which, although the molecules repel each other very strongly when very close together, there is a weak attraction between the molecules when they are relatively far apart. For rarified gases, the "ideal" relationship is a good approximation. For denser gases, when the gas expands, the average distance between the molecules is increased, and their potential energy is increased. This results in an accompanying drop in kinetic energy in addition to the changes produced by the means discussed in part (b). Thus, even in a "free expansion", the potential energy of the molecules of some gases will increase and the kinetic energy will decrease, i.e., the gas will be cooled.

PROBLEM 25

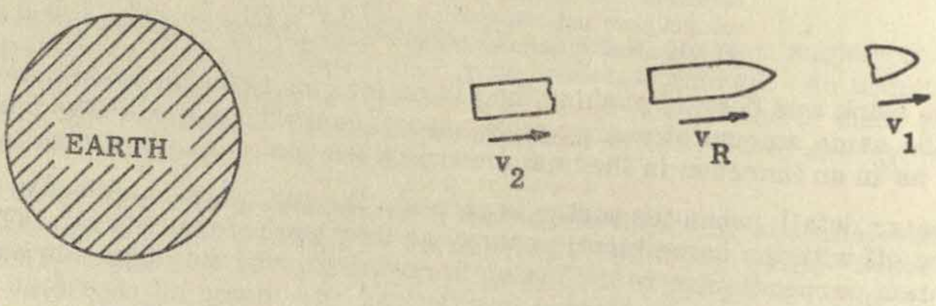
A rocket of mass 2×10^3 kg has been fired in an attempt to "escape" from the earth. However, when it is several thousand kilometers from the earth, its speed is only 30 m/sec, so it is evident that the rocket will fall back into the atmosphere. The control center on the surface therefore sends a radio impulse which fires off a small explosive charge (several kilograms) in the rocket. It explodes into two parts — one of them, with mass 0.5×10^3 kg, continues forward at 330 m/sec.

(a) Calculate the velocity of the other fragment.

(b) Calculate the kinetic energy of the original rocket, and of each piece after the separation.

(c) Why is there more kinetic energy afterwards than before?

In the problem statement of the first printing of the first edition of the text there is a minor misprint which does not affect the meaning of the problem: the first word in the eighth line should be charge, not change.



a) This is an isolated system, so momentum is conserved. We are given that one of the fragments continues on in the same direction as that of the original rocket. Therefore the momentum of the other fragment also lies on this line. As a vector equation,

$$\vec{p}_R = \vec{p}_1 + \vec{p}_2.$$

Since all the momentum vectors lie on the same straight line, we may write the above vector equation as a simple algebraic equation, simply omitting the vector signs:

$$p_R = p_1 + p_2.$$

The mass of the fragment whose velocity is unknown must be 1.5×10^3 kg, the difference between the rocket's mass and the mass of the first fragment. Substituting in the momentum equation,

$$m_2 v_2 = m_R v_R - m_1 v_1$$

$$= (2 \times 10^3 \times 30) - (0.5 \times 10^3 \times 330) \text{ kg-m/sec}$$

$$= -1.05 \times 10^5 \text{ kg-m/sec.}$$

Thus $$v_2 = \frac{-1.05 \times 10^5}{1.5 \times 10^3} = -70 \text{ m/sec.}$$

The second fragment goes backward with velocity of 70 m/sec.

b) The kinetic energy of the rocket is:

$$\frac{1}{2} m_R v_R^{\ 2} = \frac{1}{2} \times 2 \times 10^3 \times 30^2 = \underline{9 \times 10^5 \text{ joules.}}$$

For each of the fragments, the kinetic energy is:

$$\frac{1}{2} m_1 v_1^{\ 2} = \frac{1}{2}(0.5 \times 10^3 \times 330^2) = \underline{2.7 \times 10^7 \text{ joules.}}$$

$$\frac{1}{2} m_2 v_2^{\ 2} = \frac{1}{2}(1.5 \times 10^3 \times 49 \times 10^2) = \underline{0.37 \times 10^7 \text{ joules.}}$$

$$(E_K')_{\text{total}} = (2.7 + 0.37) \times 10^7 = 3.1 \times 10^7 \text{ joules.}$$

c) The kinetic energy of bulk motion after the explosion is much greater then before. That there is an increase in kinetic energy should be expected – that is the only way to separate the two fragments. The increase in kinetic energy came from the detonation of the explosive charge, which transformed chemical energy partly into this bulk kinetic energy of translation, and partly into heating the rocket fragments.

PROBLEM 26

A project: Experiment with the radiation from a glowing electric heating element. Place a sheet of asbestos tile (or a piece of damp cardboard will serve temporarily) between the heater and you. Make a small hole about 1 in. in diameter to let radiation through to you. Hold the back of your hand near the hole and feel the effect of radiation on your skin. Then try the following changes:

(a) Place a sheet of cardboard between the hole and your hand, and remove it quickly.

(b) Place a sheet of window glass between the hole and your hand and remove it quickly.

(c) Cover your hand with aluminum leaf (most easily done by clenching your fist, licking the back of your hand to make it wet with saliva, and placing a sheet of very thin aluminum leaf gently on it). Try the metal-coated skin near the hole.

(d) Keep the aluminum leaf on your hand but paint a little black paint or India ink on top of it. Again try that near the hole.

a) When the cardboard is removed, the sensation of heat will be noticed immediately. The removal of the cardboard serves as a switch to "turn on" the radiant heat. Some sort of "switch" is desirable because, as a detector of temperature changes, the skin is far from a calibrated measuring device. It "becomes accustomed" to the surrounding temperature, and while sensitive to sharp changes in temperature, is far less sensitive

APPENDIX 1

Kinematics and Dynamics of Circular Motion.

(Supplement to Teacher's Guide for Chapter 21, Sections 5, 6, and 7)

The main Guide for these sections suggested that there are three distinct levels of achievement to be attained in the understanding of the dynamics of circular motion. Briefly, these can be categorized:

I. When an acceleration occurs in the direction perpendicular to the velocity, the speed remains constant.

II. Derivation of centripetal acceleration for an object in uniform circular motion.

III. Application of Newton's law to uniform circular motion.

The plan of the text is that the student should digest the kinematics of circular motion in Chapter 6 of Part 1. With this well assimilated, the development of the dynamics can be treated as a verifiable prediction of Newton's Law.

This plan of attack has been proved in practice and it will be worth your while to try to follow it. One common problem, however, is that even for those students who clearly understood the kinematics of circular motion in Chapter 6, the associated facts and rationale may have become fuzzy by now. Therefore it is frequently advisable to spend a little time reviewing and redeveloping the purely kinematical factors; unless the kinematics is understood, the students' understanding of dynamics will be shaky.

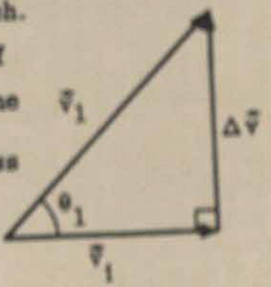
Some detailed suggestions are given below for a review of the vector ideas involved in stages I and II, the kinematics of circular motion.

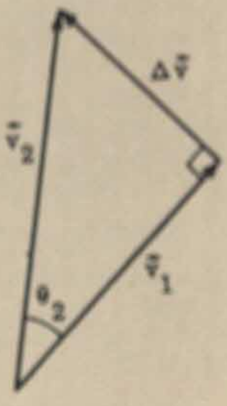
I. An Acceleration Perpendicular to the Velocity Results in Constant Speed.

One way to present this idea is to use a sequence of simple numerical examples. You can do this semiquantitatively as indicated in the examples given below. Rough computations will accomplish the purpose.

Consider an initial velocity vector, $\vec{v}_1$, of magnitude 10 m/sec. Add a perpendicular vector of magnitude Δv thus forming a resultant vector, $\vec{v}_1$, which will differ both in magnitude and direction from $\vec{v}_1$. Now add again a vector of magnitude, Δv, but this time make it perpendicular to the new velocity $\vec{v}_1$; and so forth. Let us follow a concrete example using an acceleration, $a = \Delta v/\Delta t = 10\ \text{m/sec}^2$.

1. First use 1 second intervals so that $\Delta v = 10$ m/sec. Add $\Delta\vec{v}$ perpendicular to $\vec{v}_1$ so as to create $\vec{v}_1$, as indicated in the sketch. Then apply Δv to $\vec{v}_1$, etc. This sequence of additions may be indicated graphically on the board in class. An exact treatment is not difficult but it would consume too much class time.





$$v_1^2 = v_1^2 + (\Delta v)^2 = 100 + 100 = 200$$

$$v_2^2 = v_1^2 + (\Delta v)^2 = 200 + 100 = 300$$

$$v_3^2 = 400, \quad v_4^2 = 500, \quad v_5^2 = 600, \text{ etc.}$$

The angle between $\vec{v}_1$ and $\vec{v}_1$ which we can call θ_1 is given by

$$\tan \theta_1 = \frac{\Delta v}{v_i} = \frac{10}{10},$$

$$\tan \theta_2 = \frac{10}{(200)^{1/2}} = \frac{10}{14.1},$$

$$\tan \theta_3 = \frac{10}{(300)^{1/2}} = \frac{10}{17.3}, \text{ etc.}$$

A table of these θ values and the accumulated rotation is given below:

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
	45°	35°	30°	27°	25°	23°	21°	19°	18°
Total Rotation	45°	80°	110°	137°	162°	185°	206°	225°	243°

	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}	θ_{16}	θ_{17}
	17°	16°	15°	15°	14°	14°	14°	13°
Total Rotation	260°	276°	291°	306°	320°	334°	348°	361°

You need give the class only a few values; then cite the result that in 17 steps the velocity vector rotated 360° but it grew from 10 m/sec to more than 40 m/sec, i.e., $\sqrt{1800}$.

2. Now reduce Δt from one second to 0.1 seconds. Then: $\Delta v = 1$, and $v_1^2 = 101$, $v_2^2 = 102$, $v_3^2 = 103$, etc. Tan $\theta_1 = \frac{1}{10}$, $\theta_1 = 5.7°$; tan $\theta_{100} = \frac{1}{(200)^{1/2}} = \frac{1}{14.1}$, $\theta_{100} = 4°$.. Using about 5° as an average per step would give $\frac{360°}{5°} = 72$ steps to go 360°. $v_{72}^2 = 172$ and $v_{72} = 13$ m/sec. Thus, while rotating 360°, the velocity vector in this case has increased from 10 m/sec to only 13 m/sec.

3. As a final case, use $\Delta t = 0.01$ seconds. Then for the same $a = 10$ m/sec^2, $\Delta v = 0.1$, $\Delta v^2 = 0.01$, $v_1^2 = 100.02$, $v_3^2 = 100.03$ etc. The first angle of rotation would be given by $\tan \theta = \frac{\Delta v}{v_i} = 0.01$.

This gives $\theta = 0.57$ degrees. (Recall that for such small angles, $\tan \theta = \theta$ in radians, and 1 radian = 57.3 degrees.) This angular increment does not change significantly. The number of steps for a 360° rotation would then be about 630. The magnitude of the velocity vector after these 630 steps would be $v_{630} = 10.3$ m/sec.

Whether or not you choose to use some form of this method of successive approximations to the truly perpendicular acceleration case, it is important for students to realize that an acceleration perpendicular to the motion tends to rotate the velocity vector, but does not change the magnitude.

II. The Kinematics of Circular Motion.

Considerable classroom time may be required to get students to understand fully the derivation given in Section 21-5. The following series of exercises is indicative of the background the students need before they can understand Figure 21-8 and the formulas derived from it (pages 330-331).

The time required now depends on how well the class understands the material in

Chapter 6, Sections 5-7. You may have used the exercises suggested in Appendix 1 to Part 1 when you first covered vectors while doing Chapter 6. You may wish to use or repeat some of that work here.

Six distinct steps are outlined in the development which follows. Each step should be discussed with illustrations at the board or in home exercises. These six steps involve finding the answers to the following questions. It is desirable to answer all of these graphically.

1. What is the difference between two vectors? On a vector diagram, how can you most easily manipulate the vectors to get the solution to a subtraction problem?

2. How can an average velocity be obtained from a consideration of radius vectors corresponding to the positions of an object at two different times?

3. What is the instantaneous velocity for circular motion? (Derive $v = \frac{2\pi R}{T}$.)

4. What is the average acceleration for different intervals in circular motion? Give its direction and its magnitude as intervals get smaller.

5. What is the direction of the instantaneous acceleration?

6. What is the acceleration? (Derive $a = \frac{2\pi v}{T}$.)

Students will learn best if they are led to the answers to these questions through their own graphical work. There is no substitute for the insight gained by manipulating a pencil, ruler, and protractor (or compass).

1. The Difference Between Two Vectors.

The problem is to construct $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$. Some students are not sure which vector to subtract from which; they have a tendency to take the smaller vector from the large vector.

There are two ways of remembering how to find $\Delta\vec{v}$:

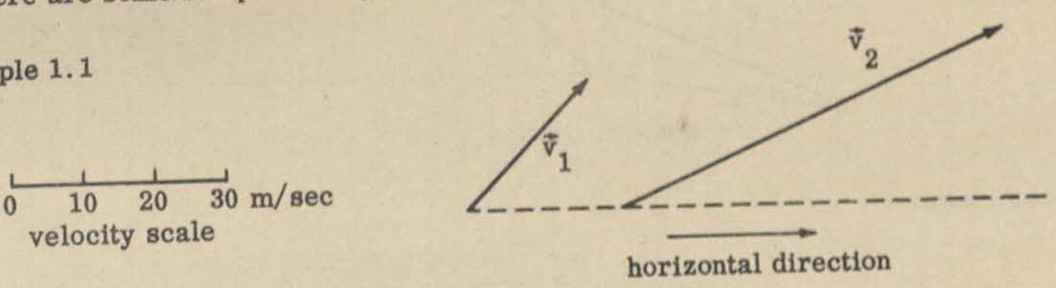
(a) To find $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, rewrite this as $\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$. Form the vector $-\vec{v}_1$ and add it to $\vec{v}_2$.

(b) To find $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, ask what $\Delta\vec{v}$ must be added to $\vec{v}_1$ to get $\vec{v}_2$.

(Note: The second technique shown below as Solution (b) is more convenient in analyzing circular motion.)

Here are some samples that you can distribute on ditto or mimeograph.

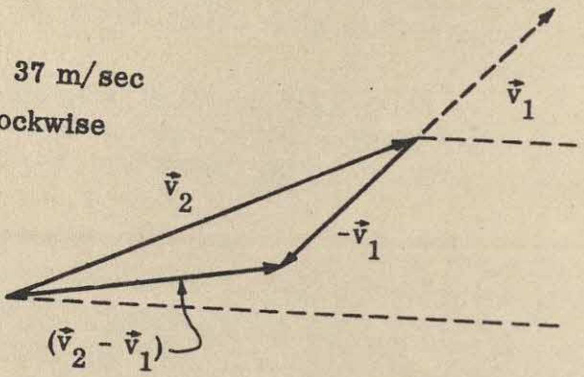
Example 1.1



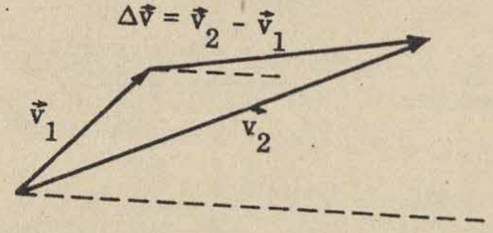
Problem: Find $\vec{v}_2 - \vec{v}_1$ graphically; find its magnitude in meters/sec, and give the direction in terms of an angle clockwise or counterclockwise from the horizontal directed to the right.

Solution (a):

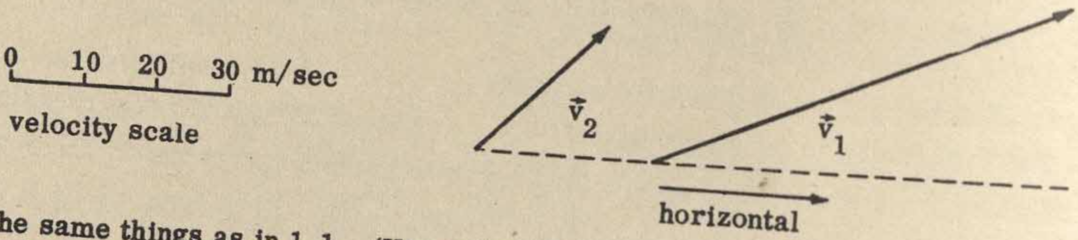
magnitude of ($\vec{v}_2 - \vec{v}_1$): 37 m/sec
direction: 8° counterclockwise from the horizontal.



Solution (b):

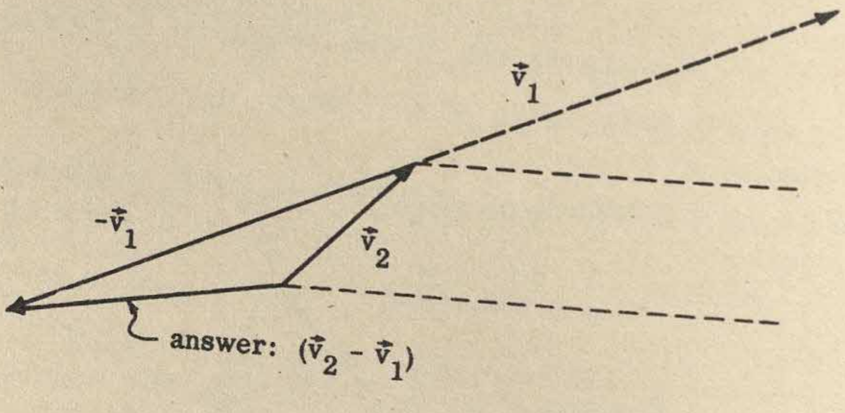


Example 1.2

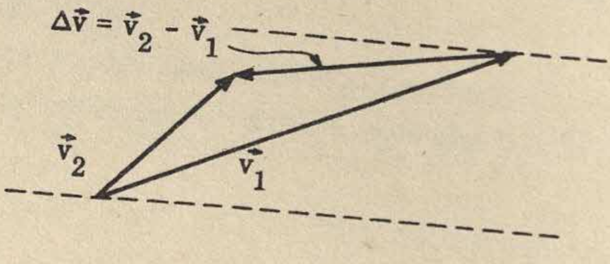


Find the same things as in 1.1. (Here since $\vec{v}_2$ is shorter than $\vec{v}_1$, students may be tempted to subtract the wrong way.)

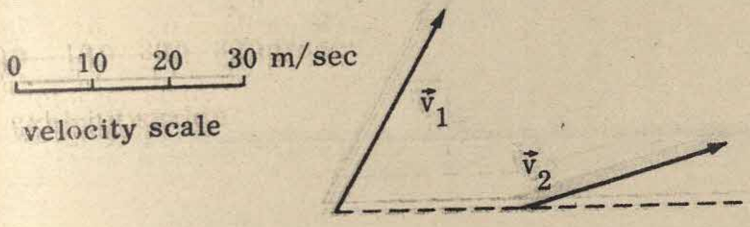
Solution (a):



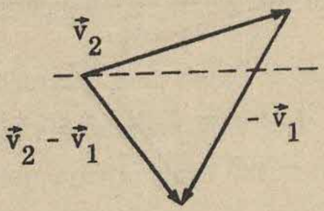
Solution (b):



Example 1.3 Same as 1.1 or 1.2 except two vectors of the same length.

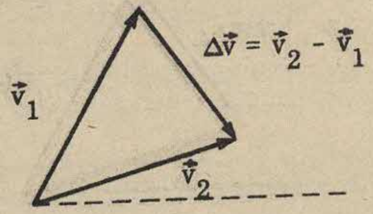


Solution (a)



magnitude of
$\vec{v}_2 - \vec{v}_1 = 22$ m/sec
direction: 51° clockwise from the horizontal to right.

Solution (b)



2. Finding an Average Velocity.

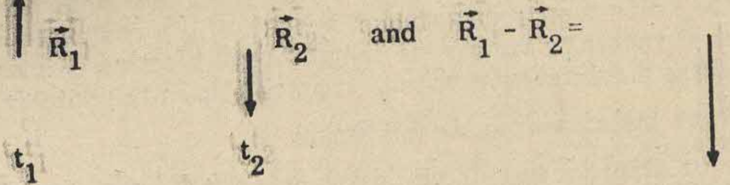
The location of an object may be designated by a vector, $\vec{R}$, drawn from some arbitrary fixed point in space to the position of the object. If the object is moving, its position at time t, might be indicated by the vector $\vec{R}$, and its position at a later time t_2, by the vector $\vec{R}_2$. Then by definition, the average vector velocity during that time interval will be:

$$\vec{v}_{av} = \frac{\vec{R}_2 - \vec{R}_1}{t_2 - t_1}.$$

(Note, for instance, that for circular motion the magnitude of the $\vec{v}_{av}$ will be less than the speed at any instant of the interval. This is not of the essence. The definition is still a useful one.)

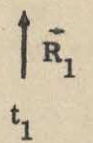
3. Finding the Instantaneous Velocity for Uniform Circular Motion.

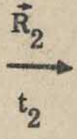
Consider the motion of the tip of a rotating pointer. If, at t_1, the hand points up, and at t_2 it points down, we will have:



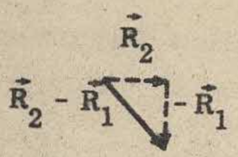
Thus $\vec{v}_{av} = \frac{\vec{R}_2 - \vec{R}_1}{t_2 - t_1}$, which has a magnitude $\frac{2R}{t_2 - t_1} = \frac{2R}{1/2\ T} = 4\frac{R}{T}$ meters/min, where T is the time required for one complete revolution.

If, instead of examining the average velocity vector over the half revolution as above, we now examine only the first quarter revolution (time interval T/4), we get:





and $\vec{R}_2 - \vec{R}_1 =$



Thus $\vec{v}_{av} = \dfrac{\vec{R}_2 - \vec{R}_1}{t_2 - t_1}$ which now has a magnitude $\dfrac{\sqrt{2}\,R}{t_2 - t_1} = \dfrac{\sqrt{2}\,R}{T/4} = 4\sqrt{2}\,R/T$ meters/min, a larger value than obtained before.

If we now shorten the interval until $\vec{R}_1$ and $\vec{R}_2$ are separated by a very small angle, $\Delta\theta$, then in the limit of very small $\Delta\theta$'s, the vector $\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$ will have a magnitude $R\Delta\theta$. Then

$$(v_{av})\text{ magnitude} = \frac{R\Delta\theta}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\frac{2\pi}{T} = \frac{2\pi R}{T}.$$

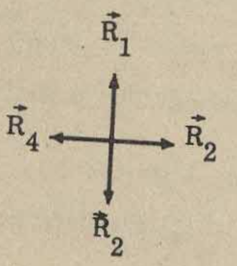
The significance of this equation is simply that the tip of the pointer, traveling in a circular path, truly covers a distance $2\pi R$ (one revolution) in a time, T, the period of one revolution.

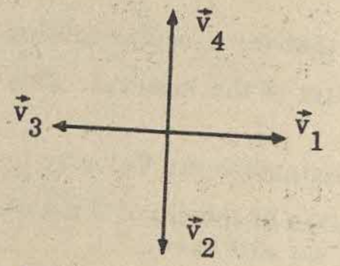
4. Finding an Average Acceleration.

The problem here is completely analogous to the one discussed above. The new idea is related to the use of velocity vectors rather than position vectors. Average acceleration is defined as:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}.$$

Consider the tip of the pointer that we have used above. When the tip is in the "up" position, for clockwise motion, the velocity vector points toward the right. Thus for the four radius vectors shown at the left, below, we have four corresponding velocity vectors drawn at the right.





The velocity vectors have the dimensions length/time, and therefore are represented in "velocity space" rather than the "configuration space" of the radius vectors. Whereas the magnitude of each R vector is simply the length of the rotating pointer, each velocity vector is the constant instantaneous speed, v, of the tip of the pointer.

5. The Direction of the Instantaneous Acceleration.

We see from the vector diagram in velocity space that the velocity vector itself rotates in a clockwise direction as the pointer rotates clockwise. Thus in a vanishingly small time interval Δt following t_1, a vector velocity increment $\Delta\vec{v}$ which is in the down direction has been added to v_1. But the direction of Δv is the direction of the acceleration, so when the pointer is is position $\vec{R}_1$, up, the acceleration of the tip is down. Thus an object in uniform circular motion is subject to a constant radial acceleration directed inward toward the center of the circle of rotation. This is called a centripetal (seeking the center) acceleration.

6. The Magnitude of the Instantaneous Acceleration.

Arguments entirely analogous to those used in (3) above now lead to the conclusion that the magnitude of the instantaneous acceleration is:

$$a = \frac{2\pi v}{T}.$$

This is because the tip of the velocity vector travels a distance $2\pi v$ in velocity space in the time interval of one revolution, T.

Using $v = \frac{2\pi R}{T}$, we can also get

$$a = \frac{2\pi}{T} v = \frac{2\pi}{T} \times \frac{2\pi R}{T} = \left(\frac{2\pi}{T}\right)^2 R,$$

or, conversely, since $T = \frac{2\pi R}{v}$,

$$a = 2\pi v \times \frac{1}{T} = 2\pi v \times \frac{v}{2\pi R} = \frac{v^2}{R}.$$

III. Newton's Law Applied to Circular Motion.

The rather detailed derivation of centripetal acceleration presented above makes it clear that this is purely a kinematical fact. The argument has been carefully developed without leaning on the fact that "the centripetal force required to maintain circular motion must result in a centripetal acceleration."

Newton's Law therefore predicts that a force must be exerted in order to maintain an object in uniform circular motion. This force should have a magnitude $F = ma = mv^2/R$. It must be directed in the direction of the acceleration, radially inward. The fact that such a predicted force is actually required to produce uniform circular motion can be verified experimentally in the laboratory.

Review the logic of the text's development in Section 21-5. Using the example given at the end of that section, ask the students what forces would have been expected (instead of 1.2 newtons)

if R had been 0.66 meters (answer: 2.4 nt)
if T had been 6.8 secs (answer: 0.3 nt)
if T had been 1.7 secs (answer: 4.8 nt)
if m had been 2.06 kg (answer: 2.4 nt).

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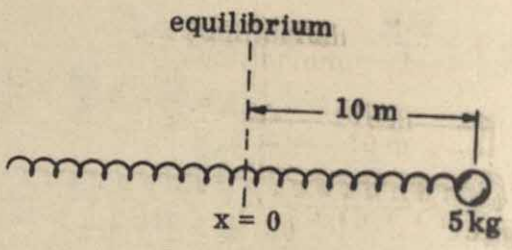
APPENDIX 2

Stepwise Solution for the Motion Where F = -kx.

A completely general approach to solving for the motion when the force is -kx is to calculate the motion numerically in many steps. For each step, $\vec{a}$ is treated as a constant. If the time intervals are chosen small enough, a very accurate picture of the motion can be obtained. Of course, we shall not use so many steps in this example.

Let us consider $F = -kx$, where $k = 1$ nt/meter. Let $m = 5$ kg.

Choose an initial displacement of 10 meters and an initial velocity of zero. Now try to find the motion, by determining x as a function of time.



(a) Choosing the time interval of the step.

Initially, at $x = 10$ m, $F = -10$ nt. Since $m = 5$ kg, $a = F/m = -2\ \text{m/sec}^2$.

We shall choose a time interval that will give some small fraction of the total distance to the equilibrium point as the distance of the motion during the first time interval. Let us call the displacement in the first interval d_1; we shall choose it equal to $\frac{1}{10}x_{\text{initial}}$, or $d_1 = -1$ meter. Using $d = \frac{1}{2}at^2$, $-1 = \frac{1}{2}(-2)t^2$, and t is found to be 1 second. We shall therefore perform the calculation as though the acceleration remained constant for 1 second intervals.

(b) Detailed calculations.

The detailed calculations are straightforward, but a bit tedious.

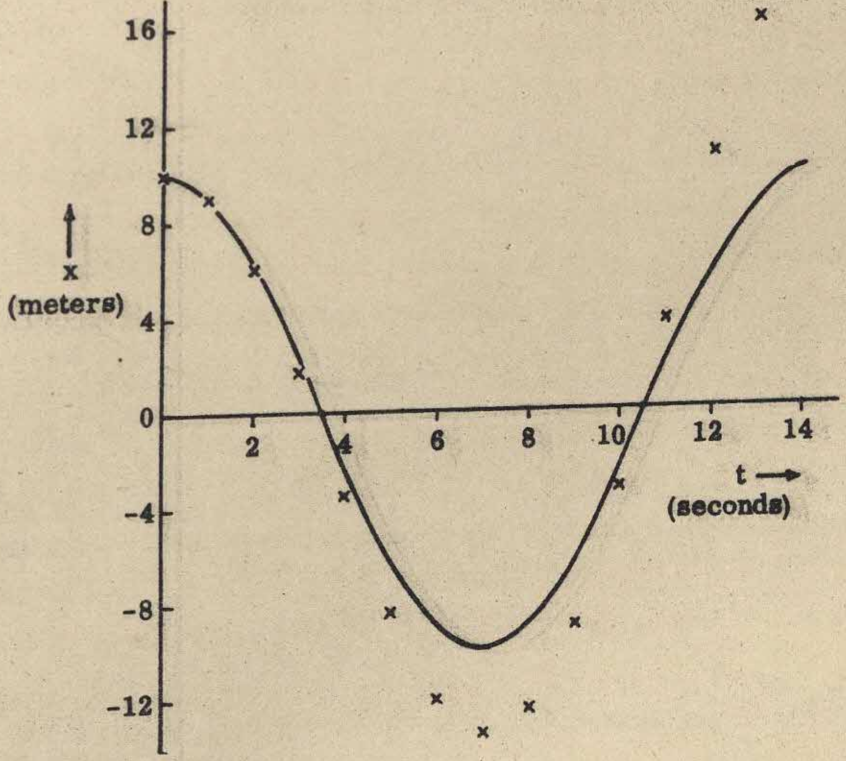
For example in interval #1, $a = -2\ \text{m/sec}^2$, $v_{1i} = 0$, $\Delta t = 1$ sec. Therefore $v_{1f} = -2$ m/sec. The average velocity is -1 m/sec, so the displacement in the first interval, d_1, is $d_1 = v_{av}\Delta t = (-1\ \text{m/sec})(1\ \text{sec}) = -1$ meter. The approximate position after 1 second is $x_{1f} = x_1 + d = 10 - 1 = 9$ m. This implies $F = 9$ nt or $a = -\frac{9}{5} = -1.8\ \text{m/sec}^2$ which is not so different from the original value of $-2\ \text{m/sec}^2$.

(c) Tabulation of Results.

Interval No.	1	2	3	4	5	6	7
x at start (m)	10	9	6.1	1.7	-3.5	-8.5	-12.1
a at start (m/sec^2)	-2	-1.8	-1.2	-3.34	+0.7	+1.9	+2.4
v at start (m/sec)	0	-2	-3.8	-5.0	-5.3	-4.6	-2.7
v at end	-2	-3.8	-5.0	-5.3	-4.6	-2.7	-0.3
v average	-1	-2.9	-4.4	-5.2	-5.0	-3.6	-1.5
d (m)	-1	-2.9	-4.4	-5.2	-5.0	-3.6	-1.5
x at end	9	6.1	1.7	-3.5	-8.5	-12.1	-13.6
a at end	-1.8	-1.2	-3.34	+0.7	+1.9	+2.4	+2.7

Interval No.	8	9	10	11	12	13	14
x at start	-13.6	-12.6	-9.1	-3.3	+3.7	+10.7	+16.3
a at start	+2.7	+2.5	+1.8	+0.7	-0.7	-2.1	-3.3
v at start	-0.3	+2.4	+4.9	+6.7	+7.4	+6.7	+4.6
v at end	+2.4	+4.9	+6.7	+7.4	+6.7	+4.6	+1.3
v average	+1.0	+3.5	+5.8	+7.0	+7.0	+5.6	+3.0
d	+1.0	+3.5	+5.8	+7.0	+7.0	+5.6	+3.0
x at end	-12.6	-9.1	-3.3	+3.7	+10.7	+16.3	+19.3
a at end	+2.5	+1.8	+0.7	-0.7	-2.1	-3.3	-3.8

(d) Graph of first approximation.



(e) Better Approximations.

This type of calculation tends to overshoot the motion because $\bar{a}$ is not readjusted enough. Taking smaller time intervals would give a better approximation, but it is not worth the effort.

For the actual motion, the object oscillates from x = +10 to x = -10. Its maximum speed would be only about 4.5 m/sec, whereas the approximate values for the coarse time intervals used above were 5.3 m/sec on the first half swing and 7.4 m/sec on the second half swing.

The exact solution of the problem requires the use of the calculus:

$$a = dv/dt = d^2x/dt^2 = F/m = -kx/m.$$

Thus, we have a simple differential equation in x and its second derivative:

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}.$$

Familiarity with solutions to differential equations enables us to propose the trial function $x = a \cos bt$. Then

$$dx/dt = ab \sin bt,$$

$$d^2x/dt^2 = ab^2 \cos bt = -b^2x.$$

So we see that $b = \sqrt{k/m} = \sqrt{1/5}$.

To find a, we see that $x = a$ at $t = 0$; thus $a = 10$ m. The solution is then

$$x = 10 \cos \sqrt{1/5}\, t \text{ m}.$$

APPENDIX 3

Solution of Differential Equation for Simple Harmonic Motion.

(For those teachers whose calculus is fresh and for whom a review of the differential equation treatment will be helpful.) We can represent the conditions imposed by the restoring force by noting that

$$F = -kx = ma = m\frac{d^2x}{dt^2}.$$

$$\text{Then } m\frac{d^2x}{dt^2} + kx = 0. \qquad (1)$$

To solve this equation, we use for a trial solution:

$$x = A \cos\left(\frac{2\pi t}{T} + \phi\right).$$

$$\text{Then, } \frac{dx}{dt} = -\frac{2\pi A}{T}\sin\left(\frac{2\pi t}{T} + \phi\right), \text{ and}$$

$$\frac{d^2x}{dt^2} = -\frac{4\pi^2 A}{T^2}\cos\left(\frac{2\pi t}{T} + \phi\right).$$

Substituting into equation (1),

$$-m\frac{4\pi^2 A}{T^2}\cos\left(\frac{2\pi t}{T} + \phi\right) = -kA\cos\left(\frac{2\pi t}{T} + \phi\right).$$

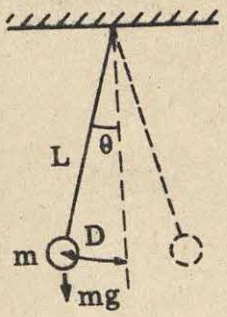
$$\text{Thus } T^2 = 4\pi^2 m/k$$

$$T = 2\pi\sqrt{m/k}.$$

The amplitude of motion A, and phase factor ϕ can only be determined if the initial conditions are specified. For example, if at time $t = 0$, x is at its maximum displacement of 5 cm, we could set $\phi = 0$ [$\cos(0 + \phi)$ is a maximum for $\phi = 0$], and $A = 5$ cm. Then,

$$x = 5\cos\left(\sqrt{\frac{k}{m}}\,t\right) \text{ cm}.$$

The mathematical treatment of the motion of a simple pendulum is somewhat more complicated, but is given below for completeness.



A massless string of length L supports a point mass, m. It is deflected from the vertical by a small angle θ, as shown. The displacement of m from its equilibrium position is $D = L\theta$.

Due to the weight of the bob, there is a force $m\vec{g}$ acting downward on the mass. This force can be resolved into components $mg\cos\theta$ in the direction of the string and $mg\sin\theta$ perpendicular to the string. The force along the string is merely a constraining force and does not enter the problem. The perpendicular force is the restoring force.

$$\text{We can write } F = -mg\sin\theta = ma = m\frac{d^2D}{dt^2},$$

$$\text{or } mL\frac{d^2\theta}{dt^2} + mg\sin\theta = 0.$$

This equation does not describe a simple harmonic motion if θ is a large angle. However, if θ is very small, we may write $\sin\theta \approx \theta$, and the equation becomes

$$m\,L\frac{d^2\theta}{dt^2} + mg\,\theta = 0.$$

Simplifying, $L\dfrac{d^2\theta}{dt^2} + g\,\theta = 0.$ (2)

Equation (2) is identical with our previous equation (1), except for relabeling the variables, and has a similar solution. It should be apparent that a solution is

$\theta = \theta_o \cos\left(\dfrac{2\pi t}{T} + \phi\right)$, with $T = 2\pi\sqrt{L/g}$.

APPENDIX 4

General Relativity

According to the basic ideas of the dynamics of Galileo and Newton, the laws of motion are valid only in an "inertial frame", that is, a system which is at rest or moving with constant velocity. These two possible states are indistinguishable, because forces are detected only when a body is accelerated. Newtonian dynamics begins to break down for bodies moving with speeds near that of light, but Einstein's theory of special relativity has satisfactorily extended the basic notions of classical dynamics to the range of all possible speeds.

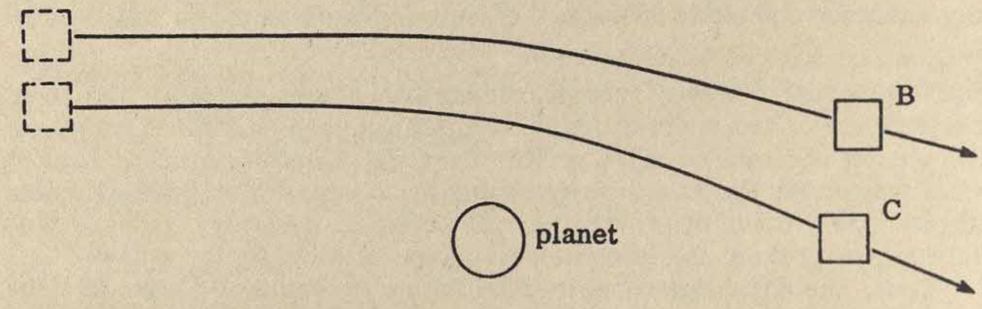
The existence of gravity, however, complicates matters considerably. We know that the surface of the earth is not an inertial frame in the Newtonian sense. If we project a body upward, it does not continue to move up but reverses direction and falls back to the floor. We generally describe the situation by saying that we are in an inertial frame, but that there is a force – the "force of gravity" – also acting on the body. Indeed, it is impossible to isolate any body from the gravitational forces exerted by the mass of the rest of the universe. Thus, the definition of an inertial frame in terms of a state of "rest" or "uniform motion" is only an approximation in a universe which contains mass, since all bodies in the universe interact with one another to a certain extent.

Einstein, in his theory of general relativity, attempts to restate the laws of physics in a manner which takes into account the equivalence of gravitation and acceleration. This theory is, as is well known, of enormous mathematical complexity, and it is hardly to be recommended as an exercise for high school students. However, the basic concepts of the theory are simple enough to be illustrated by a few examples.

Suppose we were drifting along in space in a rocket ship in the "weightless" condition which every student of modern "comics" understands. In this state we could perform dynamical experiments on ourselves and the surrounding objects floating in our room and quickly decide that we were in a true inertial frame, since the laws of motion would be obeyed exactly. If we jumped up from the floor, we would most certainly hit the ceiling, no matter how gently we jumped. Now imagine, during the course of these experiments, that this "weightless" interlude suddenly comes to an end. We would find ourselves standing on the floor, feeling our own weight. All of the surrounding objects which a moment earlier had been floating around the room would suddenly fall to the floor. What could we conclude had happened?

Without looking out the window of the rocket ship (this, incidentally, wouldn't really help), we could conclude that one of two things must have happened. Either our rocket ship had suddenly accelerated upward or we have just come to rest (with the floor "down") on the surface of some planet. In either event, it would be obvious that the laws of physics had suddenly changed and we were no longer in an "inertial frame". We would be hard put to devise an experiment to tell whether we had in fact been accelerated or had entered a gravitational field, since either occurence would have had the same effect on our observations. This is precisely what is meant when we say that the "inertial mass" of a body and its "gravitational mass" are inextricably interrelated. It is this "principle of equivalence" which lies at the heart of general relativity – that is, that the effects of an acceleration are in principle indistinguishable from the effects of a gravitational field. But what about our inertial frame? We had a perfectly good way of measuring this earlier, simply by observing the validity of the laws of motion. Evidently, during this time interval, when our rocket ship was an "inertial frame", we must have been in uniform motion in the absence of a gravitational field (impossible in our universe), or in "free fall" in a gravitational field. Thus, the true inertial frame is not one which is in uniform motion in our space, but rather one which is in free fall.

General relativity, then, is simply the description of nature as viewed by an observer in a true inertial frame – patiently doing experiments in a freely falling box. To this observer, the laws of dynamics would be exact within his box, but he would quickly conclude that the space outside his box had some very peculiar properties. To illustrate, imagine that we are in free fall in such a box, moving at a fairly high speed through space. Through our window we observe another person moving along parallel to us at the same speed and also doing dynamical experiments. We could compare results by flashing messages back and forth and agree that our results were identical and both of us were in inertial frames. If we then arrived in the vicinity of a large planet (in such a manner that we didn't conclude the experiment by hitting the planet), both boxes would be deflected by the gravitational field, as shown below.



Our box, labeled B in the figure, would be deflected somewhat, and the other box, labeled C would be deflected rather more since it is closer to the planet. Since both we and the observer in C are in free fall, within the confines of our own boxes neither of us would feel or be able directly to measure any effects due to the acceleration. We are both still in inertial frames as far as our internal measurements can tell us. However, since we are now obviously drifting apart, we would have to conclude that the distance between us suddenly started to increase without either of us apparently having accelerated. Instead of describing what has happened in terms of some "force" which we could not detect, we might rather describe the occurence in terms of a distortion in space and time which we could observe.

This is the essence of the theory of general relativity. The gravitational field, according to this viewpoint, causes a "warping" of space and time in the vicinity of a mass. Gravity then enters not as a "force" but as a property of space and time.

This whole procedure would be rather fruitless if it did not permit the extension of the laws of physics and the prediction of certain observable events. In fact, general relativity predicts certain facts which are not correctly explained either by Newtonian dynamics or special relativity. One of these involves the curvature of light in a gravitational field. In general relativity, the "shortest distance between two points" is not a straight line but is a "geodesic", the path which light would follow in traveling between the two points, bent by the gravitational field. According to special relativity, light has energy, and since mass and energy are equivalent, light will be deflected by a gravitational field. However, the amount of deflection is not given correctly by this simple argument because the force of gravity on a moving object depends on its speed. We can actually observe the bending of light which travels from distant stars and passes very close to the sun during a solar eclipse. The effect is very small, about two seconds of arc, but it is accurately measurable, and agrees precisely with the predictions of general relativity.

The theory of general relativity is of considerable importance in physics, but is more the working tool of the cosmologist in his study of the universe. The cosmologist is concerned with such questions as "Is the universe finite?" – that is, would a geodesic directed outward from the known universe go on forever; or traveling always out, return to its starting point? Much of the effort of modern astronomers is directed toward answering this question. At present, we do not know whether the space we know is finite or infinite, but it is certainly "warped", in the relativistic sense.

APPENDIX 5

(Supplement to Chapter 23, Section 5 – Derivation of Equation Relating m, m_o, v and v_e for Rockets.)

This relationship can be easily derived by using integral calculus. Consider a rocket of mass, m, and velocity, v. The rocket ejects a bit of mass in the backward direction. Let dm symbolize the small mass ejected, and let v_e be the velocity of ejection of dm. The small mass will be ejected with momentum $p = (dm)v_e$. Then, since momentum must be conserved, the change in the rocket's momentum will be $-(dm)v_e$. The remainder of the rocket, mass m, must speed up by an amount, dv, and then we have the relationship;

$$-(dm)v_e = m\,dv$$

or

$$-\frac{dv}{v_e} = \frac{dm}{m}.$$

Note that v_e is the constant velocity (relative to the rocket) with which mass is ejected, m and v are variables.

This equation can be integrated, giving (remember v_e is a constant):

$$\frac{v}{v_e} = -\ln m + c_1$$

where $\ln$ stands for the logarithm to the base e.

An alternate form is:

$$m = c_2 e^{-v/v_e}.$$

To evaluate either constant, we must insert the initial values. When the rocket was fired, its mass was $m = m_o$ and its speed was $v = 0$. This gives either $c_1 = \ln m_o$ or $c_2 = m_o$.

Thus, we get the equivalent forms of the final equation

$$\frac{v}{v_e} = \ln \frac{m_o}{m}$$

or

$$m = m_o e^{-v/v_e}.$$

APPENDIX 6

The Product of Two Vectors

Work involves the product of a force and a displacement. These are both vector quantities, but work itself is a scalar. This particular kind of product of two vectors is called a scalar product, or "dot product", and is written:

$$W = \vec{F} \cdot \vec{d}$$

The scalar product is the product of the magnitudes of two vectors times the cosine of the angle between them. This may also be expressed as the magnitude of one vector times the component of the second vector along the first.

In Figure (a), we see that in general the component of $\vec{F}$ which is along the direction of $\vec{d}$ has the magnitude $F \cos \theta$, where θ is the angle between F and D. Therefore, $W = \vec{F} \cdot \vec{d} = (F \cos \theta)d$. Note also that we could group the factors differently writing:

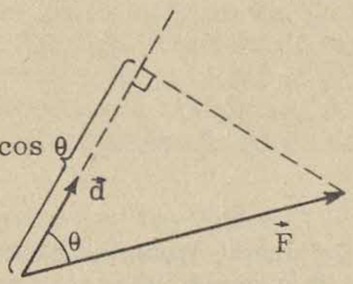
$$W = F\ (d \cos \theta).$$

We see in Figure (b) that $d \cos \theta$ is simply the component of the displacement which lies along the direction of the force. Thus we can say, equally well

Work = total displacement × component of force acting along direction of displacement,
or

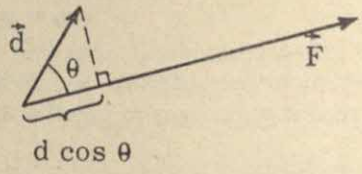
Work = total force acting × component of displacement along direction of this force.

(a)



$W = (F \cos \theta)\, d$

(b)



$W = F\,(d \cos 0)$

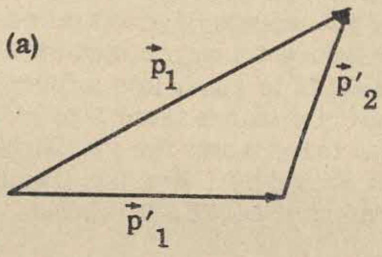
APPENDIX 7

Proof of the 90° Trajectory Angle in an Elastic Collision

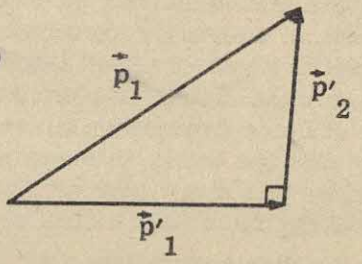
In an elastic collision between equal masses, one initially at rest, the final trajectories are at an angle of 90°, one with the other.

If $\vec{p}_1$ is the momentum of the moving ball before the collision and $\vec{p}_1{}'$ and $\vec{p}_2{}'$ are the momenta after the collision, the conservation of momentum gives $\vec{p}_1 = \vec{p}_1{}' + \vec{p}_2{}'$. Thus in the most general case, the three vector momenta must form a triangle as shown in (a).

(a)



(b)



The energy before the collision is

$$\frac{1}{2}mv_1{}^2 = \frac{p_1{}^2}{2m},$$

and if kinetic energy is conserved during the collision,

$$\frac{p_1{}^2}{2m} = \frac{p_1'^2}{2m} + \frac{p_2'^2}{2m}.$$

But this just says:

$$p_1{}^2 = p_1'^2 + p_2'^2,$$

which is simply the Pythagorean theorem for the triangle shown at (a). The triangle must then be a right triangle as at (b) and the two balls go off at 90°.

In Figures 23-9 and 23-10 the angle between p_1' and p_2' is about 85.5° rather than 90°. This is due to the fact that the collision is not truly elastic. Momentum must be completely conserved in the collision, and the fact that $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ are not on a straight line in Figure 23-10b (by about 1°) is an experimental error probably due to friction in the system (an external force).

The angle between final trajectories should be less than 90° if energy is lost in the collision. This can be seen by considering the case in which the balls stick together. In this case, the angle is 0° which is certainly less than 90°.

When the collision is head-on, the incident ball stops dead and the struck ball goes straight ahead with the speed of the incident ball. In this case, the angle between p_1' and p_2' is not defined.

APPENDIX 8

The Relationship Between Potential Energy and Force

When a mass moves under the action of a linear restoring force $F = -kx$, we know that its potential energy is $U = \frac{1}{2}kx^2$, and since the change of kinetic energy is minus the change of potential energy and is measured by the work done transferring energy from one form to the other, we obtain

$$W = \Delta E_K = \frac{1}{2}k(x^2 - x'^2).$$

Now let us suppose that somebody gave us this expression and we wanted to know if it was correct, just as somebody gave us an expression for the change of potential energy or kinetic energy in a gravitational interaction. The procedure we use to test whether our expression for the change of potential energy is correct is to put it into a form where it is a product of some expression times the small change in distance from x to x′. Since we are dealing with the work, this expression must be the force times the small change in distance and we can check whether the force is what it should be. For the example of the potential energy for a linear restoring force we therefore proceed as follows:

We factor the above expression as,

$$W = \frac{1}{2}k(x - x')(x + x'),$$

we recognize that the change in distance is $\Delta x = x' - x$. Therefore

$$W = \frac{1}{2}k(-\Delta x)(2x - x + x')$$
$$= -k\frac{(2x + \Delta x)}{2}\Delta x.$$

For a small change in distance Δx, the Δx inside the parentheses may be neglected compared to 2x, the distance, and the result is

$$W = -kx\Delta x.$$

This result agrees with our expectation, for it says that the force must be $F = -kx$.

Suppose, on the other hand, we were given a wrong expression for the potential energy, one which really belongs to a different force. By going through the same procedure, we would then find the force to which it belongs. For example, suppose that someone asserted that the potential energy associated with a linear restoring force is really $U = kx$. From this we would conclude that the work, W, which changes the kinetic energy by a small amount when the mass goes from x to x′, is

$$W = \Delta E = k(x - x') = -k\Delta x.$$

We see immediately that the force corresponding to this potential energy is just a constant $F = -k$. We should have known that we would get the constant force because our expression $U = kx$ is of the same form as the potential energy in a constant gravitational field.

In other words, any expression for a potential energy or for the work in moving from one place to another implies a definite pattern of force, and the potential energy belongs to that pattern of force and to no other. A possible method of finding the potential energy associated with any force pattern is to investigate all possible expressions that might be chosen to represent the potential energy and then pick out the one which reproduces the actual force.

APPENDIX 9

Calculations for Escape Velocities and Binding Energies

1. The escape velocity is derived in the text to be:

$$v_e = \sqrt{2GM/r_e}\,.$$

v_e can then be evaluated by substituting in this expression, or the computation can be simplified by noting that GM/r^2 is the gravitational force at the earth's surface. Therefore,

$$GM/r = gr, \text{ and}$$

$$v_e = \sqrt{2gr}$$

$$= \sqrt{2 \times 9.8\,\text{m/sec}^2 \times 6.4 \times 10^6\,\text{m}}$$

$$= 11.2 \times 10^3\,\text{m/sec}.$$

2. To compute the binding energy of the earth to the sun we need some of the pertinent constants of the solar system (text, pages 350 and 359):

$G = 6.670 \times 10^{-11}\,\text{nt m}^2/\text{kg}^2$ (Gravitational Constant)

$M = 2.0 \times 10^{30}\,\text{kg}$ (Mass of sun)

$m = 6.0 \times 10^{24}\,\text{kg}$ (Mass of earth)

$R = 1.5 \times 10^{11}\,\text{m}$ (Radius of earth's orbit)

$1\text{ yr.} = 3.2 \times 10^7\,\text{sec}$ (Number of seconds in a year)

$v = 2\pi R/1\text{year} = 3.0 \times 10^4\,\text{m/sec}$ (Speed of earth in orbit).

Then, the kinetic energy of earth relative to the sun is

$$\frac{mv^2}{2} = \frac{6.0 \times 10^{24}\,\text{kg}\,(3.0 \times 10^4\,\text{m/sec})^2}{2} = 2.7 \times 10^{33}\text{ joules.}$$

The potential energy of earth relative to the sun is

$$-\frac{GMm}{R} = -\frac{6.67 \times 10^{-11}\,\text{nt-m}^2/\text{kg}^2 \times 2.0 \times 10^{30}\,\text{kg} \times 6.0 \times 10^{24}\,\text{kg}}{1.5 \times 10^{11}\,\text{m}} = -5.3 \times 10^{33}\text{ joules.}$$

Thus, the binding energy of earth to the sun is

$$e = 2.6 \times 10^{33}\text{ joules.}$$

3. In computing your binding energy to the earth, we shall first neglect your motion due to the rotation of the earth. Then,

$v = 0,$

$r = 6.4 \times 10^6\,\text{m},$ (radius of earth)

$m =$ your mass, say, 50 to 90 kg

$g = 9.8\,\text{m/sec}^2,$ (acceleration due to gravity).

Since

$$\frac{GMm}{r^2} = mg = \underline{\text{YOUR}}\text{ weight,}$$

then

$$e = \frac{GMm}{r} = mgr - 3.1 \text{ to } 5.6 \times 10^9\text{ joules which covers the range given in}$$

the text for the binding energy of you to the earth – the exact value depending on whether you are a mere slip of a girl or a great hulk of a man.

The effect of the earth's rotation on your binding energy can be evaluated as follows:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}60\left(\frac{2\pi R}{T}\right) = \frac{1}{2}60\left(\frac{2\pi \times 6.4 \times 10^6}{24 \times 60 \times 60}\right)^2 = 6.5 \times 10^6 \text{ joules.}$$

This is only a 1/10 percent correction to the binding energy calculated above.

4. We can find a quantitative answer to the question raised in the first paragraph of Section 4 about the gravitational potential energy of a satellite. Let us consider a 1-ton satellite that has a 2 hour period of revolution around the earth, and assume a circular orbit (as has been done in the earth-sun example above). Eliminating v between

$$\frac{mv^2}{r} = \frac{GMm}{r^2}, \qquad \text{(Newtons law)}$$

and

$$T = \frac{2\pi r}{v}, \qquad \text{(period of rotation)}$$

(since the earth is now the center of motion, we let M stand for its mass, and m for the mass of the satellite) we have

$$r^3 = \frac{GMT^2}{4\pi^2}. \qquad \text{(Kepler's third law)}$$

For

$$T = 2 \times 60 \times 60 \text{ sec}, \qquad \text{(period of satellite)}$$

$$M = 6.0 \times 10^{24} \text{ kg}, \qquad \text{(mass of earth)}$$

$$G = 6.670 \times 10^{-11} \text{ nt-m}^2/\text{kg}^2,$$

then

$$r = 8.1 \times 10^6 \text{ m}. \qquad \text{(radius of orbit of satellite about Earth)}$$

Substituting this value of r into the equation for T we have, solving for v,

$$v = \frac{2\pi r}{T} = 7.1 \times 10^3 \text{ m/sec}. \qquad \text{(speed of satellite in its orbit)}$$

Taking $m = 9 \times 10^2$ kg for the mass of the satellite, its kinetic energy is

$$\frac{mv^2}{2} = 2.3 \times 10^{10} \text{ joules}, \qquad \text{(kinetic energy of satellite in orbit relative to earth)}$$

$$\frac{-GMm}{r} = -4.4 \times 10^{10} \text{ joules}, \qquad \text{(potential energy of satellite in orbit relative to earth)}$$

Hence,

$$\epsilon = 2.1 \times 10^{10} \text{ joules}. \qquad \text{(binding energy of satellite in orbit)}$$

It would take this much energy to remove the satellite from the influence of the earth.

Now let us find the energy required to put the satellite into this orbit in two ways, first in an indirect way and then in a more direct way.

We note that the binding energy of the satellite to the earth, when it is at rest before launching, is found in the same way as the binding energy of you to the earth. It is (again neglecting the very small initial kinetic energy)

$$\epsilon_{\text{ground}} = \frac{GMm}{r_{\text{earth}}} = 5.6 \times 10^{10} \text{ joules.}$$

The difference between this binding energy and the binding energy in orbit must be the total energy required to put the satellite into the orbit from the ground.

$$\text{Energy required} = \epsilon_{\text{ground}} - \epsilon_{\text{orbit}} = 3.5 \times 10^{10} \text{ joules.}$$

[It is $GMm\left(\frac{1}{r_{\text{earth}}} - \frac{1}{r_{\text{orbit}}}\right) = 1.2 \times 10^{10}$ joules plus the kinetic energy that it must have in this orbit in order to keep from falling to the earth. We have seen before that this kinetic energy is 2.3×10^{10} joules, and therefore the total energy required is $(1.2 + 2.3) \times 10^{10}$ joules, as above.]

APPENDIX 10

Energy of Motion of Polyatomis Gas Molecules

The extension of the theory to polyatomic gases is reasonably simple. We note from the equation,

$$\frac{3}{2}kT = \frac{1}{2}mv^2,$$

that a thermal energy, $\frac{1}{2}kT$, is associated with each of the three degress of freedom of motion of the center of mass of the molecule. If we consider the case of any molecule which is more complicated than a single atom, there are more degrees of freedom available. For instance, a diatomic molecule is a dumbbell-shaped unit. Consider the two atoms to lie on the x axis. Their center of mass can move independently in the x, y and z directions. In addition to these motions, the dumbbell can rotate independently about both its center of mass and about axes parallel to the y and z directions. It could also spin about the x axis in the same way that a regular dumbbell can spin about its own axis. However the dynamics for this mode of rotation is clearly very different from the other two. For point atoms, no energy could reside in this spinning motion. There is one more independent mode of motion for a diatomic molecule. This involves vibration of the two atoms toward and away from each other. This motion, unlike the others, involves both potential and kinetic energy and this fact turns out to introduce (in effect) two degrees of freedom associated with vibration instead of only one.

Classical physics tells us that the energy of such a system will, on the average, be equally distributed among all the degrees of freedom. Thus if 8 joules of heat energy are transferred to a monatomic gas, the energy will all go into the 3 translational degrees of freedom. If the same quantity of energy is transferred to a diatomic gas, the Law of Equipartition states that it will be equally divided among the seven degrees of freedom specified above (neglecting spin). Thus only 3/7 of the energy goes into temperature-determining motions of translation. The specific heat of such a gas would be 3/7 that of the monatomic gas previously discussed.

Actually, quantum mechanics changes this picture in that it specifies a minimum energy of excitation that can reside in any one of the degrees of freedom. In general, it is lowest in the translational degrees, higher in the rotational degrees, and highest in the vibrational degrees. The result is that at low temperatures, only the translational motions are excited. At higher temperatures, rotations may be excited. At still higher temperatures, vibrations will set in.

At room temperature for most diatomic gases, molecular rotation has set in but vibration has not. Therefore there are only five effective degrees of freedom. and the specific heat will be 3/5 that of a monatomic gas. Thus, in general, specific heats are dependent both on the structures of the molecules and on the temperature.

APPENDIX 11

Historical Supplement on Heat

The story of man's quest for an understanding of the nature of heat is a fascinating one. Of course, people knew about heat long before concepts about energy were formulated. Early speculations concerning the nature of heat rested on two rival hypotheses: first, that heat is somehow connected with motion (fire by friction, etc.), and second, that heat is a material substance.

In 1620 Francis Bacon stated in his Novum Organum that "Heat itself. . . . is motion and nothing else." Later in the seventeenth century both Robert Boyle and Robert Hooke expressed similar ideas, but the theory held little weight with most scientists of the following century mainly because no one was able to explain why, if heat were motion, it would be conserved in experiments where two bodies, initially at different temperatures, reach thermal equilibrium when placed in contact.

In general, during the eighteenth century, a theory of heat came into use which assumed heat to be a subtle, elastic fluid present within any "hot" body. The particles of the "fluid" supposedly repelled each other, but were attracted to the particles of ordinary matter. This "fluid" heat came to be known as "caloric", and the concept of heat as a material substance was accordingly called "the caloric theory".

The foundation of the caloric theory was the notion that heat is conserved. Most of the observations and experiments of the calorists were performed under limited conditions which allowed the total quantity of heat to remain invariable, and which suggested that heat is a conserved quantity. It was thus convenient to think of heat as a substance which could neither be created nor destroyed, but which could flow from one material to another.

It is interesting to examine explanations for heat phenomena offered by the calorists. They assumed, as was customary at that time, that the basic units of matter were impenetrable. Despite the mutual attractions of the particles of a substance, they argued that these particles could not be in actual contact; otherwise contraction could not occur when a body was compressed. Therefore, there must be some counter-balancing repulsive force, and this force was attributed to the caloric present between the particles. Because of the self-repulsion of the caloric substance, heat would flow from a hot material to a cold one. Whether a substance existed in the solid, liquid, or gaseous state depended upon the quantity of caloric which entered into its composition. When it contained large amounts of caloric, the substance would assume the form of a gas; solids and liquids would contain less caloric and thus occupy less volume. As a substance was cooled, it was thought to be drained of caloric; the removal of caloric (which supplied the repulsive force) explained the fact that most substances contract upon cooling.

Although the caloric theory of heat is no longer used, some of its language has been carried over into the modern description of heat. This is particularly convenient when one discusses heat "flow" and "transfer" of heat. We still speak of an object "soaking up heat". And, of course, our unit of heat, the calorie, was originally used as a measure of caloric substance.

Count Rumford's (1753-1814) experiments, begun about 1787, were designed to attack the caloric theory at its very foundations. Rumford set about, at first, to show that heating a body did not increase its mass. One might argue that if caloric were matter, then it must possess the fundamental property of all matter: it must have mass. By a series of extremely careful experiments, he showed that the weight of a body did not depend on its temperature. However, this offered no serious challenge to the calorists. They merely answered that caloric was not ordinary matter, and therefore was not necessarily subject to gravitational forces.

Rumford then turned to experiments with the heat produced by friction. While supervising the cannon-boring operation at the Munich arsenal, he noticed the very high temperatures attained by a cannon while it was being bored. On the basis of the caloric theory, the attractive force that was supposed to exist between caloric and the molecules of the metal must have been diminished when the metal was broken into chips releasing caloric which appeared as heat. However, the heat released in these experiments seemed to be inexhaustible. From this Rumford concluded: "It appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner in which heat was excited and communicated in these experiments, except it be MOTION."

The physical basis upon which the caloric theory was founded was the notion that heat is a conserved quantity. It was just at this point that critics of the theory attacked. They showed that in friction experiments, such as Rumford's cannon, heat could be produced in seemingly endless quantity.

Apparently Julius Robert Mayer, who lived from 1814 to 1878, was the first to realize the importance of an identification of heat as energy. Mayer had the idea that energy could not be destroyed and that one form of energy was in some sense equivalent to another. He pointed out:

> "In numberless cases we see motion cease without having caused another motion or the lifting of a weight; but an energy once in existence cannot be annihilated, it can only change its form; and the question therefore arises, what other forms is energy.... capable of assuming?"

He argued that since work can be converted to heat, heat must be a form of energy.

> "If potential energy and kinetic energy are equivalent to heat, heat must also naturally be equivalent to kinetic energy and potential energy."

Here, Mayer made his most penetrating observation. If heat was simply a converted form of kinetic or potential energy, and if energy as a whole is conserved, then a given quantity of heat must be the result of a proportional amount of mechanical energy. Work done on an object must produce a proportional amount of heat in the object, if its kinetic and potential energy is unchanged. From experiments already done on gases, Mayer was able to derive a quantitative relation between mechanical work and heat, which is in fairly good agreement with the best values that have been obtained since. The definitive work of James Prescott Joule (1818-1898) came a bit later and spelled the end of the caloric theory.

APPENDIX 12

Supplement on the Three Laws of Thermodynamics

The text refers briefly to the first and second of the three laws of thermodynamics. The first law derives from the kinetic theory and is concerned with the energy of the molecules which comprise any substance. This law is usually expressed in the form

$$\delta Q = dU + \delta W,$$

which states that the increment in heat which flows into a body, δQ, is equal to the sum of the increase in the internal (thermal) energy of the body, dU, and the increment of work, δW, done by the body against an external force. If work is done on the body by an external force (e.g., by compressing a gas with a piston), then δW is negative. Similarly, δQ is negative for heat which flows out of the body. The first law is simply a formal statement of the conservation of energy, where we have explicitly defined heat as a form of energy.

The first law tells us a great deal about the thermal behavior of matter. For example, it says that if we thermally isolate a system so that no heat can flow in or out, so that $\delta Q = 0$, the work done on the system (e.g., by compressing a gas) will raise the thermal energy of the system, and conversely if the system does work (e.g., by expanding against a piston), its thermal energy will be reduced. We can see readily that it takes more heat to raise the thermal energy of a system at constant pressure (since the system can expand and do work) than of a system at constant volume (if the displacement is zero, δW is zero).

The first law, however, does not tell us everything we know about heat. Consider the following situation. Suppose that we could cause a large group of molecules to enter an evacuated box, with all molecules moving in the same direction and at the same speed. If we looked inside the box a day later (or even a second later, for that matter) we should expect to find that the occurrence of random collisions would have completely disorganized the molecular motions. The molecules would be traveling every which way with a Maxwellian distribution of speeds. At no later time would we expect to be able to look into the box and find that all the molecules were traveling in the same direction with the same speed. The expectation of a trend from order toward disorder is dictated by experience. That is the way systems tend to evolve. This direction of evolution is not dictated by the first law of thermodynamics. Energy could be conserved in a trend from disorder to order just as well as in a trend of the reverse direction.

If we dip a hot piece of metal into a bucket of cold water, we expect the metal to become cooler and the water to become warmer. We would be surprised to find the metal getting still hotter, while the water froze. Yet the latter transformation would not violate energy conservation.

The direction in which reactions like the above tend to go is defined by the second law of thermodynamics. The parameter that is introduced to serve as a measure of this trend is called entropy.

The second law of thermodynamics may be stated in many forms. All with essentially the same meaning. One such statement is, "In thermal equilibrium, the entropy of a system is a maximum." In other words, there is a "direction" to nature, from order to disorder. If we start with a roomful of air but with one mole of gas in a bottle in one corner of the room (a highly ordered state) and remove the stopper from the bottle, the "direction of nature" is such that we shall eventually have a random distribution of molecules throughout the entire room (a highly disordered state). But if we start with the disordered state, the mole of gas molecules randomly distributed throughout the room, there is not much chance that they will eventually all come together and go back into the bottle from which they came.

Similarly, if we start with two separated bodies, one hot and one cold, and bring them into thermal contact with one another, the entropy of the combination will increase if heat flows from the hot body to the cold body so that they both reach the same final temperature. If heat flowed from the cold body to the hot body, so that the cold one cooled and the hot one grew warmer, the entropy of the combination would decrease. According to the second law, the "direction of nature" is such that we can exclude this second possibility for bodies of reasonable size. Indeed, we must do work to make heat flow from a cold body to a hot body (e. g., a refrigerator).

The second law has far reaching consequences, and tells us much more than just that heat flows from hot bodies to cold bodies. It explains why we find "noise" in radio resistors, why all measurements are subject to random errors, and much more.

The third law of thermodynamics is concerned with the absolute zero of temperature. In effect it tells us that we can bring the temperature of an object arbitrarily close to this absolute zero, but that we can never quite reach the limit.

APPENDIX 13

Supplement on Machines

Machines such as simple levers, pulleys, wedges, screws, and gears, or their almost infinite number of permutations and combinations, are frequently treated at some length in elementary physics courses. They are omitted from the PSSC course partly because of space limits, and also because they are aside from the main ideas of physics. All machines can be simply understood on the basis of the laws of thermodynamics. We often ignore thermal effects, in which case the only constraint is that imposed by the first law. When we put a certain amount of work, $F_i s_i$, into a machine, we can expect to get, at most, the same amount of work out. If we let $F_o s_o$, the product of the output force and displacement, be the work out, then for all such machines, $F_i s_i \geq F_o s_o$. The equality sign holds only for an "ideal" machine where there are no frictional losses. This "machine equation" is simply a statement of the conservation of energy. The usual definition of the "mechanical advantage" of the machine, F_o/F_i, while frequently a useful concept, is hardly a fundamental one. It is obvious that for an ideal machine, $F_o/F_i = s_i/s_o$, and we may determine the mechanical advantage of any machine simply by noting the ratio of the distances through which the input and output forces act.

Over the centuries, many men have sought to devise "perpetual motion" machines, devices which will produce useful work while consuming less than an equivalent amount of energy. These have usually served only to transfer money from the pockets of gullible people to those of the inventor or promotor. Perpetual motion machines which operate by the expedient of violating the law of conservation of energy are generally referred to as perpetual motion machines of the first kind, since they are incompatible with the first law of thermodynamics. Some perpetual motion machines are more subtle. They work without violating energy conservation, usually by having heat flow the wrong way. These devices, which violate the second law, are called perpetual motion machines of the second kind. All perpetual motion machines, no matter how cunningly contrived or ingeniously built, have one main feature in common: They don't work!

TEACHER'S GUIDE FOR EXPERIMENTS, PART III

As in Part II, it is very important to schedule the experiments in conjunction with specific sections of the text.

Number	Experiment	Best Time	Priority
III-1	A Variation on Galileo's Experiment	Before Section 20-2	*
III-2	Changes in Velocity with a Constant Force	Before Section 20-3	***
III-3	The Dependence of Acceleration on Force and Mass	Before Section 20-3	***
III-4	Inertial and Gravitational Mass	After Sections 20-5 and 20-6	**
III-5	Forces on a Ball in Flight	After Section 21-4	**
III-6	Centripetal Force	Before Section 21-5	***
III-7	Law of Equal Areas	During middle of Chapter 22	*
III-8	Momentum Changes in an Explosion	During Section 23-2	***
III-9	The Cart and the Brick	During Section 23-4	**
III-10	A Collision in Two Dimensions	After Section 23-4	***
III-11	Slow Collisions	During Section 24-5	**
III-12	Changes in Potential Energy	After Section 25-3	***
III-13	The Energy of a Simple Pendulum	After Section 25-3	**
III-14	A Head-on Collision	During Chapter 26	*

*** essential

** desirable

* optional

III-1. A VARIATION ON GALILEO'S EXPERIMENT

This is a modification of Galileo's "idealized" experiment described in Section 20-2. It is best done before that section has been studied in the text, and after raising the question whether objects can keep moving without being pushed. To save time, this experiment may be done as a demonstration or omitted in favor of the more important experiments on Newton's law.

Galileo reasoned that since a ball launched down a ramp tended to rise to the same height on an opposing ramp, a ball launched onto a horizontal surface should roll forever if there were no friction. The purpose of this experiment is to help the student reach the same conclusion.

The arc that the pendulum traces as it moves from its release position to its lowest point is analogous to Galileo's launching ramp (Fig. 20-4). The arc it swings through from the lowest point after leaving the barrier corresponds to the plane of gradual slope. Raising the clamp is analogous to diminishing the slope which the ball climbs in Galileo's experiment. It is best to use as long a pendulum as possible, but reasonably good results can be obtained with one about one meter long.

Testing the pendulum first without a barrier (Fig. 1) shows that the distances to either side of the center point and the heights reached are approximately equal under these conditions. This is not surprising. It takes more insight to predict what will happen when the barrier is used (Fig. 2).

To improve the estimation of the bob's height of rise, a horizontal string can be put at the level of release. This will also help to duplicate the release level.

- - - - - - - - - - -

A Dry Ice puck is a very useful device to demonstrate uniform motion in the absence of forces. The construction of the puck is shown in Fig. (a). Since the layer of gas between the puck and the table is very thin, it is necessary to have a table with a smooth surface of glass or metal. It is essential that the surface of the table, as well as the surface of the puck, be very clean and dry. Wipe the table and the puck with a dry cloth to remove the condensation accumulated during the demonstration.

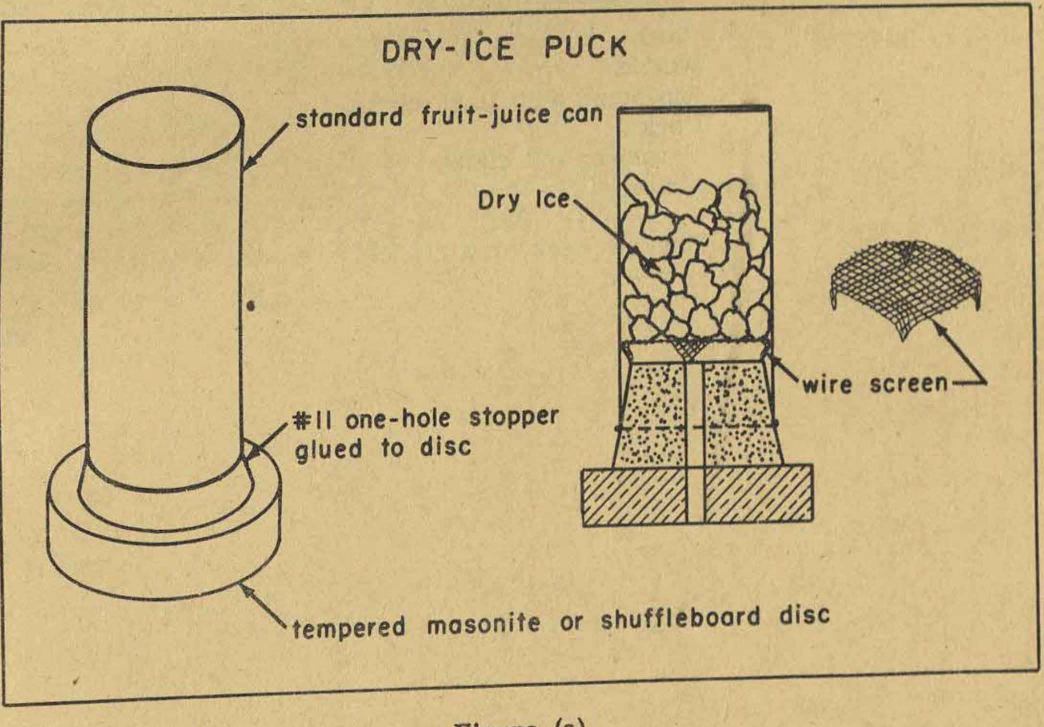

DRY-ICE PUCK
standard fruit-juice can
Dry Ice
wire screen
#11 one-hole stopper glued to disc
tempered masonite or shuffleboard disc

Figure (a)

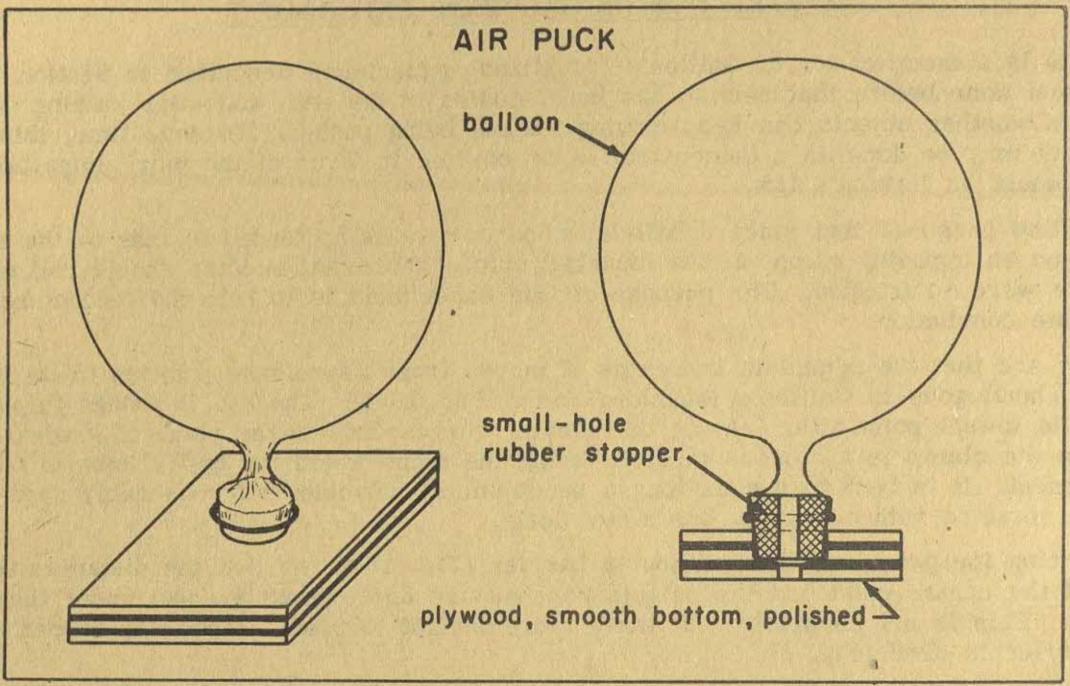


Figure (b)

If the table is level, the puck will slide the length of it at very low velocities without slowing down. If the table is not level, the puck will accelerate from rest. Experimenting with these pucks makes Galileo's law more real.

If Dry Ice is not available, an air puck made as shown in Fig. (b) can be used. Its performance is inferior to that of the Dry Ice puck, but it is useful nevertheless.

APPARATUS

Cord
Weight
Ringstand with flask clamp
Cork
Crossbar and clamp
Meter stick

Dry Ice puck or air puck
Dry Ice

III-2. CHANGES IN VELOCITY WITH A CONSTANT FORCE

This experiment and the following one (III-3, The Dependence of Acceleration on Force and Mass) are very important and should not be omitted. Since they lead the student to discover Newton's law, they should be done before Section 20-3.

As a result of this experiment the student will discover that, within his experimental error, a constant force acting on an object changes its velocity at a constant rate - that is, its acceleration is constant. The unit of force in this experiment is that produced by a rubber loop stretched a constant amount, and the velocity is expressed in meters per "tick." Notice that the units used are not important in discovering or expressing the fundamental law.

We do not use gravity as a pulling force on the cart because it raises too many questions that cannot be answered at this stage of the course. The student has no real way of knowing that gravity supplies a steady force to a moving object; and, in the next experiment, where masses must be measured, it is hard for him to see that the mass of the falling weight must be included as part of the total mass being accelerated. Furthermore, the rubber loop gives the student a feeling for what a constant force really is.

The experiment is best done on a smooth, level table about 2 meters long. If the table is not level, this will show up when the tapes made by a coasting cart are examined. The distance covered between two successive ticks of the timer at the beginning and end of the tape can be compared by holding the two ends of the tape side by side. There is no need here for accurate quantitative measurements. It is sufficient to see that a rapidly coasting cart moves at a reasonably constant velocity on a horizontal surface.

The timing tape should pass smoothly through a timer fastened to the table with a C clamp. The total frictional drag on the cart will be about 0.3 newtons, a small force compared with the force applied through the rubber loop. The tapes should be labeled for future identification as soon as they are made. If the laboratory period is short, the analysis of the tapes can be done as homework.

Practice in using the apparatus is necessary before making any quantitative runs. It is not easy to keep the rubber loops stretched to the same length at the start and while the cart is accelerating at high velocities near the end of the run. In starting the cart, it is important to synchronize the towing with the release.

A final word of warning: The rubber loops are ideal for shooting paper clips and miscellaneous missiles about the lab!

Answers to Questions

The velocity of a coasting cart is more uniform at high than at low speed. The reason for this can be explained as follows: a gentle depression in the middle of the table, only 1 mm below the surface and many centimeters long, not detectable to the eye and scarcely detectable with a good spirit level, will change the velocity of a cart starting from rest by 14 cm/sec! ($v = \sqrt{2gd} = \sqrt{2 \times 9.8 \times 10^{-3}} = 0.14$ m/sec). If the cart has an initial velocity v_o,

$$v^2 - v_o^2 = 2\,gd$$

$$(v + v_o)(v - v_o) = 2\,gd$$

$$\Delta v = v - v_o = \frac{2\,gd}{v + v_o}$$

if $v_o \gg 2\,gd$

then $v \approx v_o$

$$\Delta v = \frac{gd}{v_o}$$

and the fraction change in velocity will be

$$\frac{\Delta v}{v_0} = \frac{gd}{v_0^2}$$

which decreases rapidly as v_0 increases.

Be sure, however, that the student does not analyze the end of the tapes representing the highest velocities, where he was probably unable to keep the applied force constant.

Within experimental error, the data when plotted on the graph will show that a constant force produces a constant acceleration [Fig. (a)]. The deviations from a straight line result primarily from the difficulty of keeping the rubber loops stretched a constant amount.

The force exerted by the rubber loops is not the only force acting on the cart. The drag of the timing tape and the friction between the wheels and the table exert forces opposite to the motion. These forces produce negligible velocity changes compared with those produced by the applied force, as we can conclude from the coasting run.

The acceleration of the larger mass is less than that of the smaller one with the same force acting on the cart. Quantitative measurements of the effect of mass on acceleration will be done in the next experiment.

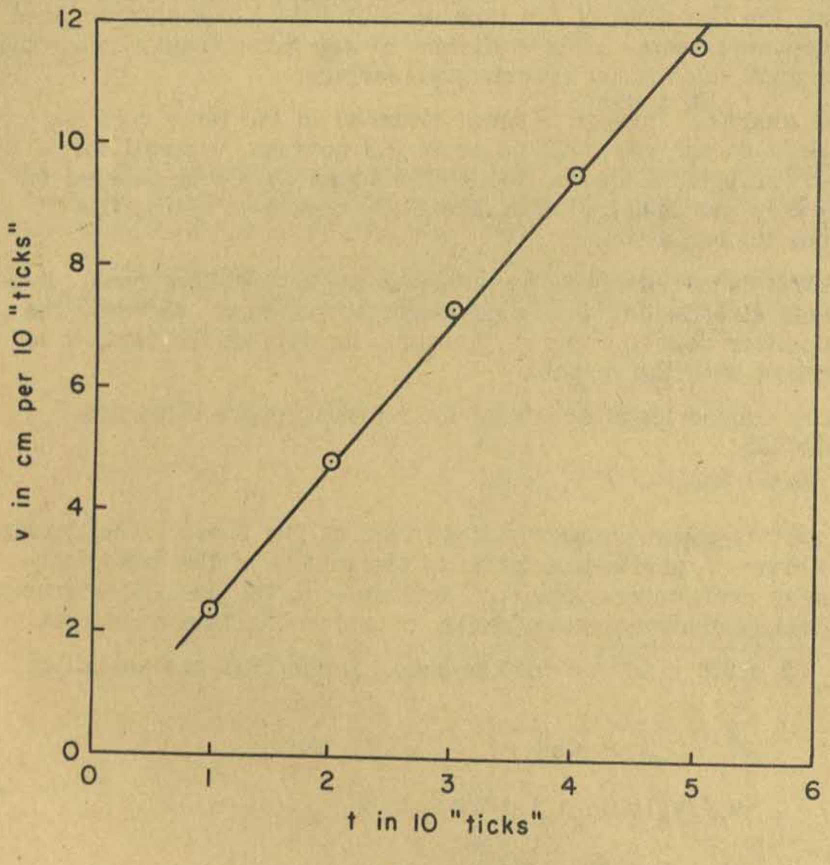


Figure (a)

APPARATUS

1 Dynamics cart
1 Rubber loop
4 Bricks
1 Timer
Paper tape
Carbon-paper discs
2-3 C clamps
1 $1\frac{1}{2}$-volt dry cell
1 Sheet graph paper
Meter stick
Table stop

Cellulose or masking tape

III-3. THE DEPENDENCE OF ACCELERATION ON FORCE AND MASS

The purpose of this experiment is to discover the relation between force, acceleration, and mass (Newton's law). It is most appropriately done before studying Section 20-3.

The technique is the same as in the previous experiment, and the same precautions should be followed. Extra care is needed in the first part, where large forces are exerted. (The meter stick should not be used for archery practice, since it easily covers the length of the classroom and curves erratically in its flight! Be sure students slow the cart gently at the end of the run to prevent damage to it, and to their toes from falling bricks.) If it is necessary to improvise rubber loops to replace the standard ones supplied, five No. 19 rubber bands chained together are approximately the equivalent of one loop of the supplied strand.

In analyzing the motion, the student may assume, after doing Experiment III-2, that the acceleration is constant throughout each run; he need measure only the distance traveled in a given number of ticks to find the acceleration from $a = \frac{2d}{t^2}$.

Often it is impossible to resolve the dots on the tape at the beginning of the run [Fig. (a)].

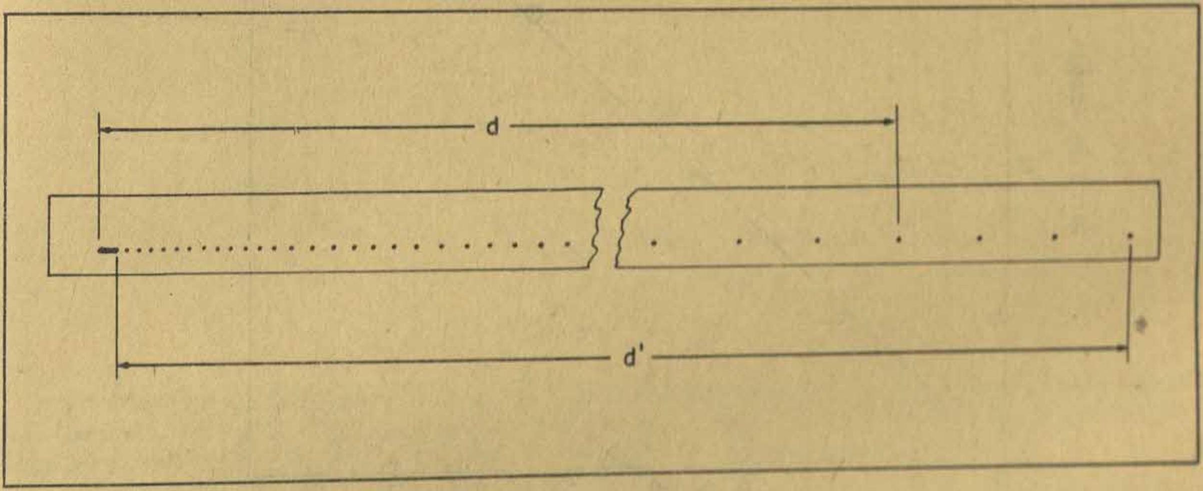


Figure (a)

If possible, the student should estimate the number of ticks in the fuzzy region. Starting from the first resolvable dot will yield an apparent distance d' and an apparent acceleration $a' = \frac{2d'}{t^2}$ instead of $a = \frac{2d}{t^2}$. This causes a fractional error $\frac{a' - a}{a} = \frac{d' - d}{d}$. If this error is above 5 per cent the acceleration may be found by determining first the velocities v_1 and v_2 and times t_1 and t_2 and from these, $a = \frac{v_2 - v_1}{t_2 - t_1}$. To get an idea of the error in the latter method, it is best to repeat the determination of a, using different values for t_1 and t_2.

If students work in groups, each student can analyze a different tape and the results can be pooled for drawing the graph.

This experiment is long, and probably all the analysis will have to be done as homework unless the experiment is divided into two parts and done in two consecutive laboratory periods.

Answers to Questions

The plot of the acceleration as a function of force comes close to a straight line and shows that force and acceleration are proportional [Fig. (b)]. From the plot, we can say that the ratio of force to acceleration is constant in this part of the experiment. With no friction, the curve will pass through the origin. With friction, the graph will show a positive value of force at zero acceleration because the applied force is equal and opposite to the friction force, and the net force is zero. In this experiment, the friction is so small compared to the pull of the rubber loops that failure to maintain a constant pull may obscure the force of friction. As a result, the curve may cross the force axis slightly above or below the origin and cannot be depended upon to measure the force of friction [Fig. (b)].

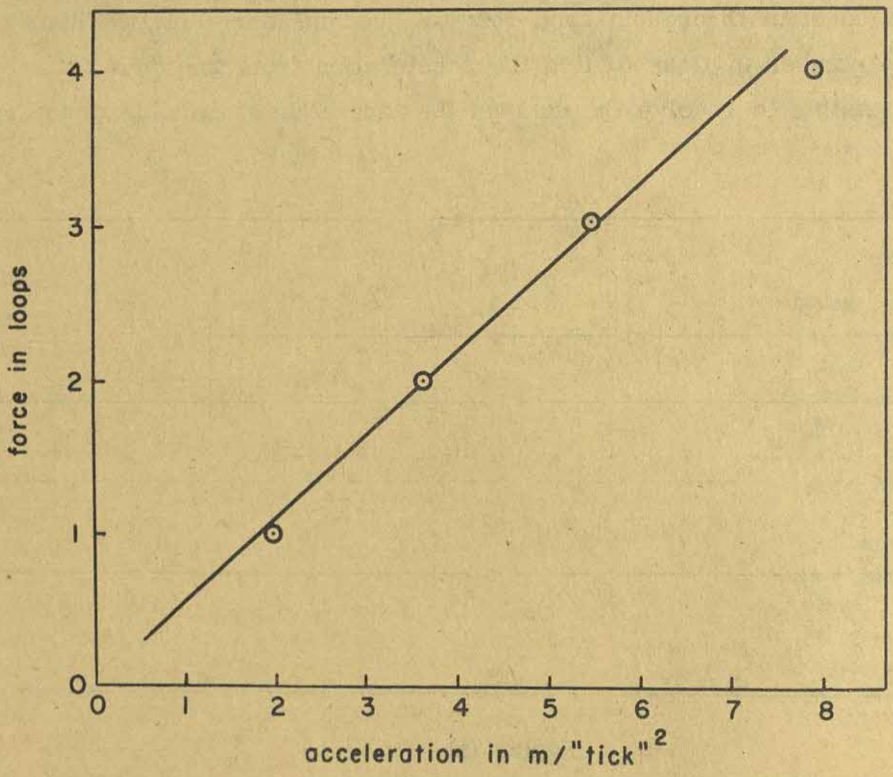


Figure (b)

The plot of the ratio of force to acceleration as a function of the number of bricks comes close to a straight line. If only three points are plotted on the graph, the results will not be convincing [Fig. (c)].

The intercept of this straight line with the "number of bricks" axis gives the mass of the cart in units of bricks. This is only approximately true, since the wheels of the cart are given rotational motion, and their rotational inertia may show up on the graph as equivalent to a mass of about 0.2 kg. The experimental data will not be accurate enough to show this.

Notice that at this point we have not yet discussed the relation of inertial and gravitational mass. If you do not plan to assign Experiment III-4, you can use this experiment to introduce the discussion of inertial vs. gravitational mass by asking the students to compare the ratio of the gravitational masses of the cart and a brick with the ratio of their inertial masses found from the graph [Fig. (c)].

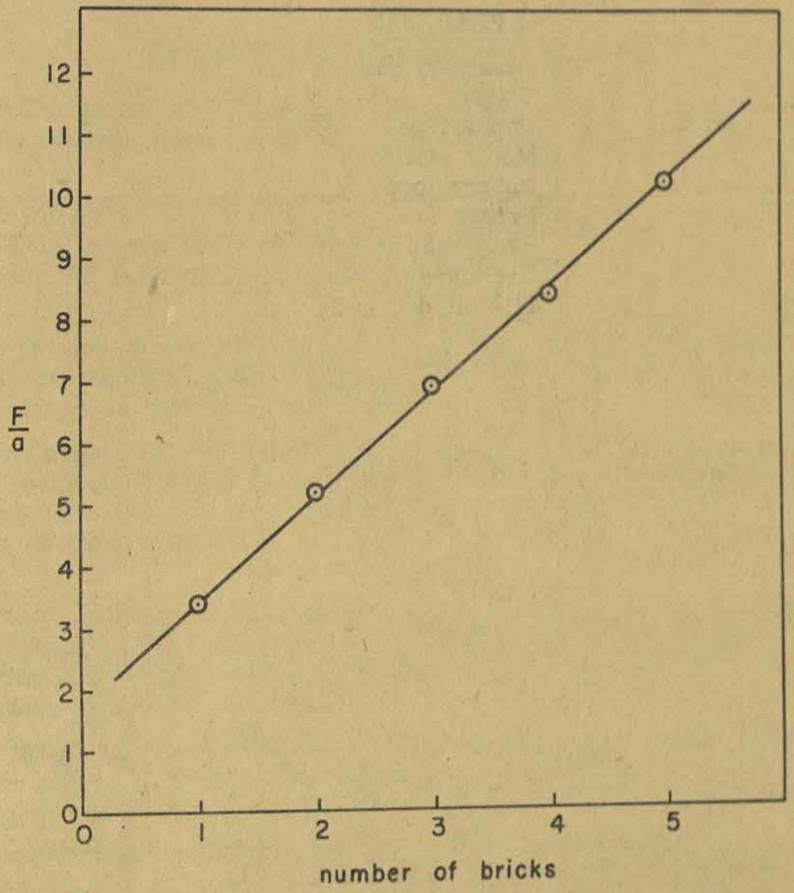


Figure (c)

To find the inertial mass of a chunk of lead or a heavy stone, pull the unknown mass on the cart with one rubber loop. From the ratio of the force to acceleration, the mass can be found by interpolation (or extrapolation) from the "f/a vs. m" plot.

The two experiments lead to the conclusion that the equation relating force, mass, and acceleration is $F \doteq kma$. The value of k depends upon the choice of units. The answer $F = ma$ is correct only if the student uses correctly related units like kilograms for mass, meters/(n ticks)^2 for acceleration, and consequently kilogram-meters/(n ticks)^2 for the force. (We use n to represent the number of ticks for a fixed time interval in making the analyses and drawing the graphs.)

The timer may be calibrated in seconds per tick and the acceleration expressed in m/sec^2. If there are 0.02 seconds per tick of the timer and 80 ticks were taken as the time interval, then an acceleration of 0.4 meters per (80 ticks)^2 is equal to

$$a = \frac{0.4 \text{ m}}{(80 \times 0.02 \text{ sec})^2} = 0.16 \text{ m/sec}^2$$

The accelerating force in newtons would be this acceleration times the accelerated mass in kilograms.

APPARATUS

1 Dynamics cart
1 Timer
Paper tape
1 Meter stick
4 Rubber loops
4 Bricks
Graph paper
C clamps
1 $1\frac{1}{2}$-volt dry cell

III-4. INERTIAL AND GRAVITATIONAL MASS

This experiment gives quite accurate results and will help clarify the relation between inertial and gravitational mass. It is best done after the discussion of Sections 20-5 and 20-6.

The experiment shows that gravitational and inertial mass are conceptually two different things. This is seen from the ways in which they are measured. Nevertheless, given two objects, the ratio of their inertial masses equals that of their gravitational masses.

The apparatus used in this experiment gives results accurate to about 1 per cent. Such a small error warrants careful consideration of the sources of error and their magnitude, expressed as fractions or per cents.

Two-inch C clamps are convenient units of mass and can easily be attached to the platform of the balance. This gives acceptable results, since the clamps are reasonably uniform. If time permits, you may want to adjust the clamps to weigh the same to about 1 per cent. To do this, take the lightest clamp as a standard and file or grind the others to match.

Other objects of similar size, shape, and composition can be used as standard masses. A piece of sandpaper glued to the platform of the inertial balance prevents these masses from sliding when the balance vibrates. If they slide, the balance will come to a stop before an adequate count can be made.

One of the things for the student to learn from this experiment is that the standard unit for inertial mass can be completely independent of the gravitational unit of mass. Therefore, it is advisable not to adjust the units of inertial mass to exactly 100 grams.

Any object of different material from the standard can be used as the unknown mass. But, to allow for accurate interpolation, it should have a mass that does not fall too near either end of the range for which the balance has been calibrated (100 to 600 grams). The unknown mass can be clamped to the balance with a unit C clamp.

In finding the period of the unloaded balance, it may not be possible for the student to count enough vibrations to get the period with an error of only about 2 per cent. This less accurate value is still worth plotting on the graph, but it should not be given too much importance when the curve is drawn.

When the load on the balance is too large (above about 750 grams) there is a visible up-and-down motion as well as horizontal motion. This makes the plotted relationship between T and m diverge from $T = 2\pi\sqrt{\frac{m}{k}}$, the relationship that holds for simple harmonic motion. (Do not discuss this relation in class at this time, as it will come up later in Chapter 21 of the text.) The more complicated graph [Fig. (a)] that one actually gets is still perfectly good as a calibration curve.

Unlike a simple pendulum, the frequency is independent of amplitude over a very wide range of amplitudes.

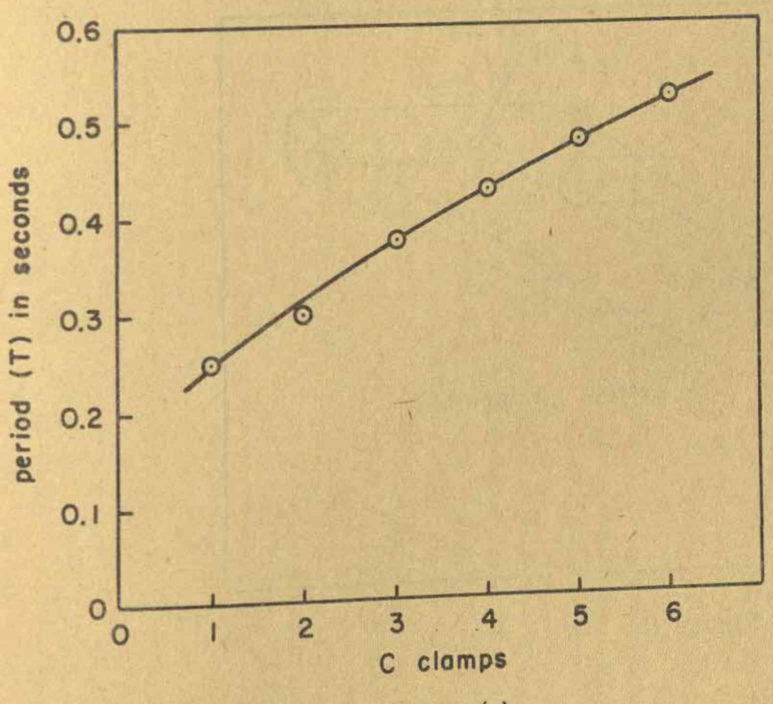


Figure (a)

Answers to Questions

The qualitative examination of the acceleration for a given amplitude and different masses shows that, even though the acceleration is not constant for a given mass, it is greater at each point in the swing for smaller masses. That is, it is the inertial effect of the mass, the same effect that occurs in Newton's law, that changes the period of the balance.

If the vibrations of the balance are measured for 100 seconds, the error in timing, assuming the time can be determined to the nearest second, will be $\frac{1}{100}$ = 1%. If 100 whole vibrations are timed, the error in counting will also be 1 per cent, because the number of vibrations can be determined to within one cycle. Now, if in 100 seconds the balance completes more than 100 vibrations, the ratio of $\frac{\text{no. vib.}}{100 \text{ sec}}$ will be accurate to within 2 per cent. If, on the other hand, it completes 100 vibrations in less than 100 sec, it is necessary to count 100 vibrations to get a ratio accurate to within 2 per cent.

The gravitational mass of the "unknown" is found from its inertial mass m_i:

$$\frac{(m_g) \text{ unknown}}{(m_i) \text{ unknown}} = \frac{(m_g) \text{ standard}}{(m_i) \text{ standard}}$$

In our experiment (m_i) standard = 1 by choice.

Inertial and gravitational mass are proportional within the experimental error. They are not equal since different units (C clamps and grams) have been used. They are, of course, equal in most physical work since the same standard piece of matter, the standard kilogram, is used for both scales.

On the moon we would get the same results. Neither gravitational nor inertial mass changes in going from earth to moon.

If the apparatus is used as shown in Fig. 4, it will have a shorter period for a given mass since the force of gravity now acts as a restoring force in addition to the restoring force of the bent blades when the balance is pushed to one side. This part of the experiment is optional.

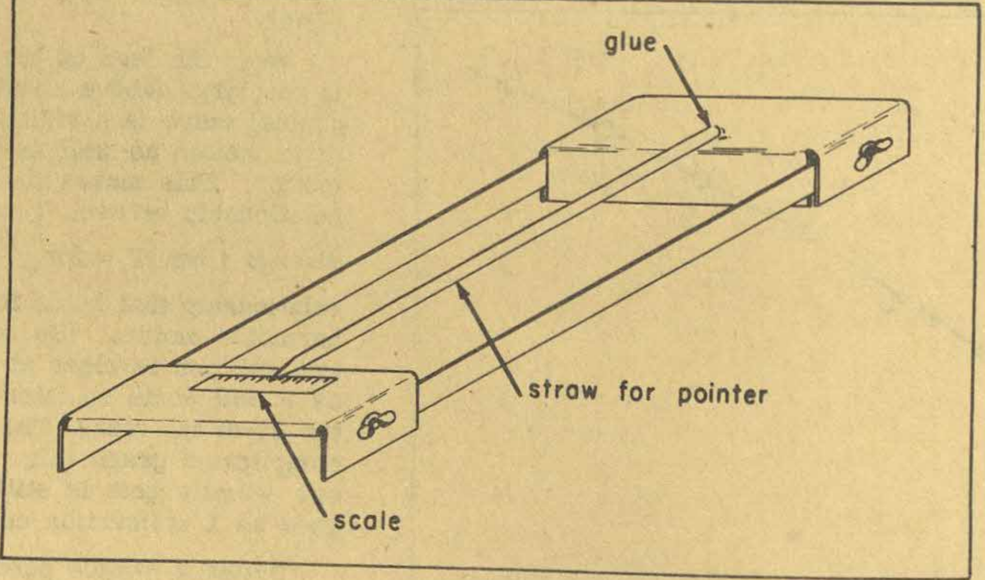


Figure (b)

This apparatus can be used as an accelerometer by fastening a straw or other indicator to the moveable platform and marking a force scale on the base [Fig. (b)]. The balance can then be clamped to a table and pulled by a calibrated spring balance to give a force scale in newtons. With a known mass (mass of object plus mass of platform) in place, the device can be calibrated in acceleration units from Newton's law ($a = \frac{F}{M}$). The answer to this question as well as the actual calibration and use of the apparatus as an accelerometer should be considered only as a project for particularly interested students. It is not meant to be answered by all.

The accelerometer can be held or mounted in an automobile so it is horizontal with the hacksaw blades at right angles to the direction of motion. Upon acceleration of the car, the accelerometer will show a deflection but there may be some oscillation back and forth if the acceleration is rapid. The balance should be damped by attaching several C clamps to prevent this. (The driver's attention should be focused on the road and not on the accelerometer!)

APPARATUS

1 Inertial balance
6 C clamps, 2" (can be shared to some extent)
1 C clamp, 3"
1 Metal slug
Cord
Ringstand with clamp or crossbar
1 Sheet graph paper
Stop watch or clock with sweep second hand

Pan balance
Several unknown masses (mass of 2 to 4 units)

III-5. FORCES ON A BALL IN FLIGHT

Students are so often instructed to "neglect friction" or "neglect air resistance" that they may think only simple, idealized motion can be analyzed. In this experiment the student can see how an application of what he has learned about simple projectile motion is the starting point which leads to an understanding of more complex motion. This is the most important thing for him to learn from this experiment. He also makes a discovery for himself; by analyzing Fig. 1 he learns that air resistance acts in a direction opposite to the motion and that it increases as the speed increases.

This experiment, although important, involves no experimental equipment and can be done as a homework assignment after projectile motion has been studied in the text.

Centimeter graph paper provides a convenient scale, although any tracing paper will suffice. Fig. (a) is a sample analysis of Fig. 1. Students may have trouble reducing the change in velocity resulting from Δv_g to a vector they can subtract on their diagram.

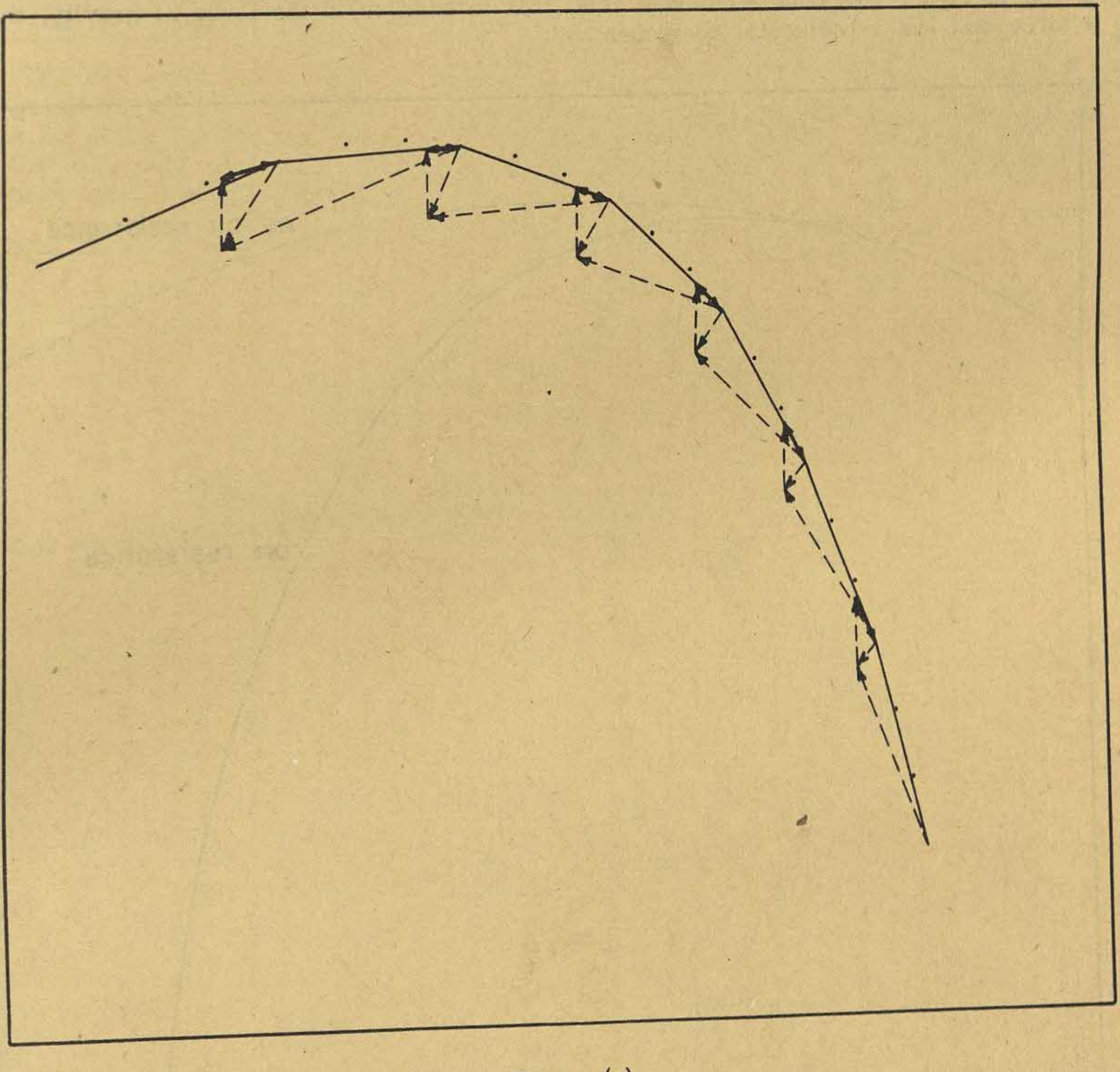


Figure (a)

The first step is $\Delta v_g = g \Delta t = 9.8(0.1) = 0.98$ m/sec. This must then be converted to meters per tenth of a second. This will be $0.98(0.1) = 0.098$ m/0.1 sec. This is equivalent to converting the acceleration of gravity from meters per second squared to meters per (tenth of a second)^2, $9.8 \text{ m/sec}^2 = \frac{9.8}{100} = 0.098 \text{ m/(0.1 sec)}^2$. This value is then reduced to the scale of Fig. 1. Since the photograph is $\frac{1}{10}$ actual size, the length of the velocity change vector due to gravity Δv_g that must be subtracted will be $\frac{0.098}{10} = 0.0098$ m. This change is, of course, in the vertical direction and is subtracted from the velocity change Δv by adding its negative as shown in Fig. 2(b).

If a student is interested in analyzing Figs. 3 and 4, he should check the scale reduction by measuring the meter stick in the photographs.

Answers to Questions

A qualitative examination of the photograph shows that the horizontal velocity decreases from left to right. One concludes from this, that in addition to the force of gravity, there is a force that has a horizontal component.

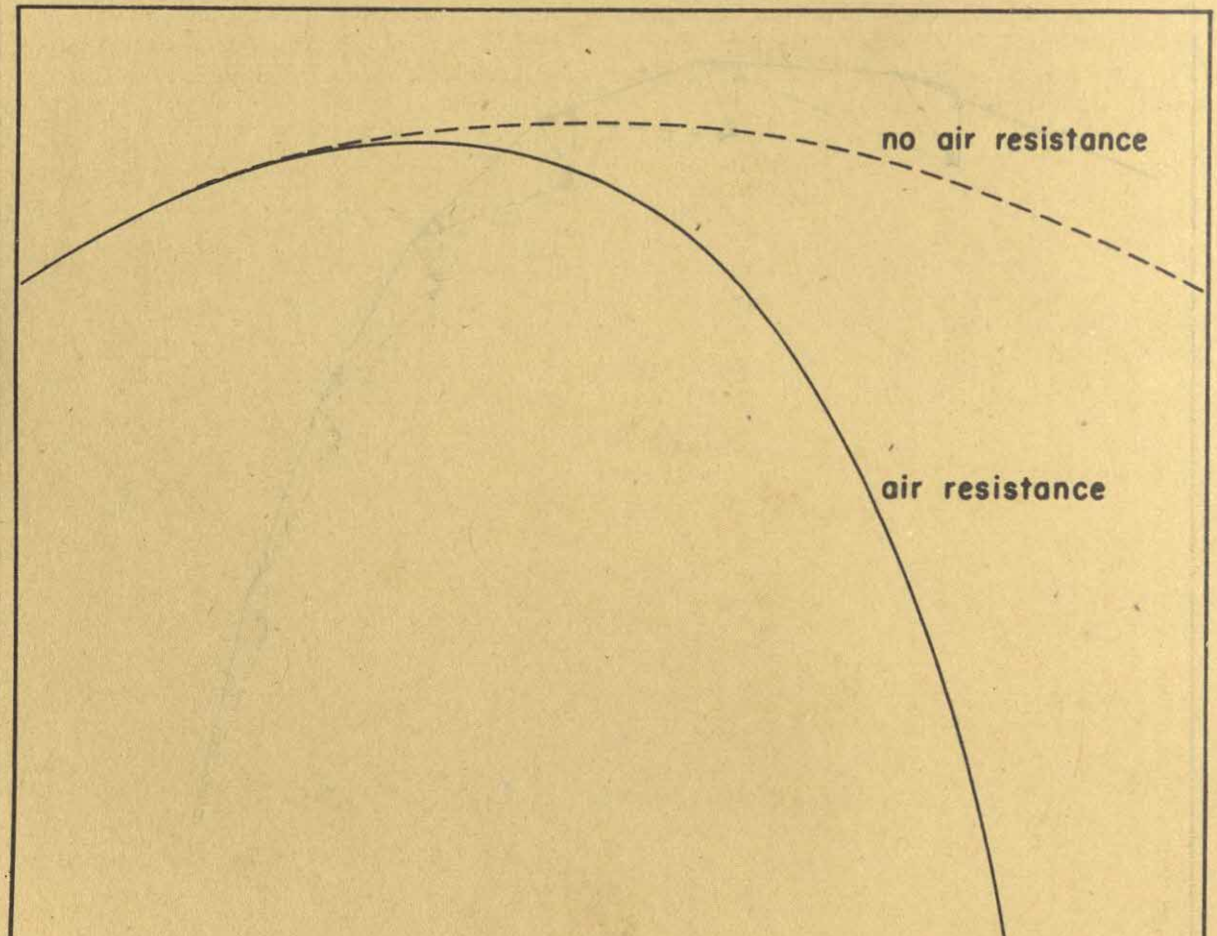


Figure (b)

The total velocity changes found by subtracting velocity vectors on the diagram vary from interval to interval in both direction and magnitude. The magnitud of the Δv decreases during the motion and the direction slowly becomes more horizontal. Since velocity changes are in the direction of the forces which produced them, there is at least a horizontal component of the net force acting on the sphere. Without further analysis it is impossible to tell how the unknown force changes.

The change in velocity of the ball in 0.1 sec. caused by gravity is 0.98 m/sec and it acts vertically downward. In meters per tenth of a second it is 0.098 m/0.1 sec.

When the effect of gravity is subtracted, the residual velocity changes due to the remaining force have both a vertical and a horizontal component and are opposite to the direction of motion. The larger the velocity, the larger is the opposing force. The retarding force is the result of air resistance. This force is proportional to the square of the velocity under the conditions of our experiment. The total variation of the velocity, however, is not great enough to show this proportionality by a graph of the force as a function of v^2, particularly with the limited accuracy with which one can measure the force in the analysis of the data.

The ball must have a very large area-to-mass ratio (a very low density) to show such large effects of air resistance at relatively low velocities. If the ball had been the same size but much heavier, it would have followed a path more like the parabolas illustrated in the text. The ball used in all three photographs was a 0.05-gm sphere 1.5 cm in diameter cut from styrofoam.

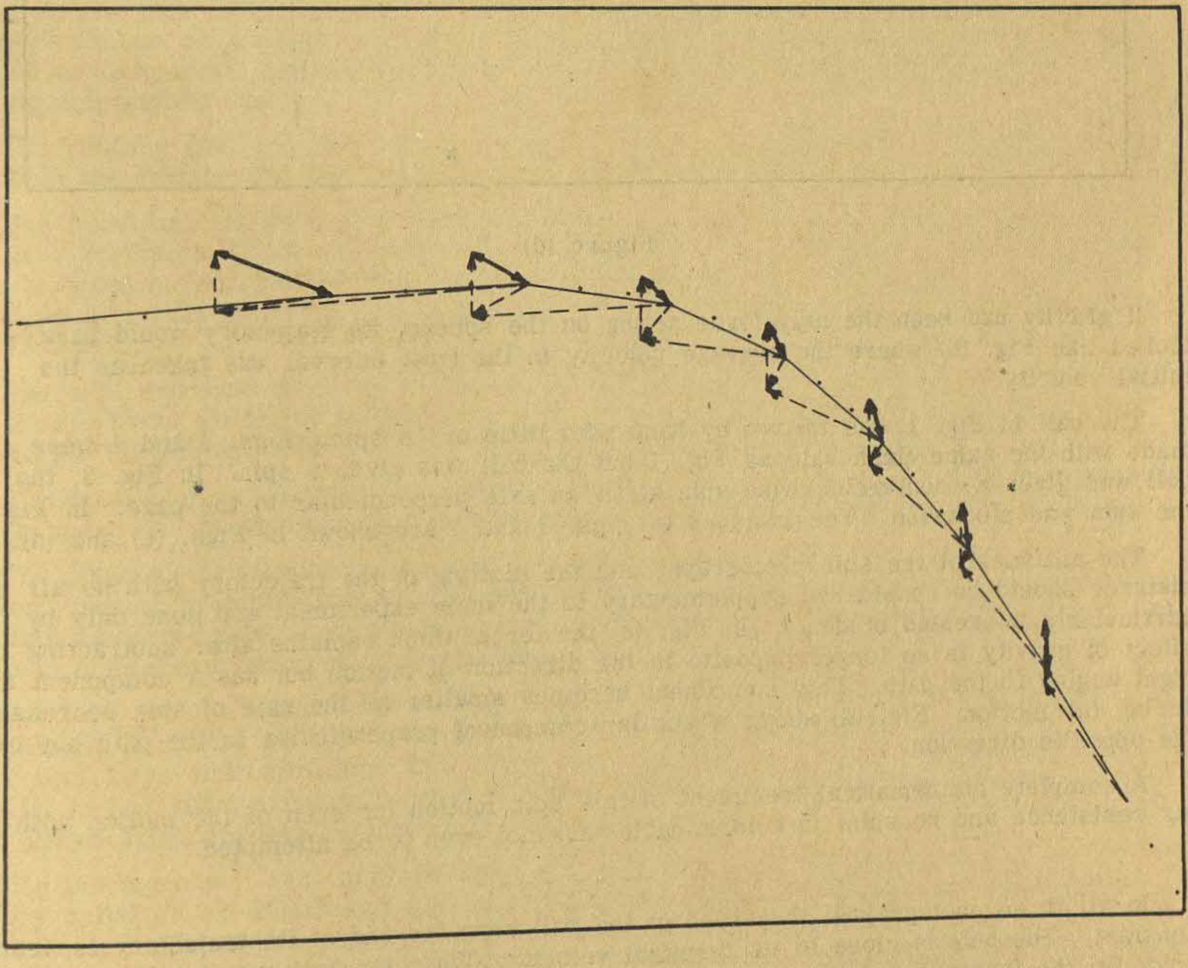


Figure (c)

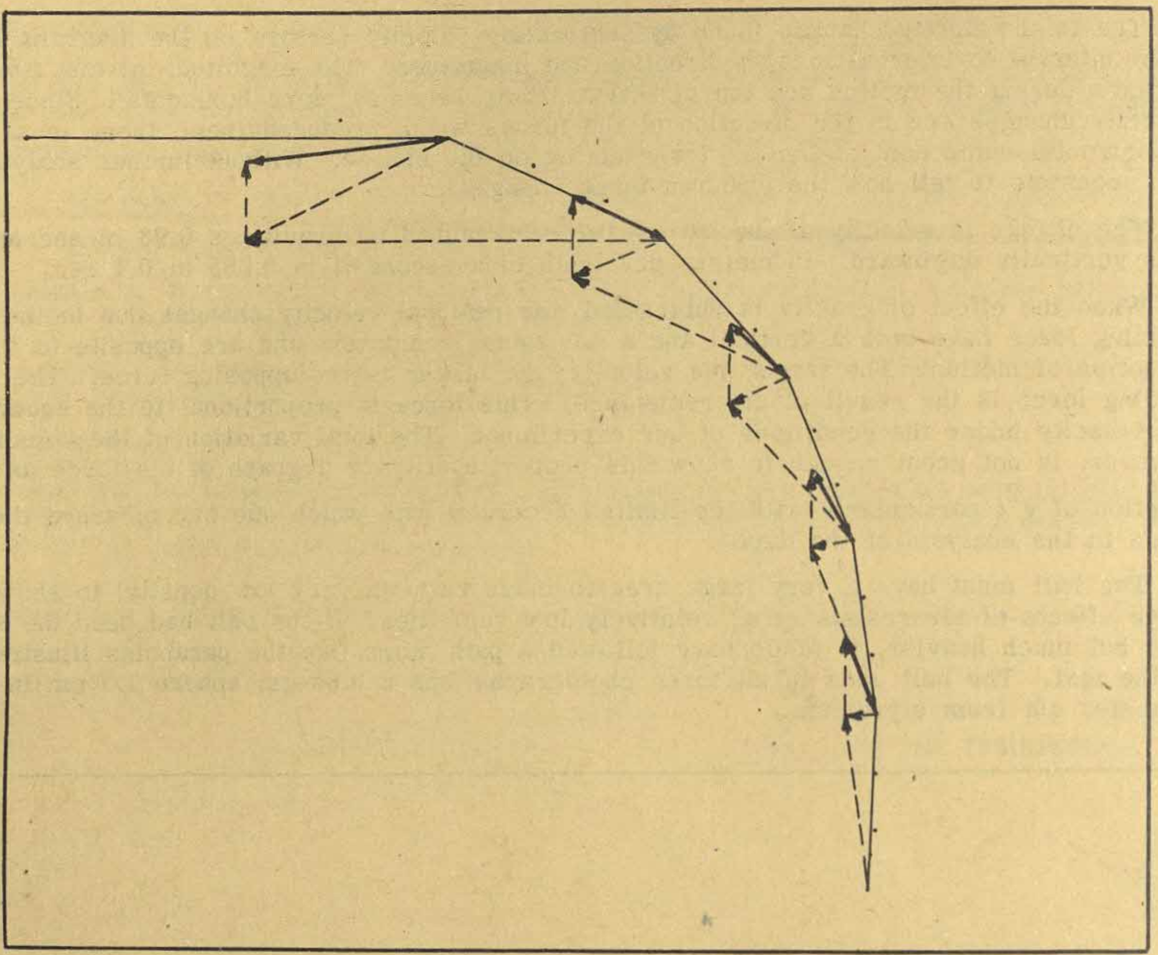


Figure (d)

If gravity had been the only force acting on the sphere, its trajectory would have looked like Fig. (b) where the average velocity in the first interval was taken as the initial velocity.

The ball in Fig. 1 was thrown by hand with little or no spin. Figs. 3 and 4 were made with the same flash rate as Fig. 1 but the ball was given a spin. In Fig. 3, the ball was given a counterclockwise spin about an axis perpendicular to the page. In Fig. 4 the spin was clockwise. The analyses of Figs. 3 and 4 are shown in Figs. (c) and (d).

The analyses of the spin photographs and the plotting of the trajectory with no air resistance should be considered supplementary to the main experiment and done only by particularly interested students. In Fig. (c) the force which remains after subtracting the effect of gravity is no longer opposite to the direction of motion but has a component at right angles to the path. This component becomes smaller as the rate of spin decreases during the motion. Fig. (d) shows a similar component perpendicular to the path but in the opposite direction.

A complete mathematical treatment of this spin motion (or even of the motion with air resistance and no spin) is a formidable task not even to be attempted.

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In all three photographs, the speed of the ball near the end of its trajectory is nearly constant. The ball is close to its terminal velocity. When the ball reaches its terminal velocity, the force of gravity on the ball equals the air resistance, the net force is zero, and it moves at constant velocity.

Students may wish to fire ping-pong balls with a spin. A cardboard mailing tube makes a good gun. Put the ball in the tube and swing it in an overhand arc or an underhand arc depending on the desired direction of spin. This method was used in making Figs. 3 and 4.

APPARATUS

Translucent centimeter graph paper, or tracing paper
Ruler

Hints on Strobe Photography

The camera in Fig. 6, Experiment I-1, records on film the position of an object each time a slot in the motor-driven strobe disk moves past the lens. This technique makes it possible to analyze two-dimensional motion which, cannot be measured with a tape timer.

In most instances, exposure rates of 5 to 30 per second will suffice. A low-speed motor such as a 300-rpm synchronous clock or instrument motor is the most versatile, since higher exposure rates can be obtained by having several equally spaced slots in the disc. A synchronous motor is best for driving the disc, since its rate of rotation, and therefore the exposure rate, is known. However, in many cases the unit of time is arbitrary and therefore a cheaper motor will be adequate.

The speed of a non-synchronous motor can be measured and checked for constant speed by photographing, through the rotating disc, a 78-rpm or $33\frac{1}{3}$-rpm phonograph disc marked with a white spot. If a synchronous motor is overloaded, it will not run at its rated speed. One should be sure, therefore, that the disc is not so large that air resistance against its surface will slow it down. When a synchronous motor is overloaded, a neon or fluorescent lamp operating on 60-cycle A.C. will flicker when observed through the rotating slotted disc.

The rotating disc should be as close as possible to the camera lens without touching it. Both the camera and the stroboscope should be held rigidly.

The individual exposure of a moving object can be varied by adjusting the disc-slot and lens apertures. Open the basic lens aperture as wide as possible. The lens should then be taped so that its aperture has the same shape as the slot and has a width equal to or smaller than that of a slot in the disc. Sharper images are obtained by decreasing the slot widths.

The total exposure time can be controlled better by using the bulb setting rather than the time-release setting of the camera. If a cable release, instead of the hand, is used to operate the shutter, there will be less chance of the camera vibrating.

A film with an A.S.A. emulsion speed of 160 or higher is adequate. In some cases, a better image can be obtained by overdevelopment and/or printing on high-contrast paper.

The plane of the film in the camera should be parallel to the plane of the motion, to prevent unequal magnifications of the different areas of view. If a scale is used to give absolute measurements, it should be placed in the same plane as the motion, so that there is a minimum error due to parallax.

For best results, erect a dark-colored backdrop of cloth, paper, or other material approximately 2 meters square. If it is necessary to paint the surface, a flat paint is best to prevent glare and reflection. The object to be photographed should be mounted to move parallel to and about a meter in front of the backdrop. The camera and stroboscope are then placed approximately $1\frac{1}{2}$ meters in front of the object.

The moving object can carry its own light source (a flashlight bulb powered by two AA dry cells) or be illuminated with two ripple-tank light bulbs (150-watt, clear glass, with shields). For objects moving horizontally, mount the two bulbs on either side of the camera, about a meter apart and pointing inward at an angle of approximately 40° to the plane of the object. To photograph objects moving vertically, mount one bulb on the floor pointing upward at an angle of 80° to the object plane, and the other bulb about two meters directly above pointing downward at an angle of 80°.

If it is desired to show a scale on the finished print, place a meter stick in the plane of the object.

Satisfactory prints using a Polaroid camera have been made with the arrangement described above and with the following specifications:

Lens aperture	f 4.7
Shutter setting	B
Strobe wheel	2, 3, or 6 slits open
Film	Polaroid, #47 (3000 ASA)

Parts and Materials

Any camera with a bulb or time exposure
Motor (preferably synchronous): 300 to 1800 rpm (mounted)
Disc (6" to 10") of heavy paper, thin metal, plastic, or similar material
Several high-wattage lamps with sockets, cords, and supports

III-6. CENTRIPETAL FORCE

The purpose of this experiment is to find the dependence of the centripetal force on the frequency of revolution, mass, and radius of an object moving in a circle. The experiment is best done before the relation is derived in the text (Section 21-5).

As a result of doing the first part of the experiment, the student will conclude that the centripetal force is proportional to the square of the frequency. If time permits and some of the students wish to go further, they will conclude from experimental evidence that the centripetal force is proportional to the mass and to the radius of the circle.

As in many of the experiments in this volume, it should be emphasized that the discovery of fundamental laws does not require the use of standard units of measure. Here we use arbitrary units of force and mass: the unit for measuring centripetal force is the force of gravity on a steel washer; the mass unit is the mass of a #4 rubber stopper.

The experiment can be done alone in the laboratory or at home, but it is best done by two or three students together - one to swing the stopper in a circle, another to count the number of revolutions and a third to do the timing. If the laboratory is too small for swinging many stoppers in a circle of one meter radius, it is advisable to do the experiment out-of-doors or in the school gymnasium.

The size of the glass tube is not critical, but the bearing surface for the string at the top end must be carefully fire-polished so it is smooth and free from bumps to keep friction at a minimum. The forces are such that the tube will not break unless it has a flaw. To prevent injury, wrap the tube with tape so there will be no danger from jagged pieces of glass if it should break.

The cord used is critical if friction is to be kept low. Fisherman's hard nylon braided casting line, 12-lb test works well.

The stopper size, radius of the circle, and size and minimum number of washers have been carefully chosen to give good results. Large variations from the recommended values may make the graph of force as a function of velocity look like a straight line.

To keep the stopper moving at constant frequency, the top of the tube must be moved in a small circle. This provides a small component of force along the path to counteract friction. However, this circle should be less than a centimeter or two in diameter or the circle described by the stopper cannot be accurately measured by the length of the cord. The radius of the circle described by the stopper should be measured from the middle of the stopper near its center of mass. Since the dimensions of the stopper are small compared to the radius of the circle, measurement from this point will give a value for the radius which is close to the correct value.

The Laboratory Guide specifies a minimum number of washers so the cord will be horizontal enough to permit the use of its length as a reasonably accurate measure of the radius of the circle. Actually, using the "slant" radius given by the length of the cord leads to correct results no matter how large the angle between cord and the horizontal. This can be verified by analyzing Fig. (a). The force of gravity $m_w g$ acting on the washers is transmitted along the cord and acts on the stopper along L. The horizontal component of this force provides the centripetal force F_c along R. We see that $R = L\cos\theta$ and $F_c = m_w g\cos\theta$. Writing the equation for the centripetal force in the form

$$F_c = 4\pi^2 m_s f^2 R = \frac{4\pi^2 m_s R}{T^2},$$

we have by substitution

$$m_w g\cos\theta = \frac{4\pi^2 m_s L\cos\theta}{T^2}$$

$$m_w g = \frac{4\pi^2 m_s L}{T^2}$$

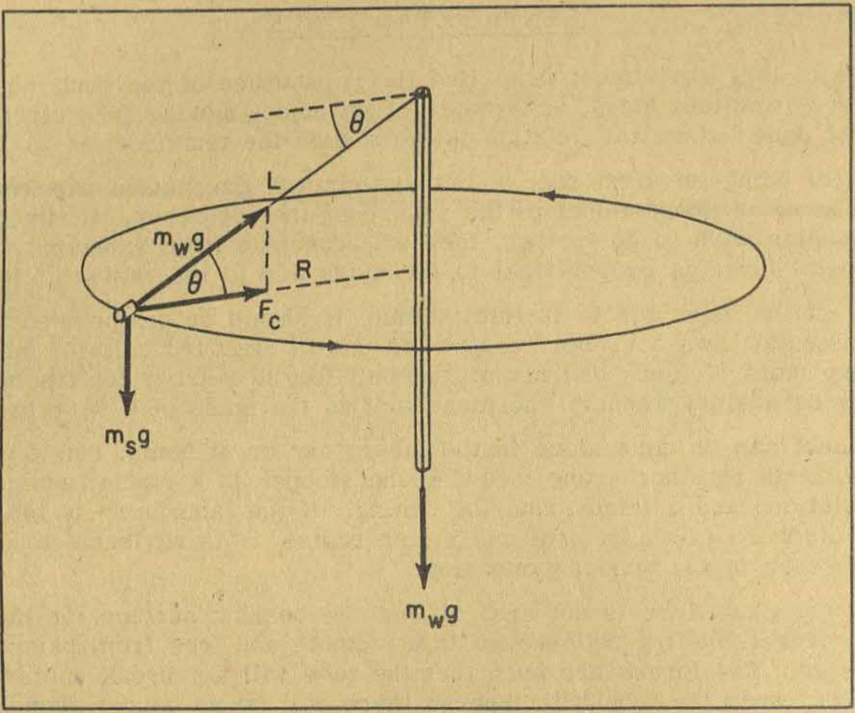

Figure (a)

This means that the force exerted by the washers is inversely proportional to the period squared for a constant length of cord L. Since the student assumes that R stays very nearly equal to L, he measures the period of rotation and assumes this to be inversely proportional to the velocity v. There is no error due to the changes in the angle θ, because θ drops out of the calculation. You will note from Fig. (a) that θ is determined only by the weight of the washers since $\sin\theta = \frac{m_s g}{m_w g}$. Using less than the recommended number of washers with a #4 stopper will therefore make θ larger.

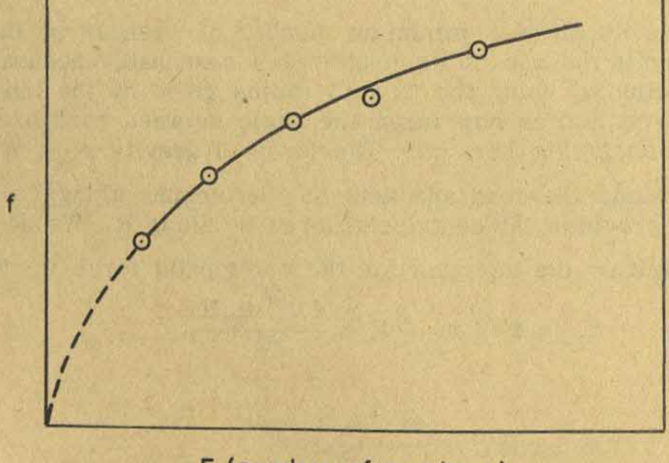

Figure (b)

Answers to Questions

The faster the stopper rotates, the harder one must pull down on the string. If one lets go, the stopper ceases to move in a circle and pulls the string up the tube behind it. By exerting the force himself, the student becomes aware that a force at right angles to the motion is necessary to make an object move in a circle.

A graph of the frequency of the stopper as a function of the number of washers looks like Fig. (b). It is, of course, not clear what function this graph represents although one might suspect that it is a parabola. The student may think, if his points are somewhat scattered, that the curve is a straight line which does not pass through the origin, but if he is asked what the frequency should be with zero force, he should see that it would have to be zero, that is, the stopper must be stationary. Hence the curve must pass through the origin, curving down from his assumed straight line.

A plot of $\underline{f}^2$ as a function of the number of washers will be a straight line passing through the origin, showing that the centripetal force varies as $\underline{f}^2$.

The student can conclude, from Newton's law F = ma, that doubling the mass whirling in a given circle with a given centripetal acceleration should require twice the force to keep it moving at the same speed in the circle. If he has time, he can try this.

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It is difficult to find out how the force depends upon the radius because it is hard to whirl the stopper at the same frequency each time the radius is changed. A better way of getting the relation between force and radius is to repeat the first part of the experiment several times, using different radii and only one value of centripetal force for each radius. Then a family of curves of $\underline{f}^2$ as a function of $\underline{F}$ can be drawn [Fig. (c)]. Since we know from the first part of the experiment that these curves are straight lines, the single value of the centripetal force for each radius suffices to establish the straight line through the origin.

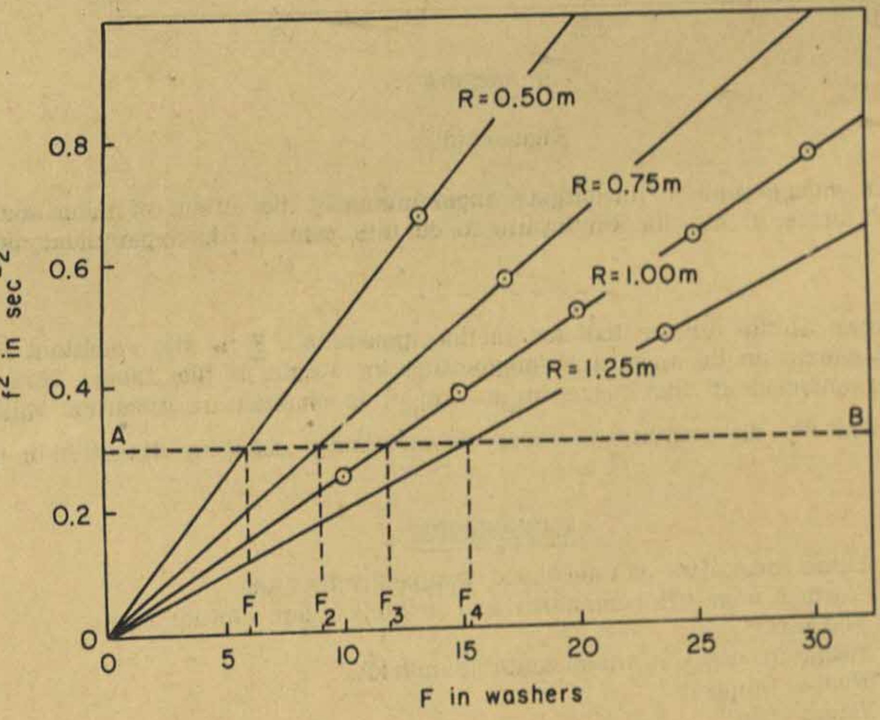

Figure (c)

For a constant value of f^2, represented by the line AB, the various values of the force (F_1, F_2, F_3, F_4) lie on curves corresponding to different values of $\underline{R}$. When these values of $\underline{F}$ and $\underline{R}$ are plotted, they give curves which resemble Fig. (d). [Note that Figures (c) and (d) do not represent actual data taken with the apparatus. They are merely approximate sketches of the results that would be obtained.] Students who are well versed in "ham" radio may recognize this method of analyzing a family of curves as similar to the method of finding the amplification factor of a vacuum tube from a plot of the plate current as a function of the plate voltage.

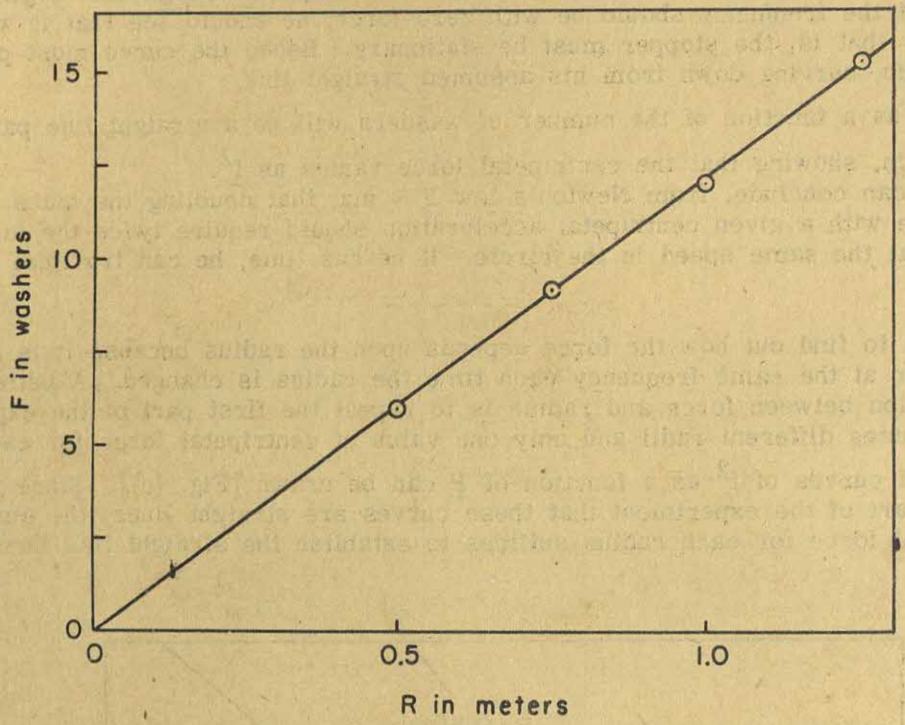

Figure (d)

If there is not enough time to investigate experimentally the effect of mass and radius on the centripetal force, it may be worthwhile to do this part of the experiment as a class demonstration.

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Figure (e) shows all the forces that act on the apparatus. $\underline{F}$ is the resultant force that the operator exerts on the apparatus (neglecting the weight of the tube). Note that the gravitational component of this force ($m_w g + m_s g$) is constant in direction while the horizontal component F_c, the centripetal force, is constantly changing direction in the horizontal plane.

APPARATUS

1 Glass tube, fire-polished and wrapped with tape
Cord, 1.5 m (fisherman's hard braided nylon casting line)
20 Washers
1 Alligator clip, or other suitable marker
1 Rubber stopper
1 Paper clip
2 Sheets graph paper
1 $\frac{1}{4}$" dowel or pencil
Meter stick
Stop watch or clock with sweep second hand

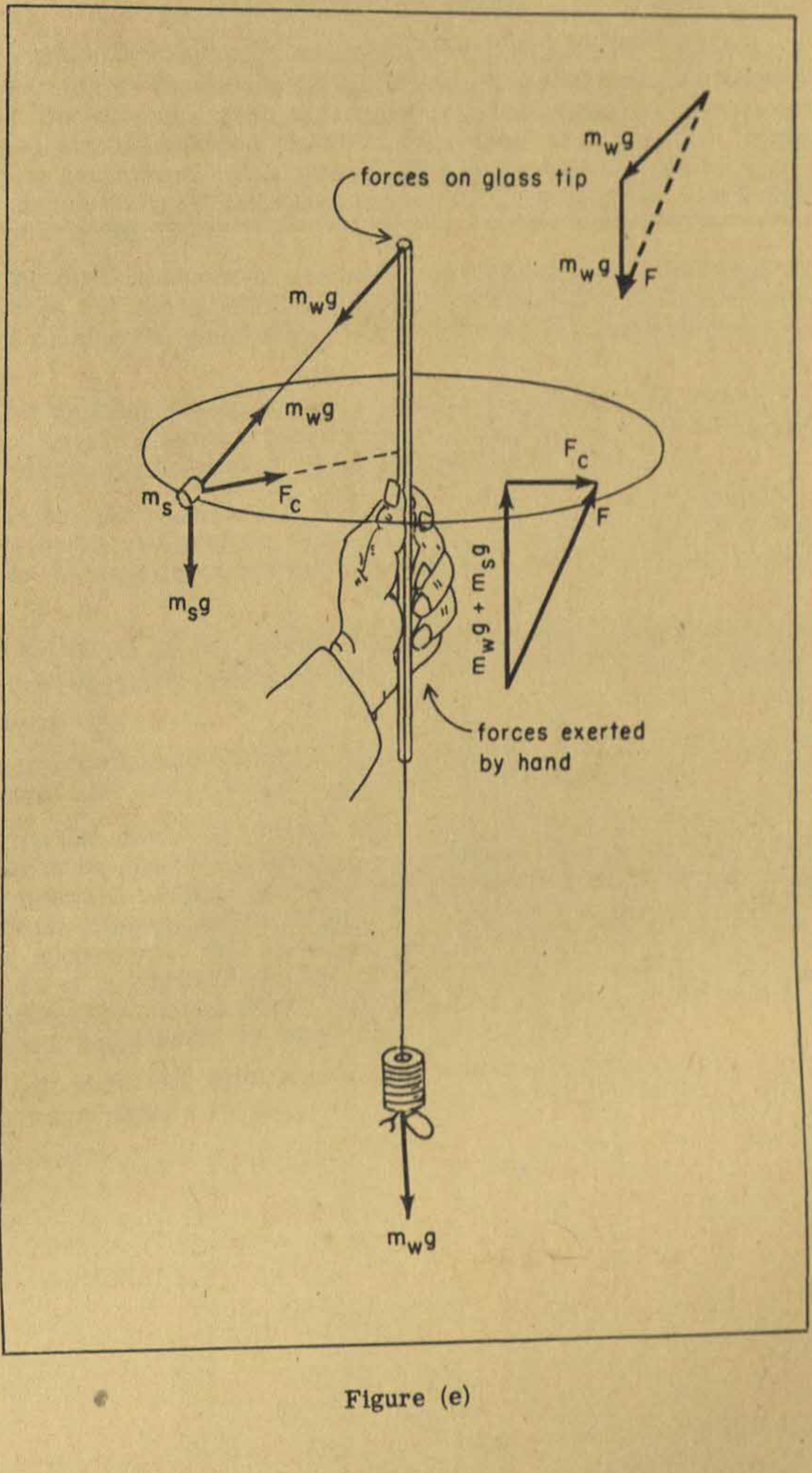
forces on glass tip
$m_w g$
$m_w g$
F
$m_w g$
$m_w g$
m_s
F_c
$m_s g$
F_c
F
$m_w g + m_s g$
forces exerted by hand
$m_w g$

Figure (e)

III-7.

LAW OF EQUAL AREAS

Whenever a central force operates, an object will move in orbit such that it sweeps out equal areas in equal times, regardless of how the force varies with the distance from the central point. The law of equal areas expresses the conservation of angular momentum, but since angular momentum is not covered in the course, it is probably unwise to discuss it in relation to this experiment. This experiment serves only to illustrate the law of equal areas and is not among the most important experiments. It is most appropriately done during the study of Chapter 22.

Fine sand is the best substance to use as a time indicator. Conical cups should be used, since sand does not run out of flat-bottomed cups evenly. To stop the flow, the hole can be plugged by pushing the pointed end of a pencil down through the sand and into the hole.

The balance for weighing the sand should have a sensitivity of at least 0.1 gm; a sensitivity of 0.01 gm would be better. More sensitive balances are not worth while, since other sources of error will be greater than those incurred in weighing.

A rigid support for the pendulum is necessary. If the support wobbles, the pendulum will precess in its orbit, reducing the accuracy when several revolutions are used to collect enough sand to be weighed accurately. Ideally, the pendulum should be supported from the ceiling.

You may find it best to let the students make several runs with the pendulum, weigh the sand, and then analyze the results of one or two of the best runs as homework.

Answers to Questions

Several complete revolutions permit a more accurate weighing of the accumulated sand. Too many revolutions will result in inaccuracies caused by precession.

The net force on the pendulum always acts toward a point directly beneath the point of suspension. This can be checked by pulling the pendulum to one side and releasing it without giving it any sideward motion. In this case, if sand is allowed to run out as the pendulum starts from different points around its orbit, all the sand tracks will intersect under the point of suspension. This question suggests to the student that this is the point toward which the force is directed and around which equal areas are swept out by a conical pendulum, unlike planetary motion where the force acts toward a focus of the ellipse and the planet sweeps out equal areas in equal times around this focus.

Careful work will show that equal areas are swept out to an accuracy of about 5 to 10 per cent, even for a narrow elliptical orbit where the variation in velocity is large.

For both the pendulum and the planets, the force is a central force acting toward a single point regardless of the position of the object. The force on the planet varies inversely as the square of the distance between it and the sun. The force on the pendulum is directly proportional to the distance of the bob from the center of the ellipse as long as the angle of the suspension string with the vertical is small.

An inverse-square force causes a planet to move in an ellipse with the center of force at one focus, whereas the linear force on the pendulum causes it to move in an ellipse with the center force at the center of the ellipse.

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Fig. 4(a) was taken by the method described in the Laboratory Guide. (See the suggestions given at the end of Experiment III-5.) The plane of the film was parallel to the plane of the ellipse. Fig. 4(b) was taken the same way with the same equipment as Fig. 4(a), but the size of the orbit was different.

If Figs. 5 and 6 are analyzed to get the ratio $\frac{R^3}{T^2}$, the ratios will not be equal for the two cases since Kepler's law of periods applies only to an inverse-square force.

APPARATUS

1 Conical paper cup
Lightweight cord 20' to 25'
Ringstand with crossbar and clamps
Sand or dry salt, with container
1 Large sheet brown wrapping paper
3 C clamps

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1 Camera
1 Motor-driven stroboscope
2 150-watt bulbs, sockets, cords and shields
(or floor lamps)
1 White ball
5 Ringstands with crossbars and clamps
String or thread
3 C clamps
Large sheet black paper
Meter stick
1 Sheet translucent graph paper

Pan balance
Dustpans and brushes
Tablet paper

III-8. MOMENTUM CHANGES IN AN EXPLOSION

This experiment is most appropriately done as soon as impulse and momentum have been studied in the text, and preferably before taking up momentum changes when two bodies interact (Section 23-3). The experiment will introduce the law of conservation of momentum.

The spring exploder can be dangerous if it is released indiscriminately. It should not be cocked without the spring safely anchored to the cart. You may want to caution your students not to release the spring when the cart is pointed in the direction of others.

Springs may differ in their strength. A weak spring acting on a heavy cart may not give it enough velocity to reduce the effect of slight variations in the table top.

The optimum distance between the two bumpers is about 1.5 m. If the distance the cart must travel is too great, the cart may, because of friction, come to a stop before reaching the bumper. On the other hand, if the distance traveled is too short and the velocities are reasonably high, the coincident timing method will not be accurate.

Both carts should be loaded with at least one brick, since some of the force of the exploder applies torque to the wheels, making them rotate. This appears as a loss of linear momentum. As long as the mass of the wheels is small compared to the total mass, this effect is insignificant.

The starting positions of the carts can be marked on the table with chalk. Make sure the student places the starting mark for each cart at the end closest to the bumper, as shown in Fig. 2.

If both carts have the same load, friction tends to cancel out, since the force of friction acts for the same time on both carts, giving equal and opposite impulses to the two carts and decreasing their momentum equally. There will, of course, be different forces of friction on two carts loaded with different masses, but the effect of friction is small and can be neglected in this experiment.

If the table is not level, one cart will gain momentum during its run, and the other will lose momentum. If the table cannot be made level, it may be necessary to run two trials for each loading of the carts, reversing their positions for the second trial and comparing the average momentum change of each cart.

Answers to Questions

If the spring is released with the cart at rest on the table, the cart does not appear to move. The student will conclude that the momentum of the cart before and after the explosion is zero. Changing the load on the cart will not change this result.

With various loads on the carts, it can be observed qualitatively that the lighter cart moves faster as a result of the interaction. Some students may guess that the momenta of the carts have the same magnitude but are in opposite directions.

In calculating the ratio of momenta, there is no need for a standard unit of momentum. The momenta can be expressed in kilogram-meters per "clank." (A "clank" is the unit of time interval, the time taken by both carts to reach the bumpers in a given run.) Of course, changing the loads on the carts will result in different speeds and hence different time and momentum units, but the time unit will be the same for both carts. Without trying to find a specific unit, the student can answer that the change in momentum of one cart is equal in magnitude and opposite in direction to the change in momentum of the other cart. (The ratio of the momenta is minus one to about 5 per cent).

The total momentum before and after the explosion is zero, and hence momentum is conserved.

If a stick of dynamite were exploded between the carts, the pieces would fly in many different directions at different speeds, but we would expect the vector sum of their momenta to be zero. This is a hard question for the student to answer before he has become thoroughly familiar with momentum changes in two dimensions, studied later in Chapter 23 and in Experiment III-10.

Another way of doing this experiment eliminates the need for listening for simultaneous collisions of carts and bumpers. It requires more elaborate apparatus and raises more questions in the student's mind, but some students may be interested in trying it.

Place the carts on a smooth board about 1.5 meters long which has bumpers at its ends [Fig. (a)]. The board should rest on 1-foot pieces of $\frac{1}{2}$" or $\frac{3}{4}$" dowel. Use three pieces of dowel, one near each end and one in the middle. Make an index mark on the board and a corresponding reference mark on the table. When the exploder sends the carts toward the ends of the board, one will strike its bumper first and move the board in the direction in which it is traveling. A moment later, the second cart strikes its bumper and stops the board. The movement of the index mark is a sensitive indicator of the relative order in which the carts strike. When the correct starting position of the carts has been found, the carts will strike the bumpers simultaneously and the board will not move at all.

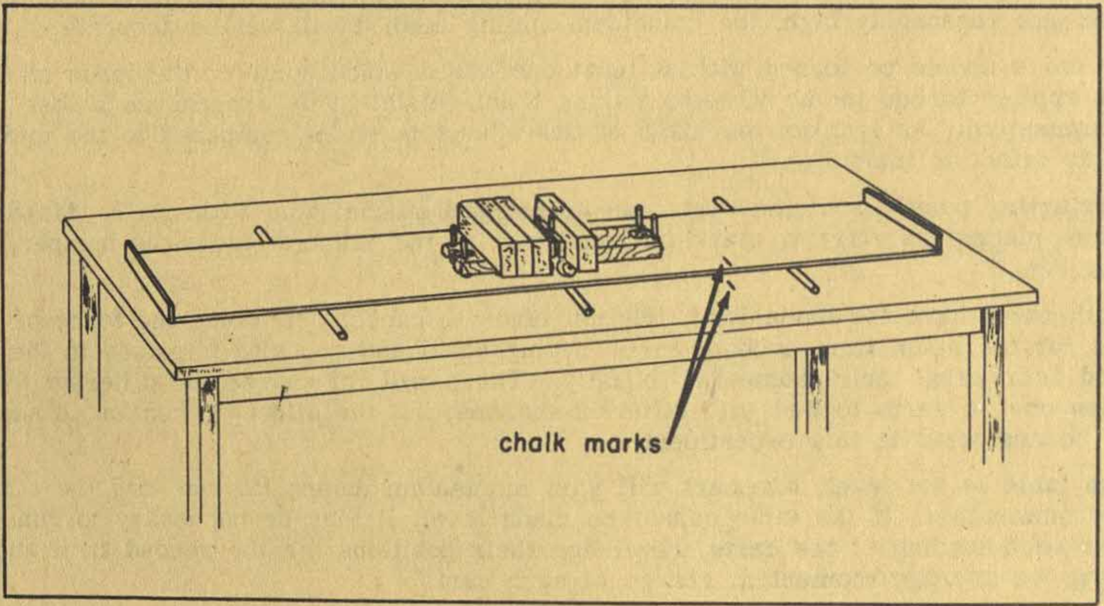

Figure (a)

APPARATUS

2 Dynamics carts
4-5 Bricks
4 C clamps
2 Table stops (wood bars)
Meter stick

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1 Smooth board, 1.5 m. long
3 Wood dowels, $\frac{1}{2}$" or $\frac{3}{4}$" by 1'

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Pan balance
Wedges
Chalk or masking tape

III-9. THE CART AND THE BRICK

Since this experiment comes after the student has discovered conservation of momentum in the explosion experiment (III-8), it is not important whether it is done before or after momentum conservation is taken up in the text. A more complex experiment than "Momentum Changes in an Explosion," it gives the student a deeper insight into momentum conservation.

The experiment is not difficult to perform, but there are several techniques to be noted. Heavy cord should be used to support the brick, because it is much easier to pinch a thick string to the support bar as shown in Fig. 1. Several practice runs should be made before recording the motion of the cart.

Since some friction will always be present, the cart will slow down throughout the run. To analyze the interaction properly, the velocity of the cart should be measured just before and just after the interaction. At least one brick should be on the cart when making measurements, to prevent the final velocity from being so low that table irregularities will cause it to fluctuate too much. Good judgment is needed in finding the velocities before and after the interaction, to choose an interval long enough to be meaningful and short enough to have fairly constant velocity.

To prevent damage to table tops from falling bricks that miss the cart, wrap the bricks in paper.

An alternate analysis of the data can be made by plotting velocity as a function of time. By extrapolating the two lines which represent the slowly changing velocities before and after impact, the actual velocities can be determined at the time of impact [Fig. (a)]. On smooth tables, practically no extrapolation is necessary. In any case, only the better students should be encouraged to use this method of analysis.

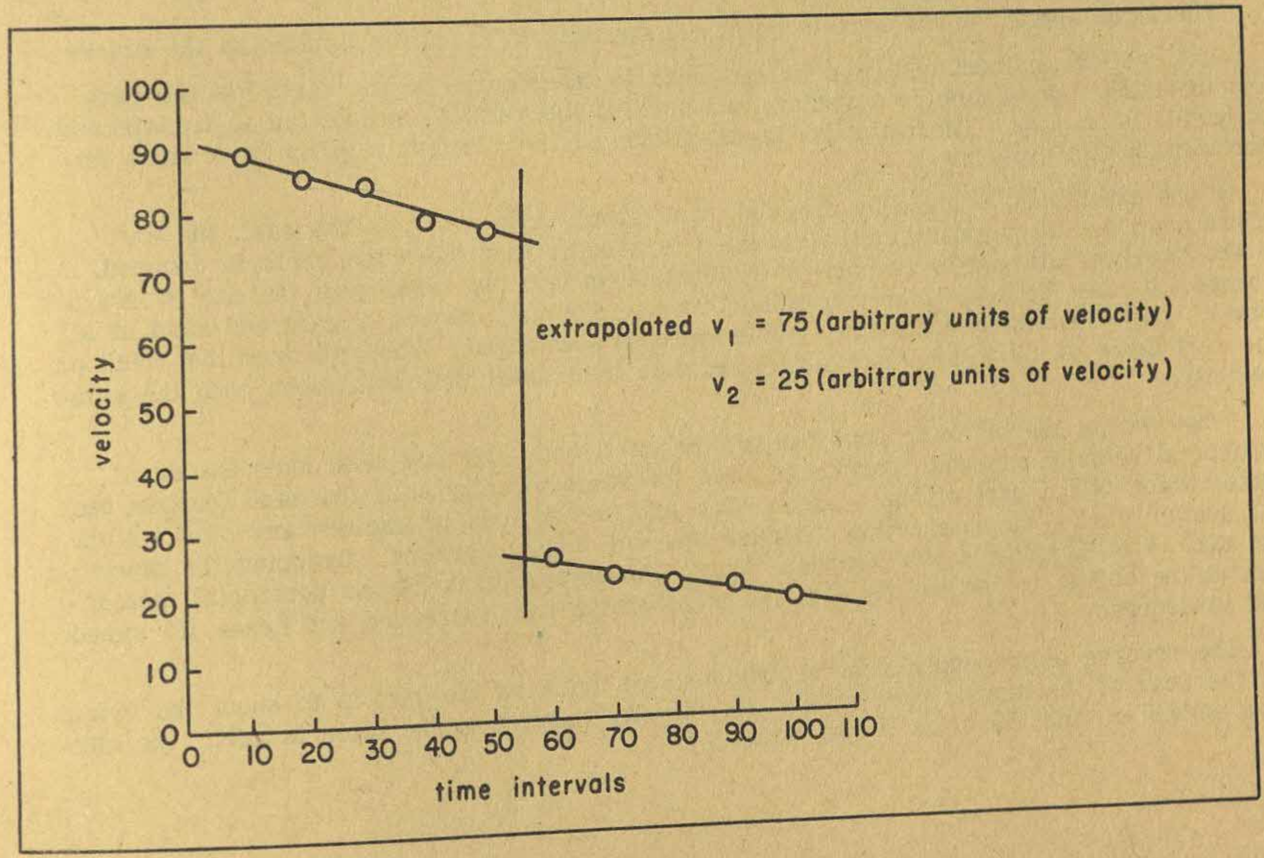

Figure (a)

Answers to Questions

The cart will slow down when the brick is dropped on it. If the mass of the cart is increased, it will appear to slow down less.

The decrease in the momentum of the cart equals the increase in the horizontal momentum of the brick, but the changes are in opposite directions.

The total horizontal momentum of the system of the cart and suspended brick before interaction is just that of the cart. Since, as a result of the collision, the cart loses the same momentum that the brick gains, the total momentum of the system of cart and brick after the interaction is the same as before the interaction. Momentum is conserved within experimental error (about 2 per cent).

The horizontal impulse applied to the falling brick is equal to its change in horizontal momentum. The length of time of the interaction can be estimated by carefully examining the tapes and estimating how many intervals show a rapidly changing velocity. In general, the whole interaction will occur in one or two ticks.

To estimate the horizontal force (in newtons) applied to the brick, "ticks" must be calibrated in seconds. This is best done by pulling a tape through the timer for a known time (4 or 5 sec) and counting the dots. Accuracy in calibrating the timer this way is not important, since the inspection of the tape will only give, to an order of magnitude, the number of intervals during which the interaction took place.

Using the known impulse applied to the brick, the average force of the interaction can be found to an order of magnitude from $F = \frac{I}{\Delta t}$ where $\underline{I}$ is the impulse and Δt is the interaction time. This is a very rough determination, and it is worth pointing out that this is an average force; in actuality, the force probably varies widely over the interaction time in some unknown manner.

The force applied to the cart is equal but opposite to the force applied to the brick.

The vertical momentum of the falling brick is transferred to the earth via the cart and the table. Momentum is a vector, and none of the vertical momentum is transferred to horizontal motion. Theoretically the magnitude of the vertical velocity is of little importance in this experiment.

If the experiment is done by dropping sand slowly into a box on the cart, the cart slows down but the duration of the collision is greater than when the brick is dropped. If the sand was allowed to run out of the cart, however, the velocity of the cart would not change. It must be kept clearly in mind that our system consists of cart and sand at all times. Only the sum of momenta of both remains unchanged. When we drop the brick on the cart there is a friction force acting between them until they both move with the same velocity.

When we let the brick or sand fall off the cart, both cart and sand have the same horizontal velocity and will continue to have the same velocity until the sand touches the table, which is not part of our system. One may not include or exclude any object from the system during the interaction, because this will lead to errors. Dropping the brick on the cart is misleading in this respect. Only because the brick has no horizontal momentum at the beginning can we say that the mass of the cart increased and hence its speed had to decrease.

The reverse of the experiment of dropping the brick on the cart is to shoot the brick off the rear of the moving cart so that its velocity relative to the table is zero; the cart then speeds up and the brick drops vertically.

APPARATUS

1 Dynamics cart
1 Timer
Paper tape
Carbon-paper discs
1 $1\frac{1}{2}$-volt dry cell
3-4 Bricks
1 Ringstand with crossbar and clamps
3 C clamps
Heavy cord
Sheet of masonite or other protective cover for tables

Pan balance
Wedges
Cellulose or masking tape

III-10. A COLLISION IN TWO DIMENSIONS

The purpose of this experiment is twofold. The first objective is to impress upon the student that momentum is a vector quantity and that it is the vector sum of the momenta, not the arithmetical sum, that is conserved. Only by analyzing a two-dimensional collision can this be made really clear. The second objective is to let the student discover that another quantity, mv^2, which is not a vector, appears to be conserved. The discovery of conservation of mv^2 in this experiment is meant only to stimulate the curiosity of the abler student, and one should not go into a deep study of energy at this time. This experiment is important and should not be omitted. It is best done after Section 23-4.

Tape timers can be used to measure velocities and momenta only in collisions occurring along a straight line. Collisions in two dimensions on a table can be analyzed by studying stroboscopic photographs, and it is done this way in the textbook. Lacking this equipment, some other method must be used to time the motions. The constant horizontal velocity of a projectile provides a simple way of measuring velocities in any horizontal direction.

When a ball rolls over the edge of a table, the table gives it a small horizontal impulse; the faster the ball moves, the smaller the impulse. For this reason the target ball is mounted on a screw away from the table edge, so that the incident ball will not be in contact with the table when the collision occurs.

The struck sphere should be to the right of the center of the ruler. This arrangement allows a wider range of positions for the target ball without its bumping into the support after being struck.

Some students may feel this is really a collision in three dimensions, since the spheres move not only in different directions in the horizontal plane but also fall vertically. You may remind them that they are finding the velocities just before and just after the collision, when the objects are moving in the horizontal plane, and that the following vertical motion is used merely as a means of measuring the velocities in the horizontal plane.

Answers to Questions

When the incident sphere is rolled down the incline ten or fifteen times with the target removed, the points where it lands on the paper will be scattered over a small area that can be enclosed by a circle whose diameter is about 3 per cent of the distance traveled by the ball. Therefore, the initial velocities in the different trials vary from the average by about 1.5 per cent.

The momenta in the case of the spheres of equal mass are represented by the displacement vectors on the paper. If the experiment is carefully done, the vector sum of the two final momenta will be equal in both magnitude and direction to the initial momentum of the incident sphere, within the experimental accuracy as determined by the scattering of the points. A sample set of data and vector diagrams is shown in Figs. (a) and (b). Momentum is conserved within experimental error.

Though the vector sum of the momenta is constant, the arithmetic sum of the momenta magnitudes is not.

When the experiment is repeated with unequal masses, the heavier ball should be used as the incident ball. Otherwise, with head-on or near head-on collisions, the lighter incident ball will rebound and bounce off the table edge.

When unequal masses are used, the displacements on the paper representing velocities no longer represent momenta. The vector sum of the final velocities will not equal the initial velocity.

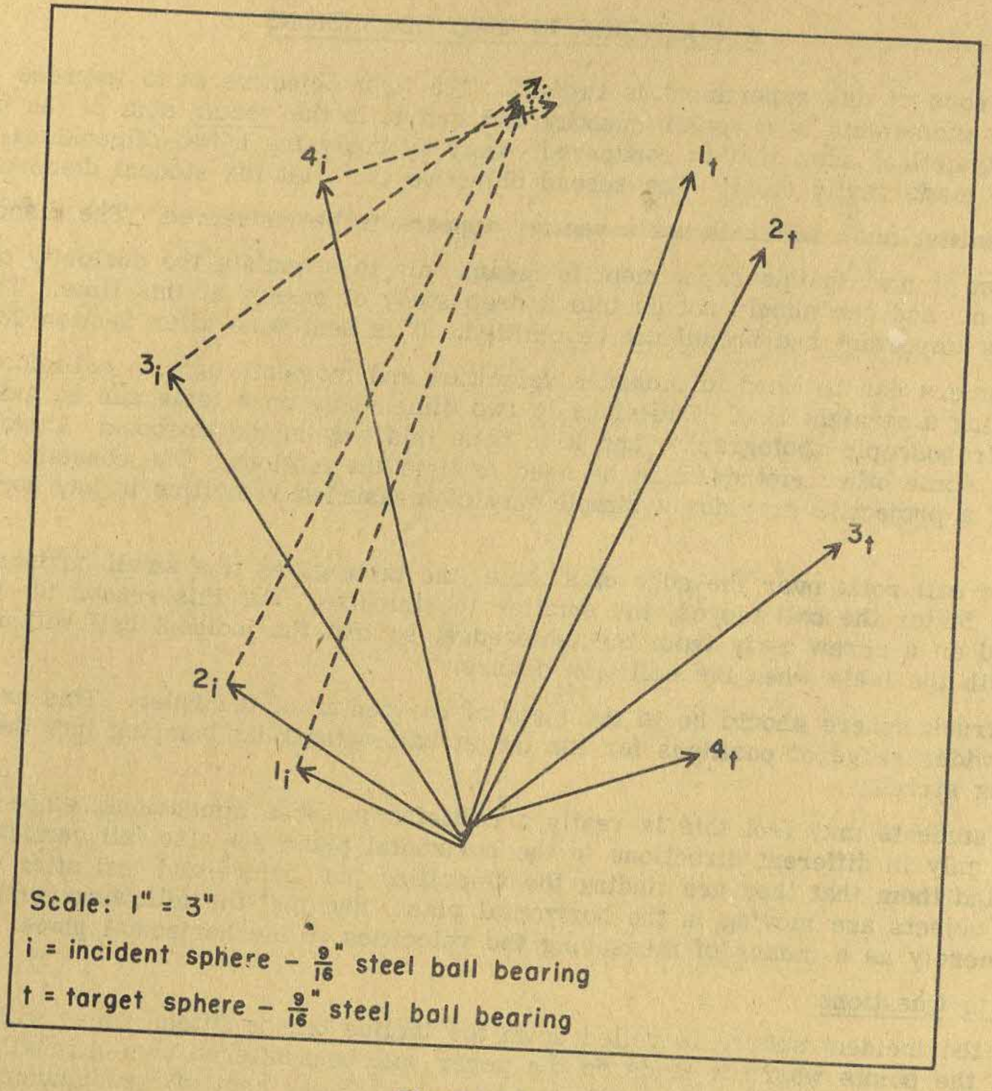
4i
1t
2t
3i
3t
2i
1i
4t
Scale: 1" = 3"
i = incident sphere – 9/16" steel ball bearing
t = target sphere – 9/16" steel ball bearing

Figure (a)

To convert the velocity vectors to momentum vectors when unequal masses are used, the velocity vector of each ball must be multiplied by the ball's mass. For simplicity we may choose the mass of the incident ball as our unit of mass. The momentum vector of the incident ball then becomes identical with its velocity vector, and the momentum vector of the target sphere is shorter than the corresponding velocity vector by the ratio $\frac{m_t}{m_i}$. Momentum is found to be conserved.

The vector components of the final momenta of the two balls in a direction at right angles to the initial momentum are equal in magnitude and opposite in direction.

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When the student compares the square of the initial and final velocities for the case of equal masses, he will see that the square of the initial velocity of the incident ball is very nearly equal to the arithmetic sum of the squares of the two final velocities. This suggests that some quantity involving the square of the velocity is conserved.

In the case of the unequal masses, one finds that the square of the velocities is not conserved, but when the squares of the velocities are multiplied by the appropriate masses, the sum of the final mv^2 equals the initial mv^2.

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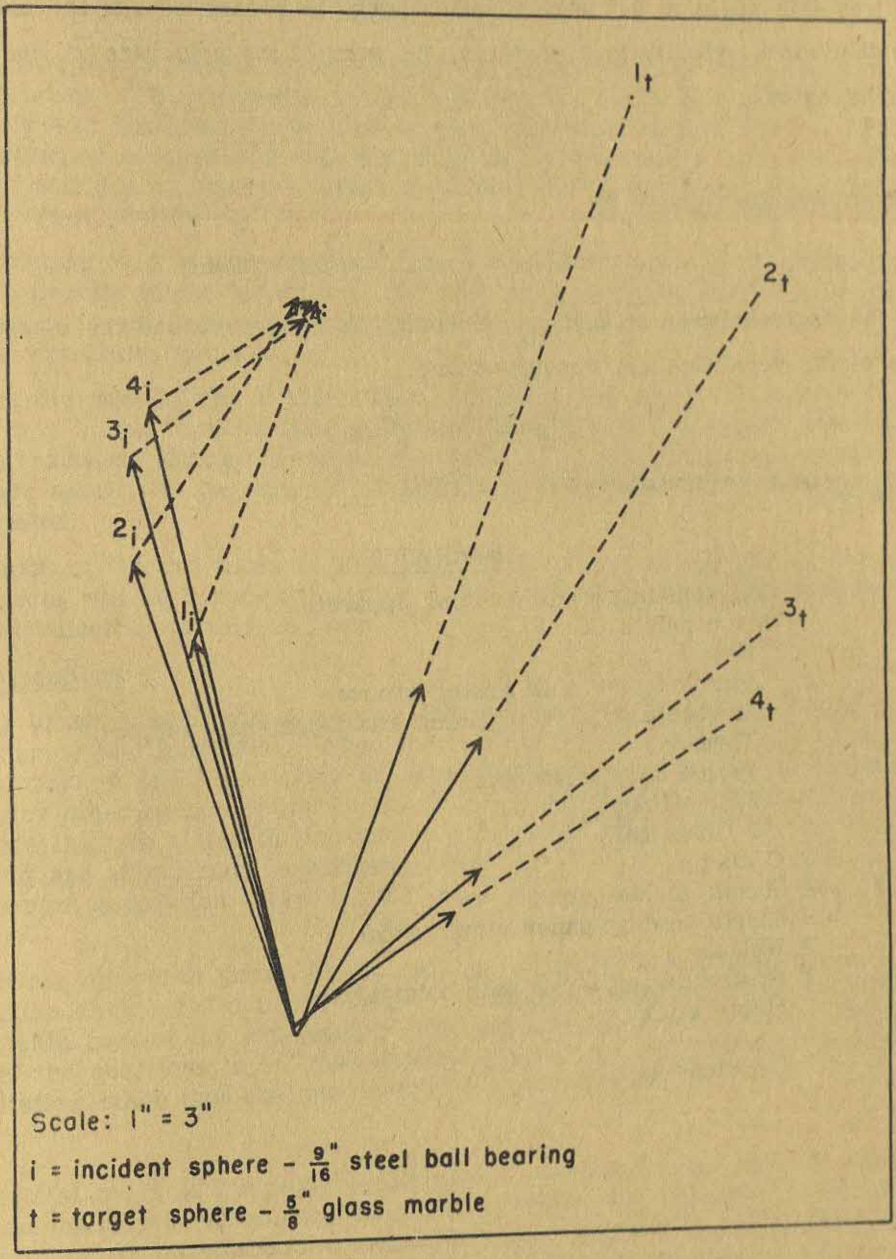
1t
2t
3t
4t
4i
3i
2i
1i
Scale: 1" = 3"
i = incident sphere - 9/16" steel ball bearing
t = target sphere - 5/8" glass marble

Figure (b)

Once the apparatus is set up, a large amount of data can be taken quickly, but the analysis is long and will probably have to be done as homework. Don't require too many collisions to be analyzed. (The calculations of the vector components of the final momenta in a direction at right angles to the initial momenta is interesting, but not an essential part of the experiment.)

The last part of the experiment, the calculations showing the conservation of kinetic energy, may be left as optional. There is very little loss of kinetic energy in these collisions, and a quick way of checking this in the case of equal masses is to examine the angle between the directions of motion of the two spheres after collision. If there is no loss in kinetic energy, this will be a right angle; the greater the loss, the smaller the angle will be.

To see why this angle is 90° when kinetic energy is conserved and the masses are equal, note that since velocity is a vector $\vec{v}$, the sum of the velocities $\vec{v}_a$ and $\vec{v}_b$ of the two equal masses is

$$\vec{v} = \vec{v}_a + \vec{v}_b$$

and the magnitudes are related by

$$v^2 = v_a^2 + v_b^2 - 2v_a v_b \cos \alpha$$

where α is the angle between $\vec{v}_a$ and $\vec{v}_b$. If energy is conserved and the masses are equal, the squares of the velocities are conserved and

$$v^2 = v_a^2 + v_b^2 .$$

Hence, $-2v_a v_b \cos \alpha = 0$ and therefore $\alpha = 90°$.

APPARATUS

1 Collision-in-two-dimensions apparatus;
  this consists of:
    Frame
    Plastic ruler with center groove
    Moveable arm (with clamp and target-ball support)
    Thread
    Plumb bob (screw eye)
    2 Steel balls, $\frac{1}{2}$"
    1 Glass ball, $\frac{1}{2}$"
1 C clamp
4 Sheets carbon paper
8 Sheets tracing paper (onion skin)
2 Weights
1 Protractor or large pair of dividers
  Meter stick

  Cellulose tape

III-11. SLOW COLLISIONS

An interaction which results in the transfer of kinetic energy from one body to another is hard to visualize. The discussion in the text (Section 24-5) is somewhat abstract, but this experiment will demonstrate, in slow motion, what occurs in a similar interaction. It is best performed concurrently with the study of the text and it is a good idea to have the carts with soft spring bumpers attached at hand during the classroom discussions of Sections 24-5, 24-6, and 24-7.

This experiment is a classroom discussion-laboratory exercise in which the text material is taught directly in the laboratory. A copy of the textbook should be at hand. A laboratory report is not necessary, since this is a qualitative experiment designed primarily to clarify the interaction process.

Emphasize the parallelism of these slow collisions with steel-ball collisions. The difference lies only in the time of interaction and the amount of distortion that takes place during the interaction. Energy is stored temporarily in the springs and released again as the carts move apart. In the case of billiard balls, energy is stored in their distortion during a collision.

The collision of the two carts is inelastic. Some of the energy is lost through friction. Frictional effects will be reduced if higher initial velocities are used. Collisions with several initial velocities should be tried.

Answers to Questions

It is easy to see that kinetic energy is lost by the incident cart and gained by the struck cart during the interaction. When the carts are at minimum separation they both move very slowly. It looks as though the total kinetic energy is less at minimum separation than at any other time, but one cannot be sure by mere qualitative observations. It is easy to believe that, as stated in the text, a good deal of energy is stored in the springs when the carts are at minimum separation. The velocities of the two carts are small and equal at minimum separation - that is, the carts are at rest for an instant with respect to each other.

The questions answered above simply call the student's attention to some of the details of an interaction as described in the text. In answering these questions, the student has finished the main part of the experiment. The experimenting that leads to the answering of the rest of the questions is an open-ended extension for students who are curious about types of collisions other than the one described in Section 24-5.

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When the total mass of the carts is increased, the interaction time increases and the distance of minimum separation decreases for a given initial velocity. When the velocity of the incident cart is increased, the interaction time appears to be constant, as indeed it should, if the compression of the springs is proportional to the force. In this part of the experiment, the minimum separation must not become so small that the bodies of the carts come into contact.

If the carts are equally loaded and one is at rest before the collision, the incident cart would end up at rest and the struck cart would move off with all the energy, were it not for energy losses due to friction in the spring bumpers. In reality the incident cart keeps moving with a small velocity but is brought quickly to rest by friction with the table, sometimes before the interaction is over.

APPARATUS

2 Dynamics carts, fitted with "soft" spring bumpers
4 Bricks
4 C clamps
2 Table stops

III-12. CHANGES IN POTENTIAL ENERGY

This experiment, like III-13, "The Energy of a Simple Pendulum," presents a situation in which energy is transformed from one form to another with little loss. It is desirable to do both experiments, but if time is short do at least one of them (preferably this one) after Section 25-3. Here gravitational potential energy is converted into the potential energy of a stretched spring. This is an extremely valuable experiment requiring some care.

A rigid ringstand about a meter high is needed to support the spring and masses. If a shorter ringstand is used, it will be necessary to arrange it so the spring and weights will hang freely over the side of the table.

If the laboratory period is short, the plotting of the extension of the spring as a function of the applied force may be deferred until all the data are obtained.

Answers to Questions

The graph of the spring extension x as a function of the force F will be so close to a straight line passing through the origin (over the range investigated) that one may infer that x is proportional to F within this range. The slope $k = \frac{F}{x}$ of the line can be substituted in the potential energy function of a linear spring, $U_s = \frac{1}{2}kx^2$, to give the potential energy stored in the spring for any extension x.

One can think up situations in which the force vs. spring-extension graph would not be a straight line. In that case, the energy stored in the spring can be found by graphical integration - i.e., finding the area under the force-extension curve from zero to different values of x. A table or graph can then be made of U_s as a function of x for use later in the experiment.

The loss in gravitational potential energy between the two extreme points $-\Delta U_g = -mg(x_2 - x_1)$ equals the gain in potential energy of the stretched spring between the same points $\Delta U_s = \frac{1}{2}k(x_2^2 - x_1^2)$ within 1 or 2 per cent. Notice that from $-\Delta U_g = \Delta U_s$ or $-mg(x_2 - x_1) = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k(x_2 + x_1)(x_2 - x_1)$ follows

$$mg = -k\frac{(x_2 + x_1)}{2}$$

That is, the weight of the mass is equal in magnitude to the force exerted by the spring when it is stretched to the midpoint between x_1 and x_2. This is the point where the mass was originally at rest before it was lifted to start the oscillations. It is the center of the oscillation.

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Setting the arbitrary zero point for the gravitational potential energy at x = 0 (i.e., at the lower end of the unstretched spring), we have for the total potential energy:

$$U = U_g + U_s = +mgx + \frac{1}{2}kx^2$$

Moving the zero point of the x axis to $\frac{x_2 + x_1}{2} = -\frac{mg}{k}$, thereby defining a new coordinate $x' = x + \frac{mg}{k}$, yields $U = -\frac{1}{2}\frac{m^2g^2}{k} + \frac{1}{2}kx'^2$. Finally, setting the zero of the total potential energy at the midpoint of the oscillation reduces U to $U' = \frac{1}{2}kx'^2$ [Fig. (a)].

The sum of the potential energies when the mass is halfway down is less than the potential energy at the start and finish of the fall. The difference equals the kinetic energy of the falling mass.

The change in kinetic energy may be investigated experimentally by taking stroboscope photographs of the falling mass or by having the heavy mass (1 kg) pull a timing tape as it falls.

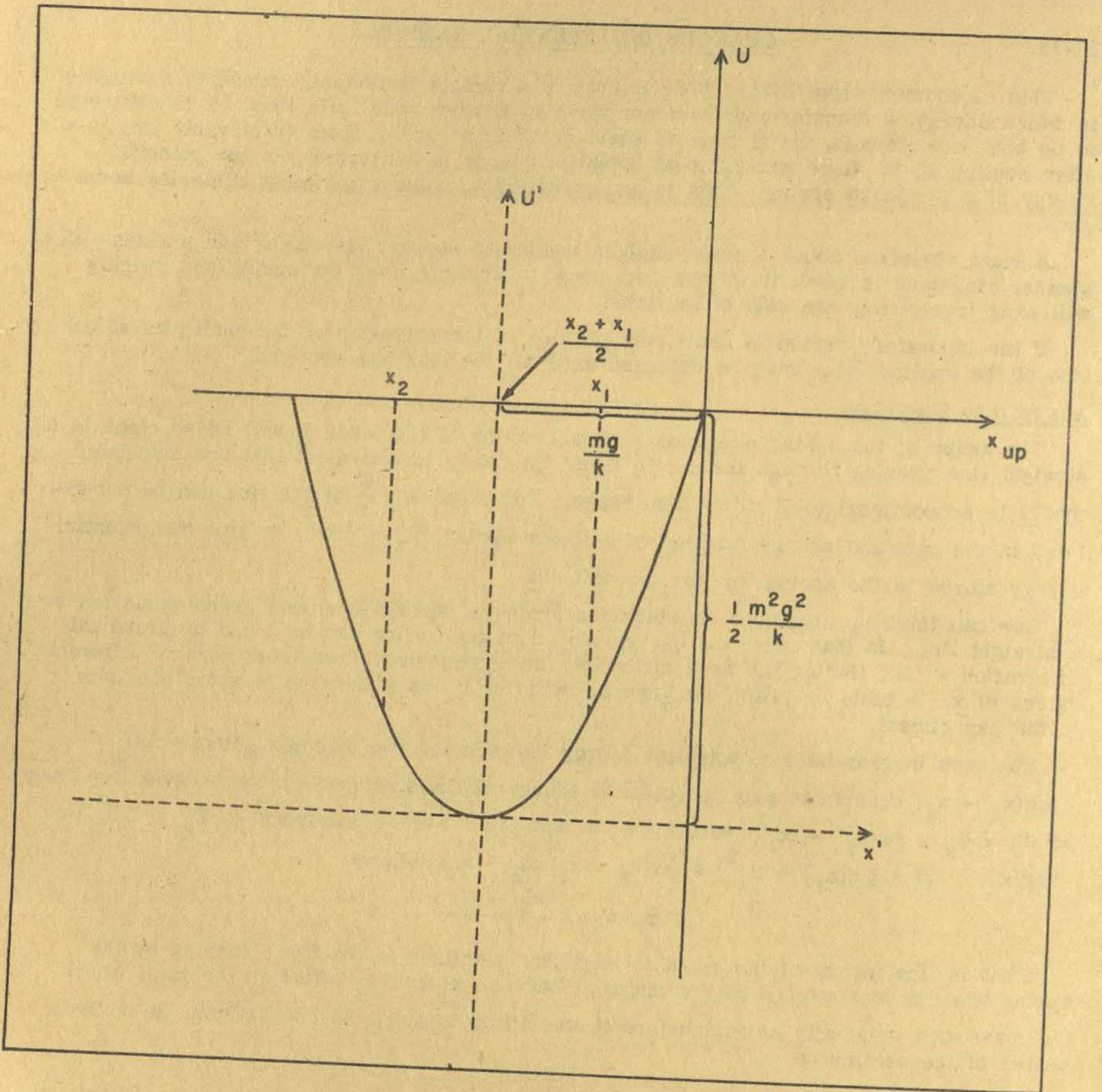

Figure (a)

APPARATUS

1 Steel coil spring (2 cm diameter by 15 cm; k approx. 4 N/m)
2 Weights (0.5 kg, 1.0 kg)
Ringstand with crossbar and clamps
1 C clamp (to support ringstand)
2 Sheets graph paper
3 Clothespins
Meter stick
Pan balance

III-13. THE ENERGY OF A SIMPLE PENDULUM

This somewhat lengthy experiment shows how potential energy is transformed into kinetic energy. Like Experiment III-12, it is best done after the study of Section 25-3 in the text.

A sturdy pendulum support with three guys is needed. If it is possible to set up an overhead suspension, this will be more stable.

Let the student practice releasing the brick until he can give it a relatively smooth swing. The line of force of the release thread should extend through the center of mass of the brick so the brick will not oscillate as it swings. Have the timing tape attached to the center of the bottom of the brick to minimize errors caused by the slow turning of the brick during the swing. It will aid the student in analyzing the tape if he marks the rest position of the brick on the tape before the run. When he does this, he must make sure that he does not move the timer after he has made the mark on the tape. The timer should be held or taped in place while the measurements are being recorded.

The velocity of the bob at different positions can be obtained from the graph of position as a function of time or, if the laboratory period is short, the velocities can be determined directly from the tape and the graph of position as a function of time can be omitted.

The velocity of the bob must be found in meters per second if the kinetic energy is to be calculated in joules. The timer can be calibrated by pulling the tape through for 5 or 10 seconds and counting the dots.

Answers to Questions

A comparison of the graphs for potential and kinetic energy clearly shows that, if both are expressed in the same units, one decreases by the same amount that the other increases [Fig. (a)]. Their sum is constant within experimental error. A systematic error may be introduced by a small inaccuracy in the calibration of the timer. In Fig. (a) this shows as a small rise in the total energy in the middle of the swing.

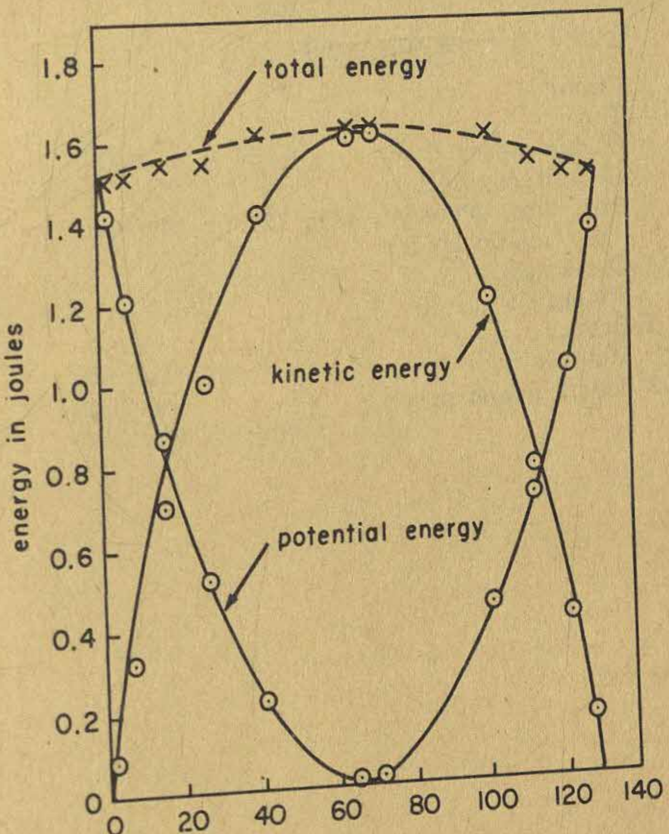

Figure (a)

The kinetic energy is at a minimum at the start and finish of the swing of the brick through the arc; it is greatest when the brick is directly below its point of suspension.

We limit the swing of the pendulum to 15° so we can use the approximate relation $h = x^2/2L$ instead of the exact relation $h(2L - h) = x^2$. For larger swings, the second formula must be used. Also, corrections must be made because the tape follows only the horizontal projection of the motion of the bob, even if the timer is on the floor far from the bob. The height h can also be measured directly instead of using the formula $h(2L - h) = x^2$. The sum of the two energies for large swings will remain constant if the above geometric corrections are made.

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Students who have had a course in trigonometry may also investigate simple harmonic motion by analyzing the graph of position as a function of time. To show that the motion for small angles is simple harmonic, the graph should be drawn with new scales as follows:

Set the origin for distance measurements at the rest position of the bob and measure the distances as positive and negative in opposite directions from this point. Call the maximum horizontal displacement x_{max} from this point ± 1. Then the distances x in this new unit will be $d = \frac{x_{max}}{x}$.

The time of the swing t_{max} is half a period and is therefore equivalent to 180°. The times can then be converted to an angular scale by calculating $\theta = \frac{t}{t_{max}} \times 180°$. If d is now plotted as a function of θ, the graph will coincide very closely with that of $d = \cos \theta$. This result holds only for small angles of swing.

The ambitious student may investigate the more complicated, nonsinusoidal motion by releasing the bob from an angle much larger than 15°, making the necessary geometric corrections in analyzing his data.

APPARATUS

1 Timer
Tape
Carbon-paper disc
1 $1\frac{1}{2}$-volt dry cell
Ringstand, 3' or 4', with 12" or 24" rod and clamp holder
4 C clamps
String, 25' to 30'
1 Brick
Meter stick
2 Sheets graph paper

III-14.

A HEAD-ON COLLISION

This experiment can be done during the study of Chapter 26. Its purpose is to emphasize the difference between the conservation of momentum and conservation of mechanical energy. Here about 25 per cent of the kinetic energy is converted into heat, whereas momentum is conserved to within 3 to 5 per cent. This experiment takes a long time to analyze and can be considered optional. Many of the conclusions can be obtained by restudying the results of Experiments III-9 and III-10. The collision in III-9 is completely inelastic (i.e., the maximum of kinetic energy, consistent with the conservation of momentum, is transformed into heat) whereas that in III-10 is almost completely elastic.

If only a short distance is available to set the first cart moving with constant velocity, it is important to give the cart a quick push. To make sure that both carts have a high enough velocity after the collision, the moving cart should have at least twice the mass of the stationary cart.

Answers to Questions

If the moving cart has a mass less than that of the stationary one, it will bounce back, thus making the tape useless.

The interaction time will be several ticks.

The momentum after the collision almost equals the initial momentum (within 3 to 5 per cent). As in earlier experiments, some inaccuracy is introduced through the rotational motion of the wheels, and some momentum is transferred to the table. The loss of kinetic energy, on the other hand, cannot be attributed to experimental error. A fair fraction is transformed into heat through friction.

The momentum is expressed in kilogram-meters per tick.

When spring bumpers are replaced by soft and inelastic material, more kinetic energy will be transformed into heat, but momentum will still be conserved.

APPARATUS

2 Dynamics carts (one with a hard spring)
1 Timer
Paper tape
Carbon-paper discs
1 $1\frac{1}{2}$-volt dry cell
3-4 Bricks
3 C clamps
1 Sheet graph paper
1 Table stop

Pan balance
Wedges
Cellulose or masking tape